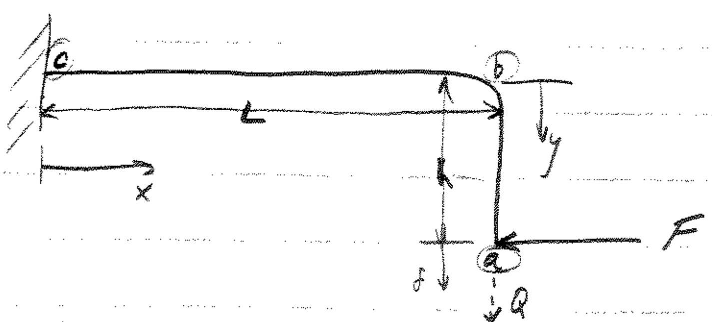
Load Type	Constant Section	General Energy	General Deflection
Axial	$U = \frac{F^2 l}{2EA}$	$U = \int_0^l \frac{F^2}{2EA}\,dx$	$\delta = \int_0^l \frac{\cancel{2}F(\partial F/\partial Q)}{\cancel{2}EA}\,dx$
Torsion	$U = \frac{T^2 l}{2GJ}$	$U = \int_0^l \frac{T^2}{2GJ}\,dx$	$\delta = \int_0^l \frac{\cancel{2}T(\partial T/\partial Q)}{\cancel{2}GJ}\,dx$
~~Pure~~ Bending	$U = \frac{M^2 l}{2EI}$ (Pure Moment)	$U = \int_0^l \frac{M^2}{2EI}\,dx$	$\delta = \int_0^l \frac{\cancel{2}M(\partial M/\partial Q)}{\cancel{2}EI}\,dx$
Bending Shear (Rect. Sect.)	$U = \frac{C}{2}$ [redacted] $\frac{V^2 l}{AG}$	$U = \int_0^l \frac{C}{2}$ [redacted] $\frac{V^2}{AG}\,dx$	$\delta = \int_0^l \frac{\cancel{2}c\,6V(\partial V/\partial Q)}{\cancel{2}5AG}\,d$

Example Find vert. displ at point a.



Assume elastic deflections

Trans. shear is negligible

4 components of energy

- Bending in ab : $M_{ab} = F(h-y)$
- Bending in bc : $M_{bc} = Q(L-x) + Fh$
- Tension in ab : Q
- Compression in bc : F

$$\delta = \frac{\partial U}{\partial Q} = \int_0^h \frac{M_{ab}\,(\partial M_{ab}/\partial Q)}{EI}\,dy + \int_0^L \frac{M_{bc}\,(\partial M_{bc}/\partial Q)}{EI}\,dx$$

$$+\int_0^h \frac{Q\,(\partial Q/\partial Q)}{EA}\,dy + \int_0^L \frac{F\,(\partial F/\partial Q)}{EA}\,dx$$

$$= \int_0^h \frac{F(h-y)(0)}{EI}\,dy + \int_0^L \frac{[Q(L-x)+Fh](L-x)}{EI}\,dx$$

$$+\int_0^h \frac{Q(1)}{EA}\,dy + \int_0^L \frac{F(0)}{EA}\,dx$$

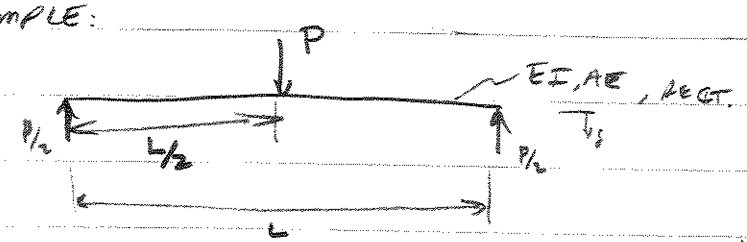
$$= \int_0^L \frac{Q(L-x)^2 + Fh(L-x)}{EI}\,dx + \frac{Qh}{EA}$$

NOW THAT PDs HAVE BEEN TAKEN, CAN SET $Q=0$

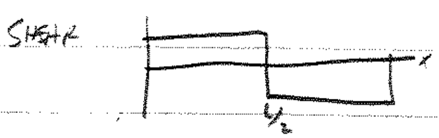
$$= \int_0^L \frac{Fh\,(L-x)}{EI}\,dx$$

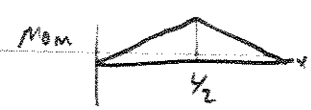
$$= \frac{FhL^2}{EI} - \frac{FhL^2}{2EI} = \frac{FhL^2}{2\cancel{EI}} \Rightarrow \boxed{\delta = \frac{FhL^2}{2\cancel{EI}}}$$

EXAMPLE:



FIND δ AT CENTER





$V_x = \pm \frac{P}{2}$

$M_1 = \frac{P}{2}x, \quad 0 \le x \le L/2$

$M_2 = \frac{PL}{4} - \frac{P}{2}\left(x - \frac{L}{2}\right) = \frac{P}{2}(L-x), \quad \frac{L}{2} \le x \le L$

$$U = \int_0^{L/2} \frac{M_1^2}{2EI}\,dx + \int_{L/2}^{L} \frac{M_2^2}{2EI}\,dx + \int_0^{L/2} \frac{V_1^2}{2AE}\,dx + \int_{L/2}^{L} \frac{V_2^2}{2AE}\,dx$$