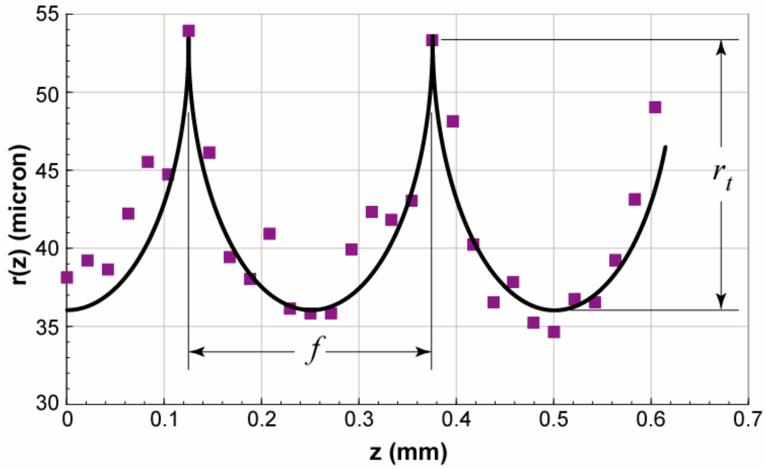
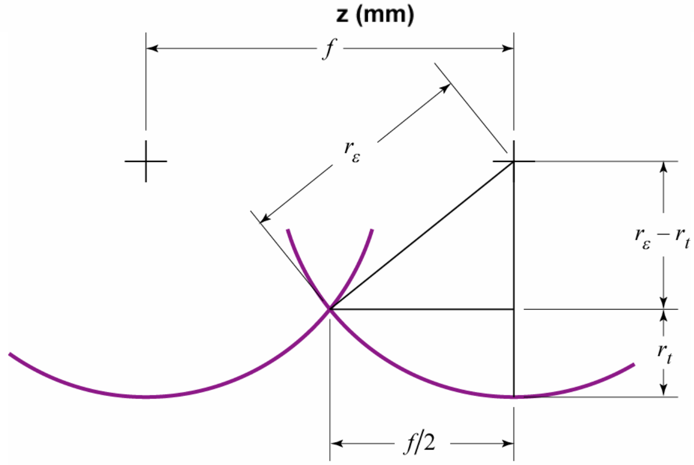
5.1

Given:	
• surface roughness data from turned surface	• sample increment is 0.021 mm

a) The data are graphed below. Continuous curves have been added to approximate the ideal profile. These curves show as elliptical, being stretched vertically, so that the vertical variation is discernable. The feed, as indicated, is the horizontal distance from cusp to cusp. The peak-to-valley is denoted as r_t.





The figure immediately above depicts the ideal case with the corner radius, feed and peak-to-valley shown. The right triangle with hypotenuse equal to r_ε obeys the relation

$$(r_\varepsilon - r_t)^2 + (f/2)^2 = r_\varepsilon^2 .$$

Given that f and r_t can be measured from the plot this relation can be solved for r_ε as

$$\cancel{r_\varepsilon^2} - 2r_\varepsilon r_t + r_t^2 + f^2/4 = \cancel{r_\varepsilon^2} \quad \rightarrow \quad r_\varepsilon = \frac{r_t^2 + f^2/4}{2r_t} .$$

Measuring from the plot, $f = 0.25$ mm(/rev) and $r_t = 17$ µm. Substituting these values, and converting r_t from 15 µm to 0.015 mm, the final result for corner radius (in mm) is

$$r_\varepsilon = \frac{(0.017)^2 + (0.25)^2/4}{2(0.017)} = \boxed{0.468} .$$

b) The equations required to compute the requested surface finish parameters are

$$r_t = r_{max} - r_{min}, \quad r_{cl} = \frac{1}{N}\sum_{i=1}^{N} r_i, \quad r_a = \frac{1}{N}\sum_{i=1}^{N} |r_i - r_{cl}|, \quad r_q^{\,2} = \frac{1}{N}\sum_{i=1}^{N} (r_i - r_{cl})^2 .$$

When applied to the entire set of discrete data given, i.e., Δ_z = 0.625 mm, the final results (in μm) are

$$r_t = \boxed{21.6}, \quad r_{cl} = \boxed{40.75}, \quad r_a = \boxed{4.54}, \quad r_q = \boxed{5.65}$$

Using a smaller cutoff of Δ_z = 0.6 mm, the final results (in μm) are

$$r_t = \boxed{21.6}, \quad r_{cl} = \boxed{40.83}, \quad r_a = \boxed{4.43}, \quad r_q = \boxed{5.60}$$

There is very little difference since the cutoff difference is small compared to the spatial frequency — the feed.

c) The peak-to-valley value is

$$r_t = \frac{f \tan \kappa_{r'}}{1 + \tan \kappa_{r'} \tan \psi_r} .$$

Substituting known values, the result is

$$r_t = \frac{0.25 \tan(15°)}{1 + \tan(15°)\tan(15°)}(1000) = \boxed{63} .$$

The centerline value is one-half the peak-to-valley value, or 31.5 μm. The roughness average value is one-quarter of the peak-to-valley value, or 15.8 μm.

The percent errors in these as compared to the data above are computed in the usual sense: (predicted – actual) divided by actual. For the centerline value, since the predicted one is with respect to the minimum of the surface profile, the one computed for the data must be shifted by subtracting the minimum of the data, yielding $r_{cl} \approx 41 - 36 = 5$ μm. Substituting known values, the final results (in μm and percent error) are

$$r_t = \boxed{63}, \quad error = \frac{63 - 21.6}{21.6}100 = \boxed{192\%}$$

$$r_{cl} = \boxed{31.5}, \quad error = \frac{31.5 - 5}{5}100 = \boxed{530\%}$$

$$r_a = \boxed{15.8} \quad error = \frac{15.8 - 4.5}{4.5}100 = \boxed{251\%}$$

d) Using Boothroyd's model,

$$r_a = \frac{0.0321 f^2}{r_\varepsilon} ,$$

substituting known values, the final result (in μm) is

$$r_a = \frac{0.0321(0.25)^2}{0.53}(1000) = \boxed{3.8} .$$

The percent error relative to that computed from the data is

$$error = \frac{3.8 - 4.5}{4.5}100 = \boxed{-15.6\%} .$$

5.2

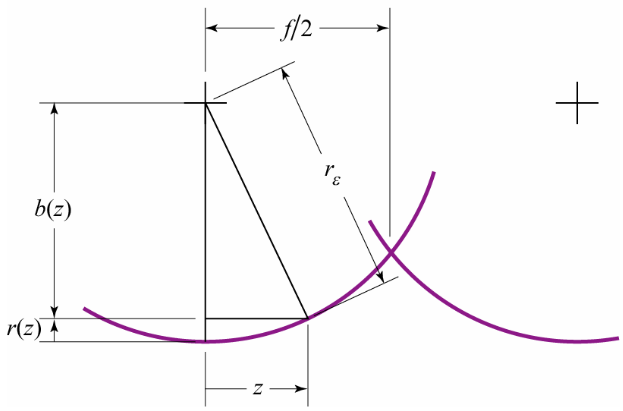
Given:	
• feed, f	• corner radius, r_ε

a) The figure to the right shows the geometry of this case. The distance $b(z)$ is found by knowing the hypotenuse of the triangle (r_ε) and the other side of length $z - 0$. From this,

$$b^2 + z^2 = r_\varepsilon^{\,2} \quad \rightarrow \quad b(z) = \left(r_\varepsilon^{\,2} - z^2\right)^{1/2}.$$

The final result, the profile height, is then

$$r(z) = r_\varepsilon - b(z) = \boxed{r_\varepsilon - \left(r_\varepsilon^{\,2} - z^2\right)^{1/2}}.$$



b) The results here can be derived using symbolic mathematics software, such as Mathematica®. It is presented here using integration table results; the steps follow the same path that would be followed using symbolic mathematics software, but present greater detail.

The centerline value is the integral of $r(z)$ divided by the interval of the integration:

$$r_{cl} = \frac{1}{\Delta_z}\int_0^{\Delta_z} r(z)dz\,.$$

Due to symmetry, the integral need be evaluated only from 0 to $f/2$, not 0 to f, of course then dividing the integral by $f/2$, not f. Substituting for $r(z)$,

$$r_{cl} = \frac{1}{f/2}\int_0^{f/2}\left[r_\varepsilon - \left(r_\varepsilon^{\,2} - z^2\right)^{1/2}\right]dz = \frac{1}{f/2}\left[\int_0^{f/2} r_\varepsilon dz - \int_0^{f/2}\left(r_\varepsilon^{\,2} - z^2\right)^{1/2} dz\right].$$

The left-hand integral is evaluated as

$$\int_0^{f/2} r_\varepsilon dz = r_\varepsilon z\Big|_0^{f/2} = \frac{r_\varepsilon f}{2}.$$

The right-hand integral is evaluated as

$$\int_0^{f/2}\left(r_\varepsilon^{\,2} - z^2\right)^{1/2} dz = \left[\frac{z\left(r_\varepsilon^{\,2} - z^2\right)^{1/2}}{2} + \frac{r_\varepsilon^{\,2}}{2}\sin^{-1}\left(\frac{z}{r_\varepsilon}\right)\right]_0^{f/2} = \frac{f\left(r_\varepsilon^{\,2} - \frac{f^2}{4}\right)^{1/2}}{4} + \frac{r_\varepsilon^{\,2}}{2}\sin^{-1}\left(\frac{f}{2r_\varepsilon}\right).$$

Subtracting this result from that of the left-hand integral, and multiplying by $1/(f/2)$,

$$r_{cl} = \frac{1}{f/2}\left[\frac{r_\varepsilon f}{2} - \frac{f\left(r_\varepsilon^{\,2} - \frac{f^2}{4}\right)^{1/2}}{4} - \frac{r_\varepsilon^{\,2}}{2}\sin^{-1}\left(\frac{f}{2r_\varepsilon}\right)\right].$$

Simplifying yields

$$r_{cl} = \frac{\cancel{2}}{\cancel{f}}\left[\frac{r_\varepsilon \cancel{f}}{\cancel{2}} - \frac{\cancel{f}\left(r_\varepsilon^{\,2} - \frac{f^2}{4}\right)^{1/2}}{\cancel{4}\,2f} - \frac{r_\varepsilon^{\,2}}{\cancel{2}\,f}\sin^{-1}\left(\frac{f}{2r_\varepsilon}\right)\right].$$

The final result (in the same length units as f and r_ε) is

$$r_{cl} = \boxed{r_\varepsilon - \frac{\left(4r_\varepsilon^{\,2} - f^2\right)^{1/2}}{4f} - \frac{r_\varepsilon^{\,2}}{f}\sin^{-1}\left(\frac{f}{2r_\varepsilon}\right)}.$$

c) The results here can be derived using symbolic mathematics software, such as Mathematica®. It is presented here using integration table results; the steps follow the same path that would be followed using symbolic mathematics software, but present greater detail.

The roughness average value is the integral of $|r(z) - r_{cl}|$ divided by the interval of the integration:

$$r_a = \frac{1}{\Delta_z}\int_0^{\Delta_z} \left| r(z) - r_{cl} \right| dz \ .$$

Due to symmetry, the integral need be evaluated only from 0 to $f/2$, not 0 to f, of course then dividing the integral by $f/2$, not f. However, due to the absolute value in the integrand, the integral must be split into two parts as

$$r_a = \frac{1}{f/2}\left[\int_0^{z_o} \left(r_{cl} - r(z)\right) dz + \int_{z_o}^{f/2} \left(r(z) - r_{cl}\right) dz\right].$$

The first is for the region where $r(z)$ is less than r_{cl}, in which case $|r(z) - r_{cl}|$ can be replaced by $r_{cl} - r(z)$. The second is for the region where $r(z)$ is greater than r_{cl}, in which case $|r(z) - r_{cl}|$ can be replaced by $r(z) - r_{cl}$. The integration limit z_o is the point at which $r(z) = r_{cl}$:

$$r(z_o) = r_{cl} \quad \rightarrow \quad r_\varepsilon - \left(r_\varepsilon^{\ 2} - z_o^{\ 2}\right)^{1/2} = r_{cl} \ ,$$

or

$$r_\varepsilon^{\ 2} - z_o^{\ 2} = \left(r_\varepsilon - r_{cl}\right)^2 \quad \rightarrow \quad z_o = +\left[r_\varepsilon^{\ 2} - \left(r_\varepsilon - r_{cl}\right)^2\right]^{1/2} .$$

Expanding within the square root,

$$z_o = \left[\cancel{r_\varepsilon^{\ 2}} - \left(\cancel{r_\varepsilon^{\ 2}} - 2r_\varepsilon + r_{cl}^{\ 2}\right)\right]^{1/2} = \left(2r_\varepsilon r_{cl} - r_{cl}^{\ 2}\right)^{1/2} .$$

Now, considering the first integral in its indefinite form for now, substituting for $r(z)$ yields

$$\int\left(r_{cl} - r_\varepsilon - \left(r_\varepsilon^{\ 2} - z^2\right)^{1/2}\right) dz = \int\left(r_{cl} - r_\varepsilon\right) dz - \int\left(r_\varepsilon^{\ 2} - z^2\right)^{1/2} dz \ .$$

The left-hand integral is evaluated as

$$\int\left(r_{cl} - r_\varepsilon\right) dz = \left(r_{cl} - r_\varepsilon\right) z \ .$$

The right-hand integral is evaluated as

$$\int\left(r_\varepsilon^{\ 2} - z^2\right)^{1/2} dz = \frac{z\left(r_\varepsilon^{\ 2} - z^2\right)^{1/2}}{2} + \frac{r_\varepsilon^{\ 2}}{2}\sin^{-1}\left(\frac{z}{r_\varepsilon}\right).$$

Adding this result to that of the left-hand integral, and multiplying by $1/(f/2)$,

$$\frac{2}{f}\left[\left(r_{cl} - r_\varepsilon\right) z + \frac{z\left(r_\varepsilon^{\ 2} - z^2\right)^{1/2}}{2} + \frac{r_\varepsilon^{\ 2}}{2}\sin^{-1}\left(\frac{z}{r_\varepsilon}\right)\right].$$

The integral from z_o to $f/2$ is identical in form to the one just considered, but opposite in sign. So, the same indefinite integral result can be used with a flip in sign. Therefore, the result for r_a is

$$r_a = \frac{2}{f}\left[\left(r_{cl} - r_\varepsilon\right) z + \frac{z\left(r_\varepsilon^{\ 2} - z^2\right)^{1/2}}{2} + \frac{r_\varepsilon^{\ 2}}{2}\sin^{-1}\left(\frac{z}{r_\varepsilon}\right)\right]_0^{z_o} - \frac{2}{f}\left[\left(r_{cl} - r_\varepsilon\right) z + \frac{z\left(r_\varepsilon^{\ 2} - z^2\right)^{1/2}}{2} + \frac{r_\varepsilon^{\ 2}}{2}\sin^{-1}\left(\frac{z}{r_\varepsilon}\right)\right]_{z_o}^{f/2}$$

Expanding by substitution of the limits, and noting that the zero limit results in zero, this yields

$$r_a = \frac{2}{f}\left\{(r_{cl}-r_\varepsilon)\left(2z_o-\frac{f}{2}\right)+\frac{2z_o\left(r_\varepsilon^2-z_o^2\right)^{1/2}-\frac{f}{2}\left(r_\varepsilon^2-\frac{f^2}{4}\right)^{1/2}}{2}+\frac{r_\varepsilon^2}{2}\left[2\sin^{-1}\left(\frac{z_o}{r_\varepsilon}\right)-\sin^{-1}\left(\frac{f}{2r_\varepsilon}\right)\right]\right\}.$$

Since the original integrals involve the integral of $r(z)$, which is the same integrand that leads to r_{cl}, it seems reasonable to expect that some terms can be combined and replaced by r_{cl}. The boxed terms in the expression

$$r_a = \frac{2}{f}\left\{r_{cl}\left(2z_o-\frac{f}{2}\right)-2r_\varepsilon z_o\boxed{+\frac{r_\varepsilon f}{2}}+\frac{2z_o\left(r_\varepsilon^2-z_o^2\right)^{1/2}}{2}\boxed{-\frac{f}{4}\left(r_\varepsilon^2-\frac{f^2}{4}\right)^{1/2}}+2\frac{r_\varepsilon^2}{2}\sin^{-1}\left(\frac{z_o}{r_\varepsilon}\right)\boxed{-\frac{r_\varepsilon^2}{2}\sin^{-1}\left(\frac{f}{2r_\varepsilon}\right)}\right\}$$

do combine to

$$\frac{r_\varepsilon f}{2}-\frac{f}{4}\left(r_\varepsilon^2-\frac{f^2}{4}\right)^{1/2}-\frac{r_\varepsilon^2}{2}\sin^{-1}\left(\frac{f}{2r_\varepsilon}\right)=\frac{f}{2}r_{cl}.$$

Making this replacement allows the following cancellation:

$$r_a = \frac{2}{f}\left\{r_{cl}\left(2z_o-\cancel{\frac{f}{2}}\right)-2r_\varepsilon z_o+\frac{2z_o\left(r_\varepsilon^2-z_o^2\right)^{1/2}}{2}+2\frac{r_\varepsilon^2}{2}\sin^{-1}\left(\frac{z_o}{r_\varepsilon}\right)+\cancel{\frac{f}{2}r_{cl}}\right\}.$$

Simplifying yields

$$r_a = \frac{2}{f}\left\{\left[2(r_{cl}-r_\varepsilon)+\left(r_\varepsilon^2-z_o^2\right)^{1/2}\right]z_o+r_\varepsilon^2\sin^{-1}\left(\frac{z_o}{r_\varepsilon}\right)\right\}.$$

Substituting for z_o,

$$r_a = \frac{2}{f}\left\{\left[2(r_{cl}-r_\varepsilon)+\left(r_\varepsilon^2-\left(2r_\varepsilon r_{cl}-r_{cl}^2\right)\right)^{1/2}\right]\left(2r_\varepsilon r_{cl}-r_{cl}^2\right)^{1/2}+r_\varepsilon^2\sin^{-1}\left(\frac{\left(2r_\varepsilon r_{cl}-r_{cl}^2\right)^{1/2}}{r_\varepsilon}\right)\right\}$$

and noting that

$$r_\varepsilon^2-\left(2r_\varepsilon r_{cl}-r_{cl}^2\right)=r_\varepsilon^2-2r_\varepsilon r_{cl}+r_{cl}^2=\left(r_\varepsilon-r_{cl}\right)^2 \quad \text{and} \quad \left(\left(r_\varepsilon-r_{cl}\right)^2\right)^{1/2}=r_\varepsilon-r_{cl},$$

the final result (in the same length units as f, r_ε and r_{cl}) is

$$\boxed{r_a = \frac{2}{f}\left\{\left(r_{cl}-r_\varepsilon\right)\left(2r_\varepsilon r_{cl}-r_{cl}^2\right)^{1/2}+r_\varepsilon^2\sin^{-1}\left(\frac{\left(2r_\varepsilon r_{cl}-r_{cl}^2\right)^{1/2}}{r_\varepsilon}\right)\right\}}.$$