

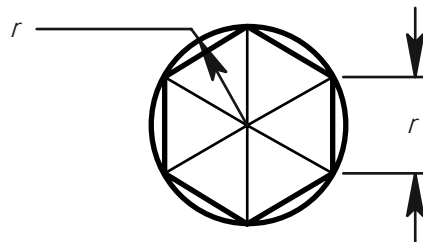
MEEM 3501

Product Realization I

HO5: Symmetry

Something is *symmetrical under an operation* if performing the operation leaves it effectively unchanged. For example, a square is symmetrical under the operations of reflection about the vertical and horizontal axes, and under rotation about any multiple of 90 degrees.

Symmetry may be the most important concept in modern physics. It is also very useful in mechanical engineering, though it is seldom formally introduced to undergraduate engineers. The most obvious example of the value of symmetry is the wheel. Wheels are symmetrical under any rotation; this allows them to roll, and makes possible the endless variety of wheel-based machinery. Bolts have hexagon heads, and angular measurements are often done in multiples of six, because a hexagon is made of highly symmetrical equilateral triangles. This means that the sides of a hexagon are equal to the radius of the circumscribing circle, a most useful fact.



A more subtle example is that conservation laws are also symmetries — the energy in a closed system is not changed by any physical process.

Symmetry in Analysis

By noticing that things are symmetrical, we can often eliminate unknown values without difficult computation. See Exercises 1 and 2.

Exercise 1: A Car Crash — You are driving a car at 50 mph toward the entrance of a one-way tunnel when suddenly you see another, identical car coming out of the tunnel at the same speed. There is no time for either of you to brake; you can only choose to hit the concrete tunnel entrance, or to hit the car. Which collision are you more likely to survive?

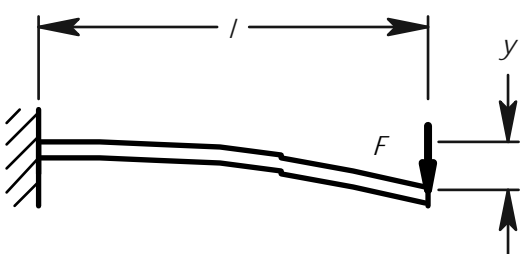
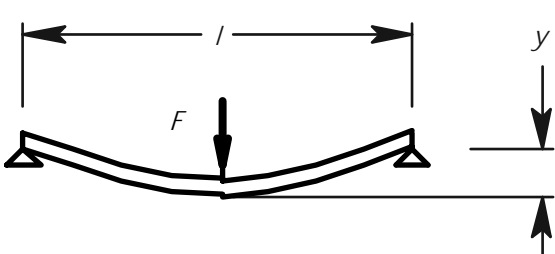
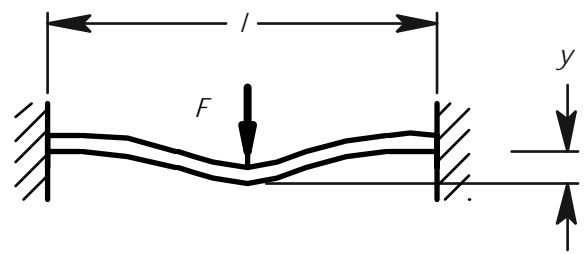
Symmetry in Manufacturing

The symmetry of three special shapes makes them important in manufacturing, partly because they allow various kinds of motions, but also because it enables them to be made by operations that are “self-improving” — that is, these shapes can be made more accurate than is the machine that makes them.

- Spheres are symmetrical under rolling in any direction. Balls for bearings can therefore be made by rolling and sliding thousands of them in a spiral groove between two large grindstones. The stones remove the high points on the bearings, making them round and similar in size; the balls wear the groove into a more precisely circular cross section.
- Flat surfaces are symmetrical under any translation in the plane, except that they are finite. One can therefore create three flat surfaces by rubbing them against each other with an abrasive power in between; this is called “lapping”. Alternatively, the high

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Exercise 2: Beam Bending — I remember the formula for deflection of a cantilevered beam. From this, symmetry gives me the deflections for many other loading conditions. Fill in the blanks in the right column with the appropriate formulae.

Beam Configuration	Deflection Formula
	$y = \frac{Fl^3}{3EI}$
	$y =$
	$y =$

points on the surfaces can be marked by putting die on one, then rubbing it against the other, and the high points scraped or polished off by hand. If you only use two surfaces, they will take on a matching curvature, but three surfaces can only match each other if they are all flat.

- Cylinders are symmetrical under rotation, and, except for having finite length, under translation along their axis. Round holes can therefore be made more round and straight by lapping them with a rotating and translating rod; the rod also grows more round and straight.

Once one has a cylinder or a flat surface, one can use them to make more. Two flat surfaces define a straight “way”. The oldest machine tools, lathes, combine cylindrical bearings for the spindle with straight ways, made by hand with the processes described. Indeed, the precision surfaces in the most sophisticated computer controlled machines are generated by one of these self-correcting methods.

HO5: Symmetry

Symmetry under Scaling — Fractals and Design Processes

Recently, there has been much enthusiasm for “fractal” objects — objects that look the same at any magnification, or are “symmetrical under magnification.” The leaves of a fern are more or less fractal — each leaf looks much like the whole plant, as do the structures within the leaves. Your lungs and blood system are fractal, because fractal geometry is an easy way to generate a large surface area, needed for oxygen exchange. Fractals are often the result of “chaotic” processes — processes that cannot be predicted because small changes in inputs produce large changes in outputs. Turbulent flow is usually chaotic, which is why clouds have fractal geometry.

Design processes are fractal. One uses the same basic process to design a bearing as to design the car that contains the bearing.