# Machining Processes Mode--Based Planning and Diagnostics 

## Problem Manual

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## Chapter 1

1.1 Consider the part shown in the figure (dimensions in mm ). Define the machining processes portion of its manufacture by answering the following questions.
a) Specify the raw stock from which the part would be machined.
b) Specify the sequence of operations, including the basic traditional machining process(es) employed in each, that would best create the features of the part. An operation is typically tied to the creation of a feature whereas a process is a particular way of creating the feature or a step in its creation. For example, creation of a precise hole would be an operation that would first employ the drilling process to create the hole, then a boring or reaming process to impart precision to the hole. Hint: There are at least six processes required, some of which may be required for more than one operation. You may be able to justify a couple more processes, but that is not necessary.


## Chapter 2

2.1 Consider orthogonal cutting using a tool that has a rake angle of $\gamma_{o}=5^{\circ}$. The cutting speed is $V=100 \mathrm{~m} / \mathrm{min}$. The friction angle is measured to be $\beta=40^{\circ}$ and the Ernst and Merchant shear angle model is assumed to apply for this material.
a) What is the shear angle $\phi_{o}$ (in degrees)?
b) What is the chip velocity $V_{c}$ (in $\mathrm{m} / \mathrm{min}$ )?
2.2 Consider orthogonal cutting using a tool that has a rake angle of $\gamma_{o}=10^{\circ}$. The uncut chip thickness is $h=0.20$ mm and the cutting speed is $V=100 \mathrm{~m} / \mathrm{min}$. The cut chip thickness is measured to be $h_{c}=0.30 \mathrm{~mm}$.
a) What is the shear angle $\phi_{o}$ (in degrees)?
b) What is the chip velocity $V_{c}$ (in $\mathrm{m} / \mathrm{min}$ )?
2.3 Consider the orthogonal cutting of AISI 304 stainless steel using an uncoated carbide tool that has a rake angle of $\gamma_{o}=10^{\circ}$. The uncut chip thickness is $h=0.20 \mathrm{~mm}$ and the cutting speed is $V=50 \mathrm{~m} / \mathrm{min}$. The chip thickness ratio is measured to be 0.5 .
a) What is the cut chip thickness $h_{c}$ (in mm)?
b) What is the chip velocity $V_{c}$ (in $\mathrm{m} / \mathrm{min}$ )?
c) What is the shear angle $\phi_{o}$ (in degrees)?

## For the remainder of the problem, assume the answer to part (c) is $\phi_{0}=30^{\circ}$.

d) What is the shear velocity $V_{s}($ in $\mathrm{m} / \mathrm{min})$ ?
e) What is the friction angle $\beta$ (in degrees), assuming the Lee and Shaffer model is valid?
2.4 Consider orthogonal cutting using a tool that has a rake angle of $\gamma_{o}=10^{\circ}$. The force magnitudes $N_{\phi}$ and $N_{\gamma}$ are known while it is also known that $N_{\phi}=1.1 N_{\gamma}$. The shear angle $\phi_{o}$ is known to be $28^{\circ}$.
a) Construct the force circle diagram showing and labeling all force components and their directions.
b) What is the friction angle $\beta$ (in degrees) based on a graphical estimation?
2.5 Consider orthogonal cutting using a tool that has a rake angle of $\gamma_{o}=15^{\circ}$. The force magnitudes $P_{\phi}$ and $N_{\gamma}$ are known while it is also known that $N_{\gamma}=1.6 P_{\phi}$. The shear angle $\phi_{o}$ is known to be $31^{\circ}$.
a) Construct the force circle diagram showing and labeling all force components and their directions.
b) What is the friction angle $\beta$ (in degrees) based on a graphical estimation?
2.6 Consider the orthogonal cutting of AISI 1045 steel using an uncoated carbide tool that has a rake angle of $\gamma_{o}=$ $20^{\circ}$. The cutting and thrust force components are measured to be $F_{C}=1000 \mathrm{~N}$ and $F_{T}=375 \mathrm{~N}$. This material is assumed to behave consistently with the Lee and Shaffer shear angle model.
a) What is the friction angle $\beta$ (in degrees)?
b) What is the shear angle $\phi_{o}$ (in degrees)?
2.7 Consider the orthogonal cutting of AISI 1018 mild carbon steel using a TiN-coated carbide tool that has a rake angle of $\gamma_{o}=20^{\circ}$. The cutting and thrust force components are measured to be $F_{C}=1000 \mathrm{~N}$ and $F_{T}=100 \mathrm{~N}$.
a) By constructing (to scale) Merchant's Force Circle Diagram, graphically estimate the normal and inplane rake-face force components, $N_{\gamma}$ and $P_{\gamma}$, as well as the resultant machining force $R$ (all in N ).
b) Assuming the Ernst and Merchant model is valid, what is the shear angle $\phi_{o}$ (in degrees)?

For the remainder of the problem, assume the answer to part (b) is $\phi_{0}=40^{\circ}$.
c) Again, using the Force Circle Diagram, graphically estimate the normal and in-plane shear-plane force components, $N_{\phi}$ and $P_{\phi}$ (in N).
d) Using the relations of both the rake-face and shear-plane force components to the cutting and thrust force components, compute $N_{\gamma,} P_{\gamma}, N_{\phi}$ and $P_{\phi}($ in N$)$ to check your graphical estimates of parts (a) and (c).
2.8 Consider the orthogonal cutting of AISI 304 stainless steel using a TiN-coated carbide tool that has a rake angle of $\gamma_{o}=10^{\circ}$. The shear angle is estimated to be $\phi_{o}=15^{\circ}$ and the rake-face force components are estimated to be $N_{\gamma}=500 \mathrm{~N}$ and $P_{\gamma}=225 \mathrm{~N}$. Use equations to compute the results.
a) What are the cutting and thrust force components $F_{C}$ and $F_{T}$ (in N)?
b) What are the shear-plane force components $N_{\phi}$ and $P_{\phi}($ in N$)$ ?
2.9 Consider orthogonal cutting where the cutting and thrust force components are measured to be $F_{C}=750 \mathrm{~N}$ and $F_{T}=600 \mathrm{~N}$. If the in-plane shear force is estimated to be $P_{\phi}=700 \mathrm{~N}$, what is the shear angle $\phi_{o}$ (in degrees)?
2.10 Consider orthogonal cutting using a tool that has a rake angle of $\gamma_{o}=15^{\circ}$. The cutting and thrust force components are measured to be $F_{C}=2000 \mathrm{~N}$ and $F_{T}=750 \mathrm{~N}$. The workpiece material is assumed to behave according to the Lee and Shaffer shear angle model.
a) What is the friction angle $\beta$ (in degrees)?

For the remainder of the problem, assume the answer to part (a) is $\beta=36^{\circ}$.
b) What is the shear strain $\gamma$ ?
2.11 Consider orthogonal cutting using a tool that has a rake angle of $\gamma_{o}=10^{\circ}$. The specific cutting energy and shear-yield strength, under machining conditions, are estimated to be $u_{C}=1000 \mathrm{~N} / \mathrm{mm}^{2}$ and $S_{s y}=150 \mathrm{MPa}$, respectively. The shear angle is measured to be $\phi_{o}=25^{\circ}$.
a) What is the shear strain $\gamma$ ?
b) What is the specific friction energy $u_{f}\left(\right.$ in $\left.\mathrm{N} / \mathrm{mm}^{2}\right)$ ?
2.12 Consider orthogonal cutting using a tool that has a rake angle of $\gamma_{o}=10^{\circ}$. The uncut chip thickness is $h=0.25$ mm , the width of cut is $w=2.5 \mathrm{~mm}$ and the cutting speed is $V=200 \mathrm{~m} / \mathrm{min}$. The shear angle and rake face coefficient of friction have been determined to be $\phi_{o}=30^{\circ}$ and $\mu=0.50$, respectively. The shear-zone tensile yield strength under these conditions is estimated to be $S_{y}=750 \mathrm{MPa}$.
a) If the specific shear energy $u_{s}$ is the shear strain energy per unit volume of material sheared on the shear plane, and the material is assumed to exhibit elastic-perfectly plastic behavior, what is $u_{s}\left(\right.$ in $\left.\mathrm{N} / \mathrm{mm}^{2}\right)$ ?
b) What is the in-plane shear force $P_{\phi}($ in N$)$ ?

For the remainder of the problem, assume the answer to part (b) is $P_{\phi}=470 \mathrm{~N}$.
c) Using equations, including the basic ones provided or any others you can derive from the FCD geometry, calculate the in-plane rake face force component $P_{\gamma}($ in N$)$. Hint: Deriving FCD relations is computationally the easiest, but if you wish to use the basic equations provided, get $N_{\gamma}$ and $P_{\gamma}$ in terms of $P_{\phi}$ (given), then solve for $N_{\phi}$ knowing $\mu$, and back-substitute $N_{\phi}$ to find $P_{\gamma}$
2.13 Consider the orthogonal cutting of AISI 4140 steel using a high-speed steel tool that has a rake angle of $\gamma_{o}=$ $20^{\circ}$. The uncut chip thickness is $h=0.30 \mathrm{~mm}$ and the width of cut is $w=1.0 \mathrm{~mm}$. The cutting and thrust force components are measured to be $F_{C}=900 \mathrm{~N}$ and $F_{T}=300 \mathrm{~N}$. Assume the Ernst and Merchant shear angle model is valid for this material.
a) What is the average effective coefficient of friction on the rake face, $\mu$ ?
b) What is the shear strain, $\gamma$, seen by the work material as it traverses the shear plane?

## For the remainder of the problem, assume the answer to part (b) is $\gamma=1.70$ and that the corresponding shear angle is $\phi_{0}=35^{\circ}$.

c) What is the specific cutting energy $u_{C}\left(\right.$ in $\left.\mathrm{N} / \mathrm{mm}^{2}\right)$ ?
d) What are the specific shear and friction energies $u_{s}$ and $u_{f}\left(\mathrm{in} \mathrm{N} / \mathrm{mm}^{2}\right)$ ?
e) Estimate the shear yield strength of the shear-zone material (in MPa).
2.14 Consider orthogonal cutting using a tool that has a rake angle of $\gamma_{o}=0^{\circ}$. The uncut chip thickness is $h=0.25$ mm , the width of cut is $w=2.0 \mathrm{~mm}$ and the cutting speed is $V=75 \mathrm{~m} / \mathrm{min}$. The cutting power is measured to be $\mathscr{P}_{C}=1000 \mathrm{~W}$, the average coefficient of friction/shear between the chip and rake face is estimated to be $\mu=$ 1.5 , and the Ernst and Merchant shear angle model is assumed to hold for this material.
a) What is the material removal rate $\dot{v}_{r}\left(\mathrm{in}_{\left.\mathrm{mm}^{3} / \mathrm{min}\right) \text { ? }}\right.$

For the remainder of the problem, assume the answer to part (a) is $\dot{v}_{r}=37,500 \mathrm{~mm}^{3} / \mathrm{min}$.
b) What is the specific shear energy of the shear-zone material $u_{s}$ (in $\mathrm{N} / \mathrm{mm}^{2}$ )?
c) What is the shear-yield strength of the shear-zone material $S_{s y}$ (in MPa)?
2.15 Consider the orthogonal cutting of A-2 tool steel using a tool that has a rake angle of $\gamma_{o}=10^{\circ}$. The coefficient of friction is estimated to be $\mu=0.75$. The shear yield strength of the shear-zone material is estimated as a function of shear plane size, $l_{\phi}$ (in mm ), to be $S_{s y}=400 \cdot l_{\phi}^{-0.25} \mathrm{MPa}$. It should be assumed that the Lee and Shaffer shear angle model is valid for this material.
a) The term used to describe the fact that the shear yield strength shows dependence on the shear plane length, via its dependence on uncut chip thickness, is called size effect. Briefly describe the multiple contributors to the size effect and when, in terms of process variables, they are predominant. The following are some of the process variables to consider: uncut chip thickness, rake angle, cutting speed, width of cut.
b) What is the specific shear energy $u_{s}$ (in $\mathrm{N} / \mathrm{mm}^{2}$ ) as a function of uncut chip thickness $h, h$ being in mm ?

For the remainder of the problem, assume the answer to part (c) is $u_{s}=950 \cdot h^{-0.25} \mathrm{~N} / \mathrm{mm}^{2}$ for $h$ in mm .
c) An experiment is conducted at a cutting speed of $V=50 \mathrm{~m} / \mathrm{min}$, an uncut thickness of $h=0.20 \mathrm{~mm}$ and a width of cut of $w=2.0 \mathrm{~mm}$. What is the shearing power $\mathscr{S}_{s}($ in W$)$ ?
d) From the information accumulated so far ( $S_{s y}, a, V, \gamma_{o}, \phi_{o}$ and $\mu$ ), can the FCD be drawn? Why/how or why not? Sketching (or attempting to sketch) the FBD may help, but is not necessary.

## Chapter 3

3.1 Consider oblique cutting, using a tool that has an orthogonal rake angle of $\gamma_{o}=10^{\circ}$ and an inclination angle of $\lambda=30^{\circ}$. The uncut chip thickness is $h=0.25 \mathrm{~mm}$, the width of cut is $w=2.5 \mathrm{~mm}$ and the cutting speed is $V=$ $200 \mathrm{~m} / \mathrm{min}$. Assume that Stabler's Rule is valid.
a) Sketch a 2-D view of the process, including all hidden lines, projected in the plane in which the orthogonal rake angle is measured. Label the uncut chip thickness $h$ and orthogonal rake angle $\gamma_{o}$.
b) Sketch a 2-D view of the process, including all hidden lines, projected in the plane in which the normal rake angle is measured. Label the uncut chip thickness $h$ and normal rake angle $\gamma_{n}$.
c) What is the normal rake angle $\gamma_{n}$ (in degrees)?

## For the remainder of the problem, assume the answer to part (c) is $\gamma_{n}=8.5^{\circ}$.

d) What is the effective rake angle $\gamma_{e}$ (in degrees)?
e) Sketch a 2-D view of the process, including all hidden lines, projected in the plane in which the chip flow angle is measured. Label the chip velocity $V_{c}$ and the chip-flow angle $\eta_{\gamma}$
3.2 Consider oblique cutting, using a tool that has a normal rake angle of $\gamma_{n}=10^{\circ}$ and an inclination angle of $\lambda=$ $20^{\circ}$. The uncut chip thickness is $h=0.25 \mathrm{~mm}$, the width of cut is $w=2.5 \mathrm{~mm}$ and the cutting speed is $\mathrm{V}=200$ $\mathrm{m} / \mathrm{min}$. The normal shear angle and rake face coefficient of friction ( $\mu$ - not a "normal" coefficient $\mu_{n}$ ) have been determined to be $\phi_{n}=30^{\circ}$ and $\mu=0.50$, respectively. The shear-zone shear-yield strength is estimated to be $S_{s y}=375 \mathrm{MPa}$.
a) Calculate the chip flow angle $\eta_{\gamma}$ (in degrees) based on Armarego's solution.

For the remainder of the problem, assume the answer to part (a) is $\eta_{\gamma}=17.2^{\circ}$.
b) What is the specific shear energy $u_{s}\left(\mathrm{in} \mathrm{N} / \mathrm{mm}^{2}\right)$ and the in-plane shear force component $P_{\phi}($ in N$)$ ?

For the remainder of the problem, assume the answer to part (b) is $P_{\phi}=500 \mathrm{~N}$.
c) What is the in-plane rake face force component $P_{\gamma}($ in N$)$. Hint: One way is to calculate $P_{\phi n}$ using $\eta_{\phi}$ and $P_{\phi}$, then calculate $P_{m}$ using $\beta_{n}$ and any FCD relations, then calculate $P_{\gamma}$ from $P_{m}$ and $\eta_{\gamma}$ In other words, use the FCD and its relations for the force components in the normal plane. See problem 0 and compare. There is another very quick way if you can find it in the text.
d) What are the specific cutting, thrust and lateral energies $u_{C}, u_{T}$ and $u_{L}$ (in $\mathrm{N} / \mathrm{mm}^{2}$ )? Hint: Calculate $F_{C}$, $F_{T}$ and $F_{L}$ from either $N_{\phi}, P_{\phi n}$, and $P_{\phi s}$, or $N_{\gamma}, P_{\eta}$, and $P_{\gamma}$, using the inverse of the transformations provided in the text.
e) What are the cutting power $\mathscr{P}_{C}($ in kW$)$ and the material removal rate $\dot{v}_{r}$ (in $\mathrm{mm}^{3} / \mathrm{min}$ )?
3.3 Consider a turning operation using a tool that has a corner radius of $\boldsymbol{r}_{\varepsilon} \approx \mathbf{0}$, a lead angle of $\psi_{r}=15^{\circ}$, a back rake angle of $\gamma_{p}=-5^{\circ}$ and a side rake angle of $\gamma_{f}=-5^{\circ}$. The feed rate is $f_{r}=0.20 \mathrm{~mm} / \mathrm{rev}$, the depth of cut is $d=$ 0.75 mm and the spindle speed is $n_{s}=1000 \mathrm{rpm}$. The initial workpiece diameter is $D_{w}=100 \mathrm{~mm}$. Empirical models of the specific cutting and thrust energies (in units of $\mathrm{N} / \mathrm{mm}^{2}$ for uncut chip thickness $h$ in mm, cutting speed $V$ in $\mathrm{m} / \mathrm{min}$, and normal rake angle $\gamma_{n}$ in radians) are

$$
u_{C}=3150 h^{-0.233} V^{-0.122} \mathrm{e}^{-1.318 \gamma_{n}} \quad \text { and } \quad u_{T}=1175 h^{-0.615} V^{-0.148} \mathrm{e}^{-1.115 \gamma_{n}}
$$

a) What is the approximate chip area $a\left(\right.$ in $\left.\mathrm{mm}^{2}\right)$ and material removal rate $\dot{v}_{r}\left(\mathrm{in}^{3} / \mathrm{min}\right)$ ?
b) What are the straight-edge equivalent cutting and thrust force components $F_{C}$ and $F_{T}$ (in N )?
c) What are the tangential, radial, and longitudinal force components $F_{T a n}, F_{\text {Rad }}$ and $F_{L o n}($ in N$)$ ?
d) The corner radius must be greater than zero in practice. Provide a brief qualitative comment regarding the expected accuracy of each of the force component predictions in parts (b) and (c) compared to what you would expect if the corner radius were not zero. Address magnitudes and directions as appropriate.
e) What is the machined workpiece diameter $D_{w f}($ in mm)?
3.4 Consider a turning operation using a tool that has a corner radius of $\boldsymbol{r}_{\boldsymbol{\varepsilon}}=\mathbf{1 . 0} \mathbf{~ m m}$, a lead angle of $\psi_{r}=15^{\circ}$, a back rake angle of $\gamma_{p}=-5^{\circ}$ and a side rake angle of $\gamma_{f}=-5^{\circ}$. The feed rate is $f_{r}=0.20 \mathrm{~mm} / \mathrm{rev}$, the depth of cut is $d=0.75 \mathrm{~mm}$ and the spindle speed is $n_{s}=1000 \mathrm{rpm}$. The initial workpiece diameter is $D_{w}=100 \mathrm{~mm}$. Empirical models of the specific cutting and thrust energies (in units of $\mathrm{N} / \mathrm{mm}^{2}$ for uncut chip thickness $h$ in mm , cutting speed $V$ in $\mathrm{m} / \mathrm{min}$, and normal rake angle $\gamma_{n}$ in radians) are

$$
u_{C}=3150 h^{-0.233} V^{-0.122} \mathrm{e}^{-1.318 \gamma_{n}} \quad \text { and } \quad u_{T}=1175 h^{-0.615} V^{-0.148} \mathrm{e}^{-1.115 \gamma_{n}}
$$

a) What is the exact chip area $a$ (to 6 decimal places in $\mathrm{mm}^{2}$ ) and material removal rate $\dot{v}_{r}\left(\mathrm{in} \mathrm{mm}^{3} / \mathrm{min}\right)$ ?
b) Use Colwell's method to predict the equivalent lead angle $\bar{\psi}$ (in degrees).
c) What is the average uncut chip thickness $\bar{h}$ (to 4 decimal places, in mm )?
d) Using equivalent lead angle in place of lead angle within the normal rake angle calculation, and average uncut chip thickness for uncut chip thickness in the specific energy relations, what are the cutting, thrust, tangential, radial and longitudinal force components $F_{C}, F_{T}, F_{\text {Tan }}, F_{\text {Rad }}, F_{\text {Lon }}$, (in N$)$ ?
e) Briefly, comment on each of the five force component predictions above relative to your answers to problem 3.3, parts (b) and (c); ignore this part if problem 3.3 was not done.
3.5 Consider a boring operation using a two-tooth tool that has a corner radius of $\boldsymbol{r}_{\varepsilon}=\mathbf{0}$, a lead angle of $\psi_{r}=15^{\circ}$, a back rake angle of $\gamma_{p}=0^{\circ}$ and a side rake angle of $\gamma_{f}=0^{\circ}$. The feed rate is $f_{r}=0.40 \mathrm{~mm} / \mathrm{rev}$ and the spindle speed is $n_{s}=600 \mathrm{rpm}$. The initial workpiece hole diameter is $D_{w}=96 \mathrm{~mm}$. The tool diameter is $D_{t}=100 \mathrm{~mm}$. Due to casting variation, the workpiece hole axis is often offset by as much as $\varepsilon_{w o}$ at an offset angle $\delta_{w o}$ relative to the $x$-direction, as shown in the figure. Consider the case of $\varepsilon_{w o}=1.0 \mathrm{~mm}$ and $\delta_{w o}=210^{\circ}$. Empirical models of the specific cutting and thrust energies (in units of $\mathrm{N} / \mathrm{mm}^{2}$ for uncut chip thickness $h$ in mm and normal rake angle $\gamma_{n}$ in radians) are

$$
u_{C}=2000 h^{-0.25} \mathrm{e}^{-1.50 \gamma_{n}} \quad \text { and } \quad u_{T}=750 h^{-0.50} \mathrm{e}^{-1.25 \gamma_{n}} .
$$


a) What are the maximum and minimum depths of cut $d\left(\theta_{\max _{d}}\right)$ and $d\left(\theta_{\text {mind }_{d}}\right)$ (in mm$)$ experienced by a tooth, and at what tool angles, $\theta_{\text {max }}$ and $\theta_{\text {mind }}$ (in degrees) respectively, do these values occur?
b) What are the minimum and maximum of the cutting force component $F_{C}\left(\theta_{\min C}\right)$ and $F_{C}\left(\theta_{\operatorname{maxc}}\right)$ (in N ) experienced by a tooth, and at what tool angles, $\theta_{\min C}$ and $\theta_{\operatorname{maxC}}$ (in degrees) respectively, do these values occur?
c) Based on the geometry of two offset circles, show that the exact solution for the depth of cut as a function of tooth angle $\theta_{i}$ is

$$
d\left(\theta_{i}\right)=R_{t}-\varepsilon_{w o}\left\{\cos \left(\delta_{w o}-\theta_{i}\right)+\left[\left(R_{w} / \varepsilon_{w o}\right)^{2}-\sin ^{2}\left(\delta_{w o}-\theta_{i}\right)\right]^{1 / 2}\right\}
$$

Then, derive a more simple approximation by neglecting certain "small" terms here.
d) Graph the $x$ - and $y$-force components $F_{x}$ and $F_{y}$ (in N ) as functions of spindle angle $\theta_{s}$ where, by convention, the angle of tooth one is $\theta_{1}=\theta_{s}$.
e) Briefly describe what you think, without calculation, would happen to the maximum and minimum $x$ and $y$-force components if the corner radius was considered to be its actual value, $r_{\varepsilon}>0$ ?
f) What is the machined workpiece diameter $D_{w f}($ in mm)?

## Chapter 4

4.1 Consider face milling using an eight-tooth cutter that has a diameter $D_{t}=700 \mathrm{~mm}$. Each tooth has a corner radius of $r_{\varepsilon}=1.0 \mathrm{~mm}$, a lead angle of $\psi_{r}=0^{\circ}$, an axial rake angle of $\gamma_{p}=0^{\circ}$ and a radial rake angle of $\gamma_{f}=0^{\circ}$. The feed rate is $f_{r}=1.6 \mathrm{~mm} / \mathrm{rev}$; the depth of cut is $d=1.0 \mathrm{~mm}$ and the spindle speed is $n_{s}=600 \mathrm{rpm}$. The workpiece centerline is offset from the feed path by $\varepsilon=+75 \mathrm{~mm}$ in the $y$-direction, as shown in the figure. The specific cutting and thrust energies are

$$
u_{C}=2000 h^{-0.25} \mathrm{e}^{-1.50 \gamma_{n}} \quad \text { and } \quad u_{T}=750 h^{-0.50} \mathrm{e}^{-1.25 \gamma_{n}} .
$$

a) What are the tooth-entry and tooth-exit angles $\theta_{e n}$ and $\theta_{e x}$ (in degrees)?
b) What is the angle range $\Delta \theta_{e 1}$ (in degrees) over which only one tooth is engaged in cutting?
c) What is the average uncut chip thickness $\bar{h}$ (in mm) at
 tooth-entry and at its maximum value?
d) Using Fu's Method for predicting equivalent lead angle, what is the radial force acting on a tooth $F_{\text {Rad }_{i}}$ as a function of longitudinal force $F_{\text {Loni }}$ acting on that tooth and the angle of that tooth angle $\theta_{i}$ ? Hint: This particular combination of conditions allows this question to be answered, given some thought, without any calculation!
4.2 Consider end milling using a four-flute tool that has a diameter $D_{t}=25 \mathrm{~mm}$ and a helix angle of $\gamma_{p}=30^{\circ}$. Each flute has a radial rake angle of $\gamma_{f}=0^{\circ}$. The feed rate is $f_{r}=0.80 \mathrm{~mm} / \mathrm{rev}$, the axial depth of cut is $d=15$ mm , the spindle speed is $n_{s}=600 \mathrm{rpm}$, radial immersion is $75 \%$ (of the cutter diameter) and the cut is upmilling.
a) What are the tooth-entry and tooth-exit angles $\theta_{e n}$ and $\theta_{e x}$ (in degrees)?
b) Draw to scale the flute-engagement diagram for the case when the angle of tooth number one, when measured at the end of the cutter $\left(\left.\theta_{1}\right|_{z=0}\right)$, is zero.
c) What is the maximum engaged wrap angle (in degrees) that a single flute can experience?
d) Assume size effect is ignored so that the specific cutting and thrust energies take on constant values, for the given speed and rake angles, of $u_{C}=1500 \mathrm{~N} / \mathrm{mm}^{2}$ and $u_{T}=1000 \mathrm{~N} / \mathrm{mm}^{2}$. If the immersion is changed to $\mathbf{1 0 0 \%}$ (slotting), what are the peak $x$ - and $y$-force components acting on the cutter, $F_{x}$ and $F_{y}$, (in N )? Hint: Think about this in terms of face milling with $D_{t} / W_{w}=1$.
e) Again ignoring size effect, what would be the dominant frequency (in Hz ) of the total $x$ - and $y$-force signatures for each case, i.e., for $75 \%$ immersion and $100 \%$ immersion?
4.3 Consider drilling a $10-\mathrm{mm}$ diameter hole using a standard conical-point twist drill that has a web thickness $2 b_{w}=1 \mathrm{~mm}$ and chisel edge angle of $\varphi=135^{\circ}$.
a) By numerically integrating $\gamma_{n}(r)$ from the web to the outer diameter (where $r=R_{t}$ ), then dividing by the integration range ( $R_{t}-r$ (at web)), calculate an average normal rake angle $\bar{\gamma}_{n}$ (in degrees).
b) At what radial position $\bar{r}$ (in mm ) does the average normal rake angle occur?
c) What is the inclination angle $\bar{\lambda}$ (in degrees) at the average normal rake angle position $\bar{r}$ ?

## Chapter 5

5.1 Consider the data in the table that were obtained from a turned surface ( $z$ in mm and $r(z)$ in $\mu \mathrm{m}$ ). Note that the sample increment is 0.021 mm .

| $z$ <br> $(\mathrm{~mm})$ | $r(z)$ <br> $(\mu \mathrm{m})$ | $z$ <br> $(\mathrm{~mm})$ | $r(z)$ <br> $(\mu \mathrm{m})$ | $z$ <br> $(\mathrm{~mm})$ | $r(z)$ <br> $(\mu \mathrm{m})$ | $z$ <br> $(\mathrm{~mm})$ | $r(z)$ <br> $(\mu \mathrm{m})$ | $z$ <br> $(\mathrm{~mm})$ | $r(z)$ <br> $(\mu \mathrm{m})$ | $z$ <br> $(\mathrm{~mm})$ | $r(z)$ <br> $(\mu \mathrm{m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0.000 | 38.1 | 0.104 | 44.7 | 0.208 | 40.9 | 0.313 | 42.3 | 0.417 | 40.2 | 0.521 | 36.7 |
| 0.021 | 39.2 | 0.125 | 53.9 | 0.229 | 36.1 | 0.333 | 41.8 | 0.438 | 36.5 | 0.542 | 36.5 |
| 0.042 | 38.6 | 0.146 | 46.1 | 0.250 | 35.8 | 0.354 | 43.0 | 0.458 | 37.8 | 0.563 | 39.2 |
| 0.063 | 42.2 | 0.167 | 39.4 | 0.271 | 35.8 | 0.375 | 53.3 | 0.479 | 35.2 | 0.583 | 43.1 |
| 0.083 | 45.5 | 0.188 | 38.0 | 0.292 | 39.9 | 0.396 | 48.1 | 0.500 | 34.6 | 0.604 | 49.0 |

a) Graph the measured surface profile and estimate the feed rate $f_{r}$ (in $\mathrm{mm} / \mathrm{rev}$ ) and the corner radius $r_{\varepsilon}$ (in mm ). Assume only the corner radius forms the surface (i.e., the lead and trail cutting edges do not contact the final surface).
b) Calculate the peak-to-valley $\left(r_{t}\right)$, center-line $\left(r_{c l}\right)$, roughness-average $\left(r_{a}\right)$, and RMS roughness $\left(r_{q}\right)$ values (in $\mu \mathrm{m}$ ) using the discrete data given. First use a 0.604 mm cutoff, then use only the data from $z=0.0$ to 0.500 mm . Comment on the differences in your results, if any.
c) Estimate the peak-to-valley $\left(R_{t}\right)$, center-line $\left(r_{c l}\right)$, roughness-average $\left(r_{a}\right)$, and RMS roughness $\left(r_{q}\right)$ values (in $\mu \mathrm{m}$ ) based on the zero corner radius analytical model, using the feed rate determined in part (a) and assuming the lead angle is $\psi_{r}=15^{\circ}$ and the end cutting edge angle $\kappa_{r^{\prime}}=15^{\circ}$. What are the percent errors in these calculated values relative to those found in part (b)?
d) Estimate the roughness-average value $r_{a}$ (in $\mu \mathrm{m}$ ) using Boothroyd's empirical model using the corner radius value determined in part (a). What is the percent error in this calculated value relative to that found in part (b)?
5.2 For arbitrary tooth feed $f$ and corner radius $r_{\varepsilon}$, assuming the machined surface is generated by the corner arc only (i.e., no portion of the surface is generated by the lead or end cutting edges), use analytical integration to derive closed-form expressions for the following values.
a) The roughness height $r(z)$, relative to the valley bottom, in terms of position $z(z=0$ at the corner radius center), tooth feed $f$ and corner radius $r_{\varepsilon}$.
b) The centerline value $r_{c l}$ in terms of corner radius $r_{\varepsilon}$ and tooth feed $f$.
c) The roughness-average value in terms of the centerline value $r_{c l}$, corner radius $r_{\varepsilon}$ and tooth feed $f$.

## Chapter 6

## Chapter 7

7.1 Consider the flank wear versus time data in the table.

| $t$ <br> $(\mathrm{~min})$ | $W(t)$ <br> $(\mathrm{mm})$ | $t$ <br> $(\mathrm{~min})$ | $W(t)$ <br> $(\mathrm{mm})$ | $t$ <br> $(\mathrm{~min})$ | $W(t)$ <br> $(\mathrm{mm})$ | $t$ <br> $(\mathrm{~min})$ | $W(t)$ <br> $(\mathrm{mm})$ | $t$ <br> $(\mathrm{~min})$ | $W(t)$ <br> $(\mathrm{mm})$ | $t$ <br> $(\mathrm{~min})$ | $W(t)$ <br> $(\mathrm{mm})$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 00.0 | 0.016 | 1.5 | 0.370 | 4.0 | 0.782 | 7.0 | 1.145 | 10.0 | 2.196 | 12.0 | 3.050 |
| 0.5 | 0.145 | 2.0 | 0.458 | 5.0 | 0.854 | 8.0 | 1.357 | 11.0 | 2.505 | 12.5 | 3.081 |
| 1.0 | 0.262 | 3.0 | 0.643 | 6.0 | 1.098 | 9.0 | 1.703 | 11.5 | 2.687 | 13.0 | 3.373 |

a) Develop an estimating model for wear level as a function of time.
b) Estimate, either graphically or mathematically, the tool life $t_{l}$ for a critical wear level of $W_{l}=0.5 \mathrm{~mm}$.
7.2 Estimate Taylor's exponent $n$ and constant $C$ (in $\mathrm{m} / \mathrm{min}$ ) for the HSS tool-life data in the table.

| $V$ <br> $(\mathrm{~m} / \mathrm{min})$ | $t_{l}$ <br> $(\mathrm{~min})$ | $V$ <br> $(\mathrm{~m} / \mathrm{min})$ | $t_{l}$ <br> $(\mathrm{~min})$ | $V$ <br> $(\mathrm{~m} / \mathrm{min})$ | $t_{l}$ <br> $(\mathrm{~min})$ | $V$ <br> $(\mathrm{~m} / \mathrm{min})$ | $t_{l}$ <br> $(\mathrm{~min})$ | $V$ <br> $(\mathrm{~m} / \mathrm{min})$ | $t_{l}$ <br> $(\mathrm{~min})$ | $V$ <br> $(\mathrm{~m} / \mathrm{min})$ | $t_{l}$ <br> $(\mathrm{~min})$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 25 | 25.708 | 100 | 4.009 | 175 | 1.503 | 250 | 1.054 | 325 | 0.672 | 400 | 0.541 |
| 50 | 10.012 | 125 | 2.949 | 200 | 1.442 | 275 | 0.804 | 350 | 0.643 | 425 | 0.483 |
| 75 | 5.183 | 150 | 2.075 | 225 | 1.186 | 300 | 0.796 | 375 | 0.609 | 450 | 0.438 |

7.3 Consider the continuous turning of a bar that has a diameter of $D_{w}=50 \mathrm{~mm}$ and length of $L_{w}=200 \mathrm{~mm}$. The feed rate is $f_{r}=0.25 \mathrm{~mm} / \mathrm{rev}$. Taylor's tool-life model parameters are $C=150 \mathrm{~m} / \mathrm{min}$ and $n=0.1$. The time to change one tooth is $t_{c}{ }^{\prime}=0.75 \mathrm{~min} /$ tooth, the material handling time is $t_{h}=0.5 \mathrm{~min} /$ part, the cost of a new cutting tooth is $c_{t}{ }^{\prime}=2 \$ /$ tooth and the overhead cost is $c_{o}=1.5 \$ / \mathrm{min}$.
a) What is the minimum unit time $t_{u_{\text {min }}}($ in $\mathrm{min} /$ part)?
b) At what spindle speed $n_{s}(\mathrm{in} \mathrm{rev} / \mathrm{min})$ is the minimum unit time achieved?
c) What is the unit $\operatorname{cost} c_{u}$ (in $\$ /$ part) associated with achieving minimum unit time?
7.4 Consider the continuous turning of a bar that has a diameter of $D_{w}=50 \mathrm{~mm}$ and length of $L_{w}=200 \mathrm{~mm}$. The feed rate is $f_{r}=0.25 \mathrm{~mm} / \mathrm{rev}$. Taylor's tool-life model parameters are $C=150 \mathrm{~m} / \mathrm{min}$ and $n=0.1$. The time to change one tooth is $t_{c}{ }^{\prime}=0.75 \mathrm{~min} /$ tooth, the material handling time is $t_{h}=0.5 \mathrm{~min} / \mathrm{part}$, the cost of a new cutting tooth is $c_{t}{ }^{\prime}=2 \$ /$ tooth and the overhead cost is $c_{o}=1.5 \$ / \mathrm{min}$.
a) What is the minimum unit $\operatorname{cost} c_{u_{\text {min }}}($ in $\$ /$ part $)$ ?
b) At what spindle speed $n_{s}$ (in rev $/ \mathrm{min}$ ) is the minimum unit cost achieved?
c) What is the unit time $t_{u}$ (in $\mathrm{min} / \mathrm{part}$ ) associated with achieving minimum unit cost?
7.5 Consider the continuous turning of a bar that has a diameter of $D_{w}=50 \mathrm{~mm}$ and length of $L_{w}=200 \mathrm{~mm}$. The feed rate is $f_{r}=0.25 \mathrm{~mm} / \mathrm{rev}$. Taylor's tool-life model parameters are $C=150 \mathrm{~m} / \mathrm{min}$ and $n=0.1$. The time to change one tooth is $t_{c}{ }^{\prime}=0.75 \mathrm{~min} /$ tooth, the material handling time is $t_{h}=0.5 \mathrm{~min} /$ part, the cost of a new cutting tooth is $c_{t}{ }^{\prime}=2 \$ /$ tooth and the overhead cost is $c_{o}=1.5 \$ / \mathrm{min}$.
a) At what cutting speed $V(\mathrm{in} \mathrm{m} / \mathrm{min})$ is maximum unit profit achieved?
b) At what cutting speed $V$ (in $\mathrm{m} / \mathrm{min})$ is maximum unit profit rate achieved?
c) What is the unit cost $c_{u}$ (in $\$ /$ part) associated with achieving minimum unit profit?
7.6 Consider face milling using an eight-tooth cutter that has a diameter $D_{t}=300 \mathrm{~mm}$ and holds square inserts each having four usable edges. The workpiece has width of $W_{w}=200 \mathrm{~mm}$, length of $L_{w}=400 \mathrm{~mm}$ and no surface voids. The feed rate is $f_{r}=2.0 \mathrm{~mm} / \mathrm{rev}$ and the depth of cut is $d=1.5 \mathrm{~mm}$. Economic information includes the following:

- Tool life model: $C=250, n=0.25$.
- Inserts cost $\$ 8$ each.
- It takes 4 minutes to change each tooth.
- It takes 5 minutes to load the workpiece and another 5 minutes to unload the workpiece.
- The overhead rate is $120 \$ /$ hour.
a) What spindle speed $n_{s}$ (in rev/min) results in minimum unit time?
b) What spindle speed $n_{s}($ in rev $/ \mathrm{min}$ ) results in minimum unit cost?
c) What spindle speed $n_{s}$ (in rev/min) results in maximum unit profit?
d) If the unit profit, when profit rate is maximized, is to be $10 \%$ of the unit revenue, what should the unit revenue $r_{u}$ be (in $\$ /$ part)? Hint: The solution is likely to require iteration, besides the iteration related to determining $V_{s}$.
e) What would be the machining time $t_{m}$ and engagement time $t_{e}$ (in min/part), as functions of cutting speed $V$, if there was a $100-\mathrm{mm}$ diameter hole in the surface, running perpendicular to the machined surface, and all the way through the workpiece?

