7.6

Given:	
• face milling • number of flutes/teeth, N_t = 8 • square inserts having 4 edges each • tool diameter, D_t = 300 mm • feed rate, f_r = 2.0 mm/rev • depth of cut, d =1.5 mm • workpiece width, W_w = 200 mm	• workpiece length, L_w = 400 mm • no surface voids • tool life constants: C = 250 m/min, n = 0.25. • inserts cost: c_t' = 8 \$/insert • tooth change time: t_c' = 4 min/tooth • handling time, t_h = 10 min/part • overhead rate, c_o =120 \$/hour.

a) Minimum unit time is achieved at a cutting speed of

$$V_{max} = \frac{C}{\left[\left(1/n-1\right)t_c k\right]^n} .$$

The tool change time (in min/tool) is the product of the time to change a tooth and the number of teeth per tool, N_t, or

$$t_c = \left(4 \text{ min}/\cancel{\text{tooth}}\right)\left(8 \ \cancel{\text{teeth}}/\text{tool}\right) = 32 .$$

The engagement fraction is

$$k = \frac{\theta_{ex} - \theta_{en}}{360°} ,$$

where, since there is no offset between the part centerline and the feed axis (by presumption),

$$\theta_{ex} = -\theta_{en} = \sin^{-1}\left(\frac{W_w}{D_t}\right) = \sin^{-1}\left(\frac{200}{300}\right) = 41.8° \quad \rightarrow \quad k = \frac{83.6°}{360°} = 0.232 .$$

Substituting known values, the cutting speed (in m/min) is

$$V_{max} = \frac{250}{\left[\left(1/0.25-1\right)(32)(0.232)\right]^{0.25}} = 115 .$$

The final result for spindle speed (in rpm) is

$$n_{s_{max}} = \frac{1000 V_{max}}{\pi D_t} = \frac{1000(115)}{\pi(300)} = \boxed{122} .$$

b) Minimum unit cost is achieved at a cutting speed of

$$V_{min} = \frac{C}{\left[\left(1/n-1\right)\left(t_c + \frac{c_t}{c_o}\right)k\right]^n} .$$

The tool cost (in \$/tool) is the product of the cost per insert, divided by the number of edges per insert, and the number of teeth (edges) per tool, N_t, or

$$c_t = \left(\frac{8 \ \$/\cancel{\text{insert}}}{4 \ \cancel{\text{edges}}/\cancel{\text{insert}}}\right)\left(8 \ \cancel{\text{edges}}/\text{tool}\right) = 16 .$$

Substituting known values, including t_c and k from part (a), the cutting speed (in m/min) is

$$V_{min} = \frac{250}{\left[\left(1/0.25-1\right)\left(32 + \frac{16}{2}\right)(0.232)\right]^{0.25}} = 109 .$$

The final result for spindle speed (in rpm) is

$$n_{s_{min}} = \frac{1000V_{min}}{\pi D_t} = \frac{1000(109)}{\pi(300)} = \boxed{116}.$$

c) Since unit revenue is independent of speed, maximum unit profit is equivalent to minimum unit cost. Therefore, the final result is the same as that for part (b).

d) It is desired that the unit profit be 10% of the unit revenue, or $p_u = 0.1r_u$. Since $p_u = r_u - c_u$, the unit revenue required to achieve this can be determined in terms of the unit cost by equating the two as

$$0.1r_u = r_u - c_u \quad \rightarrow \quad r_u = 1.11c_u.$$

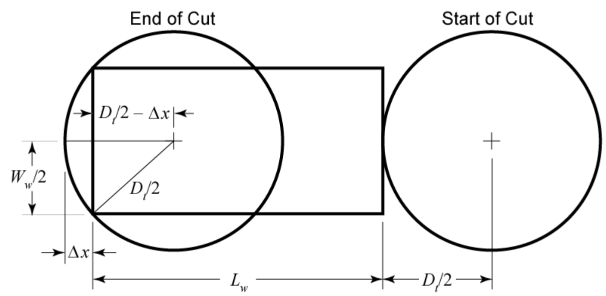
The unit cost is

$$c_u = c_o(t_m + t_h) + (c_o t_c + c_t)\left(\frac{t_e}{t_l}\right) = c_o\left[\left(\frac{\mathscr{L}}{V} + t_{np}\right) + \left(t_c + \frac{c_t}{c_o}\right)\left(\frac{k\mathscr{L}}{C^{1/n}}V^{(1/n-1)} + t_h\right)\right],$$

where the cutting speed remains as a variable not yet determined, the nonproductive time t_{np} is zero,

$$\mathscr{L} = \frac{\pi D_t(L_w + \Delta x)}{f_r},$$

and Δx is shown in the figure to the right, which is determined based on the triangle shown to be



$$\Delta x = \frac{D_t}{2} - \left(\frac{D_t}{2} - \Delta x\right) = \frac{D_t}{2} - \left(\frac{D_t^{\,2}}{4} - \frac{W_w^{\,2}}{4}\right)^{1/2}.$$

Substituting known values, the cutting length (in mm) is

$$\Delta x = \frac{300}{2} - \left(\frac{300^2}{4} - \frac{200^2}{4}\right)^{1/2} = 38.2 \quad \rightarrow \quad \mathscr{L} = \frac{\pi(300)(400+38.2)}{2.0(1000)} = 206.5.$$

Substituting into the unit cost equation, the unit cost (in \$/part) for cutting speed (in m/min) is

$$c_u = 2\left[\frac{206.5}{V} + \left(32 + \frac{16}{2}\right)\left(\frac{0.232(206.5)}{250^{1/0.25}}V^{(1/0.25-1)} + 10\right)\right] = 413V^{-1} + \left(9.81\cdot 10^{-7}\right)V^3 + 20,$$

and the unit revenue is

$$r_u = 1.11c_u = 458.9V^{-1} + \left(1.09\cdot 10^{-6}\right)V^3 + 22.2.$$

The speed that maximizes profit rate is

$$V_\$ = \left\{\left(\frac{k\mathscr{L}c_t}{nr_uC^{1/n}}\right)\frac{1}{V_\$} + \left(\frac{(1/n-1)k}{C^{1/n}}\right)\left[t_c + \frac{c_t}{r_u}\left(t_{np} + t_h\right)\right]\right\}^{-n}.$$

Combining unit revenue terms for convenience yields

$$V_\$ = \left\{\left[\left(\frac{k\mathscr{L}c_t}{nC^{1/n}V_\$}\right) + \left(\frac{(1/n-1)kc_t\left(t_{np} + t_h\right)}{C^{1/n}}\right)\right]\frac{1}{r_u} + \frac{(1/n-1)kt_c}{C^{1/n}}\right\}^{-n}.$$

Substituting known values

$$V_\$ = \left\{\left[\left(\frac{(0.232)(206.5)(16)}{(0.25)(250)^{1/0.25}V_\$}\right) + \left(\frac{(1/0.25-1)(0.232)(16)(0+10)}{(250)^{1/0.25}}\right)\right]\frac{1}{r_u} + \frac{(1/0.25-1)(0.232)(32)}{(250)^{1/0.25}}\right\}^{-0.25};$$

simplifying yields

$$V_\$ = \left\{\left[\frac{7.85\cdot 10^{-7}}{V_\$} + 5.41\cdot 10^{-9}\right]\frac{1}{r_u} + 5.70\cdot 10^{-9}\right\}^{-0.25}.$$

Substituting in for r_u as a function of $V_\$$ gives the final result from which to iterate:

$$V_\$ = \left\{\left[\frac{7.85\cdot 10^{-7}}{V_\$} + 5.41\cdot 10^{-9}\right]\frac{1}{458.9V_\$^{-1} + \left(1.09\cdot 10^{-6}\right)V_\$^{3} + 22.2} + 5.70\cdot 10^{-9}\right\}^{-0.25}.$$

A reasonable initial guess is the average of V_{min} and V_{max}, or 112 m/min. The iteration proceeds as

$$V_{\$_0} = 112 \rightarrow V_{\$_1} = 112.93 \rightarrow V_{\$_2} = 112.94 \rightarrow V_{\$_3} = 113.$$

As a side note, if 1 m/min were used as the initial guess, the iteration would proceed as

$$V_{\$_0} = 1 \rightarrow V_{\$_1} = 108.02 \rightarrow V_{\$_2} = 112.89 \rightarrow V_{\$_3} = 112.94 \rightarrow V_{\$_4} = 113!.$$

Substituting all known values, including $V_\$$, the final result (in \$/part) is

$$c_u = 413(113)^{-1} + \left(9.81\cdot 10^{-7}\right)(113)^3 + 20 = 25.07 \quad \rightarrow \quad r_u = 1.11c_u = 1.11(25.07) = \boxed{27.86}.$$

e) The machining time is the same as it would be for the enclosing rectangle (i.e., unaffected by the surface void). Therefore, the first part of the final result (in min/part) is

$$t_m = \frac{\mathcal{L}}{V} = \boxed{\frac{206.5}{V}}.$$

The engagement time is affected by the surface void. To compute it, the machined area (in mm^2) of the enclosing rectangle,

$$a'_m = W_w L_w = (200)(400) = 80,000$$

and that of that with the voids,

$$a_m = a'_m - a_{voids} = a'_m - \frac{\pi D_{hole}^{\,2}}{4} = 80,000 - \frac{\pi(100)^2}{4} = 72,146,$$

provide the engagement ratio based on that of the enclosing rectangle (k' = 0.232) as

$$k = \frac{a_m}{a'_m}k' = \frac{72,146}{80,000}(0.232) = 0.209.$$

The final result (in min/part) is

$$t_e = kt_m = 0.209\left(\frac{206.5}{V}\right) = \boxed{\frac{43.16}{V}}.$$