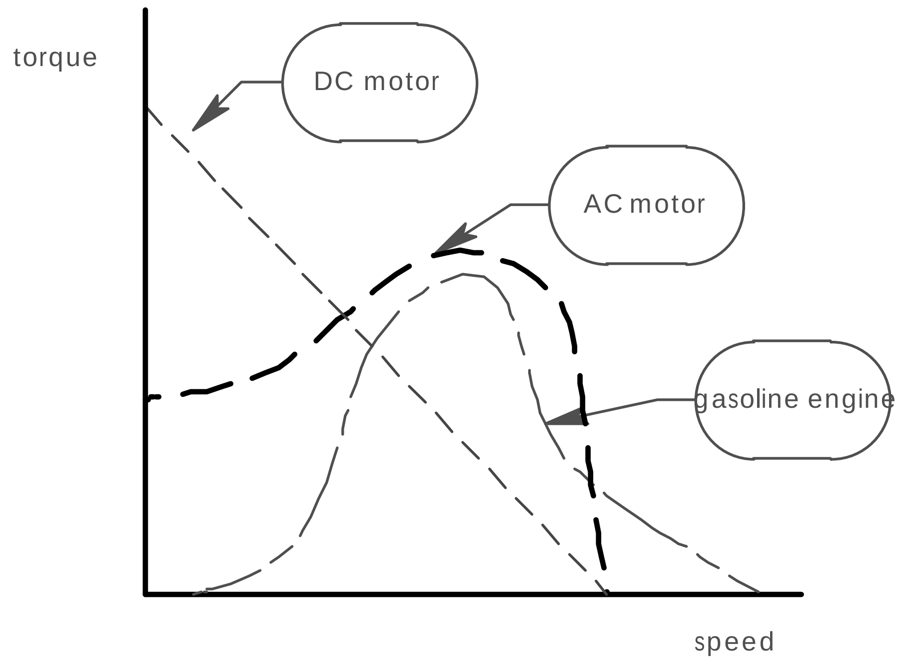
MEEM 3501
Product Realization I
HO7: Motor-Load Matching

Torque-Speed Curves

First, let us consider the torque/speed characteristics of a power source. These are usually shown as graphs (see below).



Note that the high quality (and somewhat idealized) DC "servo" motor we have shown has a wonderful property — the torque-speed curve is a straight line from the maximum zero-speed (or "stall") torque to the maximum zero-load (or "no-load") speed. "Servo" means that that motor has been designed for precise electronic, usually computer, control. This linear torque-speed curve, along with low "ripple" (change in torque as the rotor turns because of the varying angle of the coils) and low inertia for fast response are the major characteristics of servomotors.

There are many kinds of AC motors with different curves; good catalogs show the curves. In particular, some have very low torque at low speed, and cannot be used with loads that are difficult to start. Internal combustion piston engines, like automobile gasoline engines and diesel truck engines, have zero torque up to their minimum operating speeds. Gas turbines, like aircraft engines, are even worse: because they compress their fuel-air mixture dynamically using airfoils, rather than squashing it in a cylinder, they have almost no torque until they reach 90% or so of their maximum operating speed — the speed at which they explode. This is one reason, along with poor efficiency, that turbine engines have never been successful in cars; without a continuously variable transmission, the turbine would have to operate at the wrong speed too much of the time.

Of course, the motor or engine can operate anywhere on the torque-speed curve, provided the voltage or throttle setting is constant. (The throttle is a plate, connected to the gas pedal in cars, that turns to open or close the air passage into the engine.) Usually for electric motors, the voltage is the maximum the motor can safely take for long periods of time; for internal combustion engines, the throttle being wide open corresponds to its peak operation. Reducing the voltage, or partly closing the throttle, shifts the curve down toward the origin.

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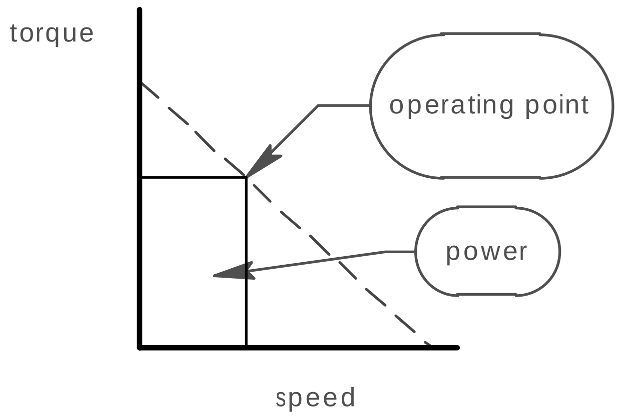
One purpose for a transmission is to allow the power source to operate at the place we want on the torque-speed curve. One common desire is to set the operating point so as to get as much power as possible out of the power source. The power is the speed (in radians per second) times the torque, so it is the area of the rectangle defined by the operating point and the axes.



Exercise 1: *Maximum DC Motor Power* — Use your geometric intuition to determine where (i.e. operating speed ω and torque T), in terms of no-load speed ω_o and zero-speed torque T_o, the power will be greatest for a DC motor?

Exercise 2: *Scenarios for Matching* — Where on the torque-speed curve would you want to run an automobile engine for highway cruising? Why?

Where on the torque-speed curve would you want to run an automobile engine for accelerating to enter a highway? Why?

You can symbolically prove your guess in Exercise 1. From the form of the torque-speed line, we have

$$T(\omega) = T_o + k\omega, \quad k = -\frac{T_o}{\omega_o} < 0.$$

The power is therefore

$$\mathscr{P}(\omega) = T(\omega) \cdot \omega = T_o\omega + k\omega^2.$$

The derivative of power with respect to speed is

$$\frac{\partial}{\partial\omega}\mathcal{P}(\omega) = T_o + 2k\omega.$$

Setting this equal to zero and solving for speed and the corresponding torque yields

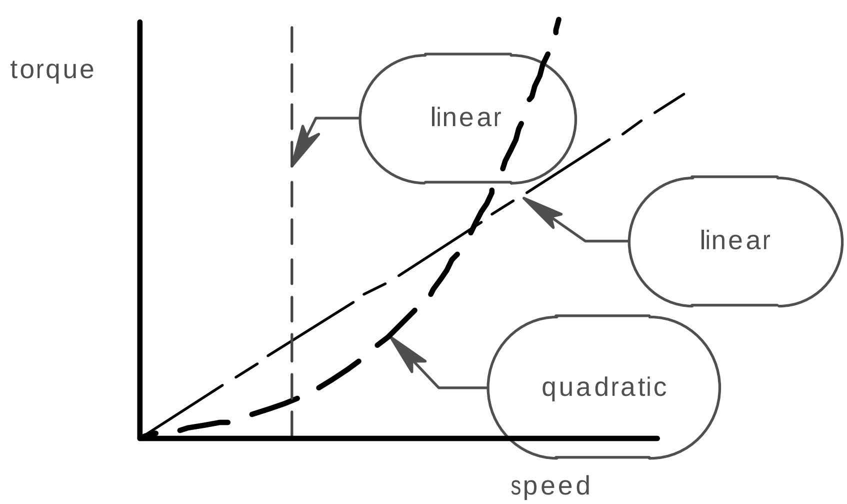
$$\frac{\partial}{\partial\omega}\mathcal{P}(\omega) = 0 \qquad \rightarrow \qquad \omega = -\frac{T_o}{2k} = -\frac{T_o}{2\,T_o/\omega_o} = \frac{\omega_o}{2} \;\;\&\;\; T = \frac{T_o}{2}.$$

Therefore, for a linear torque-speed relation, the maximum power comes when the torque and the speed are at one-half their maximum value. Assuming, then, that we know the speed at which we want to run the DC motor, we still have the problem of the load, which, for maximum power operation to occur, must be half of the zero-speed torque when measured at the motor. Several cases are of interest.

First, we may know how fast we want to run the load, and how much torque the load requires to drive it at that speed. Then, we can just pick a motor that produces sufficient power, and set our transmission ratio to produce the maximum power from the motor.

A more interesting problem arises, however, if we know what motor we are going to use, and want to run the load as fast a possible with this motor. We then need to know the torque-speed characteristic of the load. There are three common types.

Constant loads appear when the torque (or force) required is independent of speed. This occurs when a weight has to be lifted, or a load is moved against dry “Coulomb” friction (ideally; real friction is actually tricky). *Linear* loads appear when the load torque is proportional to the speed, for example, in stirring pancake batter with an egg-beater – they are often a consequence of laminar flows. *Quadratic* loads occur when the load torque is proportional to the square of the speed, and are often a result of turbulent flows; examples would include fans, the air drag on aircraft and automobiles.



Once we know how much power is available, we can then simply look at the load characteristics to determine at what speed that power will be consumed. For example, in the steady-state lifting of a weight, the work equation tells us that the available power after friction losses, divided by the weight, equals the speed:

$$\omega = \frac{\mathcal{P}_{motor} - \mathcal{P}_{loss}}{mg}.$$