

MEEM 3501

Product Realization I

H04: Springs and Dampers

You should be familiar with springs and dampers in terms of the elements that make up a dynamic system. The springs and dampers considered in dynamic system models are representations of real devices. Some real mechanical components are specifically designed for use in these respective capacities while others show such characteristics as a consequence of nature. In design, we will differentiate between components that are specifically designed for such purposes and those that exhibit these natural characteristics. This distinction will be made as we design for specific functions and is not to suggest that we forget that everything has such spring and damping characteristics, though their primary purpose is not that of a component specifically designed to provide springiness or damping. The latter is important since there may often be the opportunity to make use of the natural springiness or damping capacity of a component initially designed for some other function, hence removing the necessity for a dedicated spring or damper component.

Springs

Every solid is a spring in the sense that it deflects under load and returns when the load is released. The mechanical components we call springs give unusually large and predictable deflections under load, will accept an unusually large amount of energy (load · deflection) for their weight, and efficiently return most of the energy when released. Many springs have a linear relationship between force and deflection: $F = kx$ and therefore, of course, a quadratic relationship between energy absorbed and deflection:

$$U = \int \vec{F} \cdot d\vec{x} = \frac{1}{2} kx^2. \quad (1)$$

When the spring is not linear, one can often treat it as if it were linear for small motions around any particular deflection, using

$$k = \frac{\Delta F}{\Delta x}. \quad (2)$$

Springs have many uses in design.

Exercise 1: Purpose of Springs — Match the purpose to the application.

Purpose	Application
1. ____ To absorb and return energy.	a. Automobile suspension spring
2. ____ To allow relative motion between system elements while keeping the force relatively constant.	b. The spring inside a bathroom scale
3. ____ To establish a precise relationship between force and motion.	c. The launch spring in a pinball machine

Since every solid is a spring, there are many different ways to make springs.

- *Leaf* springs are flat beams that deflect by bending. You can determine their spring constants and stresses using standard beam equations. *Constant force springs* are mostly long thin beams that have been bent into tight coils.
- *Torsion bars* are round, straight rods with attached lever arms. When the lever arm

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moves, the rod twists. Until the development of gunpowder, some of the most powerful weapons of war worked by twisting and releasing thick cables of horsehair. You can determine their spring constants and stresses using standard equations for torsional stress and deflection.

- *Coil or helical* springs are torsion bars that have been bent into a helix. When you push or pull on the end, the rod twists. The curved shape causes a stress concentration on the inside of the spring that should be corrected for best accuracy, and you may also want to correct for the fact that the end coils partially lie flat (in compression) and do no work (see textbook). Usually, you buy helical springs from a catalog, and the manufacturer gives a spring constant and a maximum load. *Compression* helical springs have flat ends so you can push on them, often until they compress to their *solid length*; *extension* helical springs have hooks on the ends so you can pull on them.
- *Bellville, wavy, and curved* washers make very flat springs; they are generally non-linear.
- *Gas* springs are just sealed pneumatic cylinders: their force/deflection curves are determined by the gas compression equations familiar from thermodynamics.

Regardless of shape, springs are usually designed to eliminate stress concentrations and out of very strong materials. More precisely, the energy per unit volume they can store (i.e., absorb and return), for a given geometry, is proportional to the ratio of strength squared to elastic modulus:

$$U \propto Fx \propto F \frac{F}{E} \propto \frac{S^2}{E}. \quad (3)$$

To assure there is no permanent deformation, and since the deflection is proportional to elastic modulus only for elastic deformation, the strength in Eq. (3) should be the yield strength S_y , i.e., $S = S_y$. To allow large deflections, springs must be thin in at least one dimension.

Exercise 2: Energy Storage Capacity — Rank the following materials by their effectiveness at storing (absorbing and returning) energy, for a given geometry, on a per unit mass basis with 1 being the highest and 5 being the lowest. Write an example of a spring you have seen made of each material.

_____ aluminum example: _____

_____ steel example: _____

_____ fiberglass example: _____

_____ rubber example: _____

_____ wood example: _____

Why is steel the most common spring material? _____

When springs are in parallel, obviously, changing the deflection a distance Δx requires a change in force equal to the sum of the force changes required for the two separately, so the total spring constant equals the sum of the spring constants. For example, for two springs in parallel,

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$$\frac{\Delta F_{total}}{\Delta x} = k_{total} = \frac{\Delta F_1 + \Delta F_2}{\Delta x} = \frac{\Delta F_1}{\Delta x} + \frac{\Delta F_2}{\Delta x} = k_1 + k_2. \quad (4)$$

When springs are in series, obviously, a change in the applied force produces a change in the total length equal to the sum of the length changes. If we call the *compliance* the inverse of the spring constants, the total compliance equals the sum of the individual compliances. For two springs,

$$\frac{\Delta x_{total}}{\Delta F} = \frac{1}{k_{total}} = \frac{\Delta x_1 + \Delta x_2}{\Delta F} = \frac{\Delta x_1}{\Delta F} + \frac{\Delta x_2}{\Delta F} = \frac{1}{k_1} + \frac{1}{k_2}. \quad (5)$$

Exercise 3: Spring Combinations — Sally wants to make the world's longest range slingshot in order to crack Calvin's window from a safe distance. She buys six powerful hunting slingshots and cuts off the surgical rubber-tube springs. She groups all the springs side by side into two large bundles, ties the ends of the bundles to a large Y-shaped piece of wood, and attaches a pocket at the center of the bundles.

1. If she uses her normal (award-winning) slingshot shooting form, will she be able to shoot six times further with this slingshot than with a normal one?

Yes

No

2. Why?

3. How could she most easily use the materials from the six slingshots to get great range?

People often forget that the spring constant, for most springs, is the same regardless of the direction of Δx , provided the spring is within its operating range. A compression spring cannot take a tensile load, but exhibits the same magnitude of force change for a given increase in length as for the same decrease in length, provided it is still in compression.

Exercise 4: Spring Rates / Constants — Calvin has contrived to hang his friend Hobbes by the neck using the rubber from one of *his* slingshots. He notices that Hobbes falls exactly four inches as result of the stretch of the rubber, after the rubber becomes taut, and that Hobbes is exactly two inches off the floor. Hobbes weighs ten pounds. If he pulls Hobbes down until Hobbes touches the floor (to increase his suffering and win the argument), how hard will he have to pull?

Dampers

Dampers, dashpots, or shock-absorbers are designed to allow motion while exerting an opposite force; they therefore absorb energy. Usually, this is their purpose, either to damp out vibration, or to bring a moving object to a smooth stop. Sometimes they are used to control velocity. For example, automatic drills for transfer lines may be driven by a pneumatic cylinder, which provides a constant force, and kept from slamming into the workpiece by a hydraulic

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damper.

Most dampers force a liquid through small holes, producing a force proportional to the square of the velocity. They can be designed to force a liquid through a long thin tube, producing a force proportional to the velocity, or they can use spring-loaded valves to produce other velocity/force relationships. They can also be made of solids that slide against each other, in which case the force is usually highest when motion is just starting, then relatively constant regardless of velocity — this is often called “sticktion”, or stick-slip. The force produced by sliding solids is difficult to predict, especially if any lubricant gets on them, so most commercial dampers use liquids. Dampers can also be made using gases, in which case they have a built-in springiness.

Exercise 5: *Spring and Damper Systems* — Automobiles are connected to their wheels through springs and hydraulic dampers (“shock absorbers”) which act in parallel.

1. Which use of springs is this?
2. Why are dampers used in this system?
3. Assuming that the dampers exhibit a linear relationship between speed and force, sketch the relationship between force on the car and time when the car goes over a speed bump.
4. How could one design a damper to smooth out the force on the car under these circumstances?