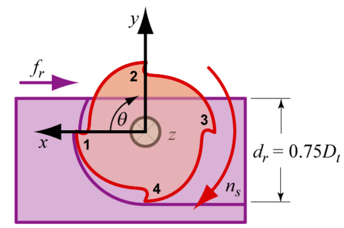
4.2

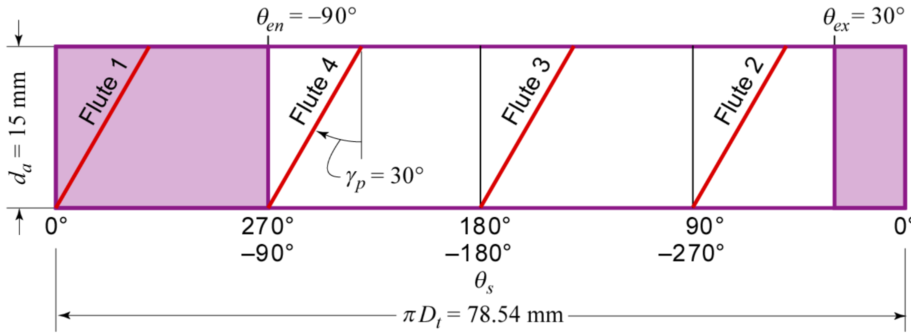
Given:

- end milling
- number of flutes/teeth, $N_t = 4$
- back rake angle, $\gamma_p = 30°$
- side rake angle, $\gamma_f = 0$
- tool diameter, $D_t = 25$ mm
- feed rate, $f_r = 0.8$ mm/rev
- spindle speed, n_s =600 rpm
- axial depth of cut, $d_a = 15$ mm
- radial depth of cut, $d_r = 0.75D_t$
- up milling

a) Referring to the figure to the right, this being up milling, by inspection $\theta_{en} = -90°$. The tooth leaves at a distance d_r in the positive y direction from the entry position, which results in one-half the radius of the cutter above the cutter center (the x axis). Therefore, $\theta_{ex} = \sin^{-1}(0.5) = 30°$.



b) The flute-engagement diagram is a rectangle of height equal to the axial depth of cut and width equal to the circumference of the tool (πD_t), which is typically labeled in terms of spindle angle rather than circumferential arc length. The result is shown below.



c) To compute the maximum wrap angle, the flute should be positioned with its end at θ_{ex}. The flute then wraps backward (relative to the rotation direction, i.e., backward toward θ_{en}). The flute maintains engagement until the flute loses engagement angularly at θ_{en} or axially at d_a up the cutter, whichever comes first. To determine which comes first, the wrap angle to the point of axial loss of engagement and compare to $\theta_{ex} - \theta_{en}$. The wrap angle at which axial loss of engagement occurs is found from

$$\frac{R_t \Delta\theta}{d_a} = \tan\gamma_p \quad \rightarrow \quad \Delta\theta = \frac{d_a}{R_t}\tan\gamma_p \, .$$

Substituting known values, this angle (in deg) is

$$\Delta\theta = \frac{15}{12.5}\tan(30°) = 0.692 \text{ rad} = 39.7° \, .$$

Since this is less than $\theta_{ex} - \theta_{en} = 30° - (-90°) = 120°$, the maximum engaged wrap angle is 39.7°. The axial loss of contact occurring before circumferential loss of contact is also observable in the flute-engagement diagram above, and observation that can be made since it is drawn to scale.

d) At 100% immersion (slotting), any axial slice of the tool looks like a face milling cut. For face milling when $D_t/W_w \leq 1$ and no size effect, the variation in the x and y forces is zero meaning that the "peak" forces are the same as the mean forces. The x and y forces as a function of tooth angle θ_i are

$$F_{x_i} = F_{Tan_i}\cos\theta_i - F_{Rad_i}\sin\theta_i \quad \text{and} \quad F_{y_i} = -F_{Tan_i}\sin\theta_i - F_{Rad_i}\cos\theta_i \, .$$

Since these are known to be constant with no size effect, they can be evaluated at any angle; choosing $\theta_i = 0$ is the easiest since cos(0) = 1, sin(0) = 0, and only one tooth is cutting. The result is $F_x = -F_{Rad}$ and

$F_y = -F_{Tan}$. For the zero lead angle of a straight end mill, $F_{Rad} = F_T$ and $F_{Tan} = F_C$. Since each axial slice exhibits the same and constant forces, F_C and F_T can be computed as

$$F_C = u_C f_t \cos(0) d_a = u_C \frac{f_r}{N_t} d_a .$$

Substituting known values, the final result (in N) is

$$F_C = 1500 \frac{0.80}{4}(15) = \boxed{4500} \quad \text{and} \quad F_T = 1000 \frac{0.80}{4}(15) = \boxed{3000} .$$

e) For anything less than 100% immersion, each cycle of tooth entry-exit creates an identical peaked force signature. As such, the dominant frequency is the tooth frequency (in rad/sec) $\omega_s N_t$, where ωs = $2\pi n_s/60$ is the spindle frequency, n_s being the spindle speed in rpm. For 100% immersion, as noted above the force are constant, meaning that the dominant frequency is simply zero. Therefore, the final result is 600(4)/60 = 40 Hz at 75% immersion, and 0 Hz at 100% immersion.

4.3

Given:	
• drilling • number of flutes/teeth, $N_t = 2$	• web thickness, $2b_w = 1$ mm • chisel edge angle, $\varphi = 135°$

a) By numerically integrating $\gamma_n(r)$ from the web to the outer diameter (where $r = R_t$), then dividing by the integration range ($R_t - r$(at web)), calculate an average normal rake angle $\bar{\gamma}_n$ (in degrees). The numerical integration goes from $r_1 = b_w/\sin\varphi$ to $r_2 = R_t$. The results are shown below.

rho r/Rt	beta_w 0	back rake angle	side rake angle	cut edge angle	inclin. angle	orth. rake angle	norm rake angle	integ elem area
0.141	45.00	-42.94	4.67	49.64	-37.31	-28.39	-23.26	-0.35
0.159	39.09	-36.54	5.23	52.25	-32.70	-20.87	-17.78	-0.27
0.176	34.68	-31.65	5.79	53.85	-29.15	-15.74	-13.83	-0.21
0.193	31.22	-27.74	6.36	54.91	-26.31	-11.93	-10.72	-0.16
0.210	28.42	-24.50	6.92	55.66	-23.98	-8.92	-8.16	-0.12
0.227	26.10	-21.75	7.48	56.21	-22.02	-6.43	-5.97	-0.09
0.244	24.15	-19.35	8.03	56.64	-20.35	-4.31	-4.04	-0.05
0.262	22.47	-17.24	8.59	56.97	-18.91	-2.44	-2.30	-0.03
0.279	21.02	-15.35	9.14	57.23	-17.64	-0.76	-0.72	0.00
0.296	19.75	-13.64	9.70	57.45	-16.52	0.77	0.74	0.02
0.313	18.62	-12.08	10.25	57.62	-15.51	2.18	2.10	0.05
0.330	17.62	-10.63	10.80	57.77	-14.60	3.50	3.39	0.07
0.347	16.73	-9.29	11.34	57.90	-13.78	4.75	4.61	0.09
0.365	15.92	-8.03	11.89	58.00	-13.02	5.93	5.77	0.11
0.382	15.18	-6.84	12.43	58.09	-12.32	7.05	6.89	0.13
0.399	14.51	-5.72	12.97	58.17	-11.67	8.13	7.97	0.15
0.416	13.90	-4.65	13.51	58.24	-11.07	9.18	9.01	0.16
0.433	13.34	-3.62	14.05	58.30	-10.50	10.18	10.02	0.18
0.451	12.82	-2.63	14.58	58.36	-9.96	11.16	11.00	0.20
0.468	12.35	-1.68	15.11	58.40	-9.45	12.11	11.95	0.21
0.485	11.90	-0.76	15.64	58.45	-8.96	13.04	12.89	0.23
0.502	11.49	0.14	16.16	58.49	-8.50	13.95	13.80	0.24
0.519	11.10	1.01	16.69	58.52	-8.05	14.83	14.69	0.26
0.536	10.75	1.86	17.21	58.55	-7.62	15.70	15.57	0.27
0.554	10.41	2.70	17.72	58.58	-7.20	16.56	16.43	0.29
0.571	10.09	3.52	18.24	58.60	-6.80	17.40	17.28	0.30
0.588	9.79	4.32	18.75	58.63	-6.40	18.22	18.11	0.32
0.605	9.51	5.12	19.26	58.65	-6.01	19.03	18.93	0.33
0.622	9.25	5.90	19.76	58.67	-5.63	19.83	19.74	0.35
0.639	9.00	6.67	20.26	58.69	-5.25	20.62	20.54	0.36

0.657	8.76	7.44	20.76	58.70	-4.88	21.39	21.32	0.37
0.674	8.54	8.20	21.26	58.72	-4.51	22.16	22.10	0.39
0.691	8.32	8.95	21.75	58.73	-4.14	22.91	22.86	0.40
0.708	8.12	9.69	22.24	58.74	-3.78	23.66	23.61	0.41
0.725	7.93	10.44	22.72	58.76	-3.42	24.40	24.36	0.42
0.742	7.74	11.17	23.20	58.77	-3.05	25.12	25.09	0.44
0.760	7.56	11.91	23.68	58.78	-2.69	25.84	25.82	0.45
0.777	7.40	12.64	24.15	58.79	-2.33	26.55	26.54	0.46
0.794	7.24	13.37	24.63	58.80	-1.96	27.26	27.24	0.47
0.811	7.08	14.10	25.09	58.81	-1.59	27.95	27.94	0.49
0.828	6.93	14.82	25.56	58.81	-1.22	28.64	28.63	0.50
0.845	6.79	15.55	26.02	58.82	-0.84	29.32	29.32	0.51
0.863	6.66	16.27	26.48	58.83	-0.46	29.99	29.99	0.52
0.880	6.53	16.99	26.93	58.84	-0.08	30.66	30.66	0.53
0.897	6.40	17.71	27.38	58.84	0.31	31.32	31.32	0.54
0.914	6.28	18.44	27.82	58.85	0.70	31.97	31.97	0.55
0.931	6.16	19.16	28.27	58.85	1.10	32.62	32.61	0.57
0.948	6.05	19.88	28.71	58.86	1.51	33.25	33.24	0.58
0.966	5.94	20.61	29.14	58.86	1.92	33.89	33.87	0.59
0.983	5.84	21.33	29.57	58.87	2.34	34.51	34.49	0.60
1.000	5.74	22.05	30.00	58.87	2.77	35.13	35.10	
							integral =	12.824
							avg. norm. rake angle =	14.936

b) Interpolating, either mathematically or visually, the average normal rake angle is 14.94° and occurs at $\bar{r}$ /Rt = 0.527. Therefore, the final result is (in mm) $\bar{r}$ = 2.63.

c) Average inclination angle from the table above is –7.8°.