3.5

Given:	
• boring • number of teeth, $N_t = 2$ • corner radius, $r_\varepsilon = 0$ • lead angle, $\psi_r = 15°$ • back rake angle, $\gamma_p = 0$ • side rake angle, $\gamma_f = 0$	• feed rate, $f_r = 0.40$ mm/rev • spindle speed, n_s =600 rpm • initial workpiece diameter, $D_w = 96$ mm • tool diameter, $D_t = 100$ mm • spindle-work axis offset (mag), $\varepsilon_{wo} = 1.0$ mm • spindle-work axis offset (dir), $\delta_{wo} = 210°$ • specific cutting and thrust energy models

a) The axis offset causes the depth of cut to vary with tooth angle about the nominal depth of cut. The nominal depth of cut is

$$d_{nom} = \frac{D_w - D_t}{2}.$$

Substituting known values, the nominal depth of cut (in mm) is

$$d_{nom} = \frac{100-96}{2} = 2.$$

The minimum and maximum are $d_{nom} \mp \varepsilon_{wo}$ and occur at δ_{wo} and $\delta_{wo} - 180°$, respectively. Substituting known values, the final result (in mm) is

$$d_{min} = d_{nom} - \varepsilon_{wo} = 2-1 = \boxed{1 \text{ at } \theta_{min_d} = 210°} \quad \text{and} \quad d_{max} = d_{nom} + \varepsilon_{wo} = 2+1 = \boxed{\text{ at } \theta_{max_d} = 30°}.$$

b) The minimum and maximum tooth-local forces (cutting and thrust) occur at d_{min} and d_{max}, respectively. The cutting force acting on a tooth at angle θ_i is

$$F_C(\theta_i) = u_C(\theta_i) f_t d(\theta_i).$$

The feed per tooth (in mm/t) is the feed per revolution (in mm/rev) divided by the number of teeth:

$$f_t = \frac{f_r}{N_t} = \frac{0.4}{2} = 0.2.$$

The specific energy requires the uncut chip thickness and normal rake angle. The normal rake angle for this “neutral rake” case ($\gamma_p = \gamma_f = 0$) is, by inspection, zero. Since the corner radius is zero, the uncut chip thickness is simply

$$h = f_t \cos\psi_r.$$

Substituting known values, the specific cutting energy (in N/mm^2) is

$$u_C(\theta_i) = u_C = 2000\left(0.2\cos(15°)\right)^{-0.25} e^{-1.50(0)} = 3018.$$

Therefore, the final result (in N) is

$$F_C\left(\theta_{min_C}\right) = u_C f_t d\left(\theta_{min_d}\right) = 3018(0.2)(1) = \boxed{603.6 \text{ at } \theta_{min_C} = \theta_{min_d} = 210°}$$

and

$$F_C\left(\theta_{max_C}\right) = u_C f_t d\left(\theta_{max_d}\right) = 3018(0.2)(3) = \boxed{1811 \text{ at } \theta_{max_C} = \theta_{max_d} = 30°}.$$

c) Based on the geometry of two offset circles, show that the exact solution for the depth of cut as a function of tooth angle θ_i is The tool and workpiece are represented by circles with the tool being centered on the x-y coordinate frame and the workpiece centered at coordinates $(x_{wc}, y_{wc}) = (\varepsilon_{wo}\cos\delta_{wo}, \varepsilon_{wo}\sin\delta_{wo})$. Given the distance from the origin to the tip of a tooth, $r_t(\theta)$, and the distance from the origin to the workpiece circle at that same angle $r_w(\theta)$, the depth of cut is

$$d(\theta_i) = r_t(\theta) - r_w(\theta).$$

For the case of work-spindle offset, $r_t(\theta) = R_t$ for all θ (actually only relevant at each tooth). The workpiece counterpart ($r_w(\theta)$) is found using the equation of the workpiece circle

$$(x_w - x_{wc})^2 + (y_w - y_{wc})^2 = R_w \,,$$

where (x_w, y_w) is a point on the workpiece circle. Expanding yields

$$\left(x_w{}^2 - 2x_{wc}x_w + x_{wc}{}^2\right) + \left(y_w{}^2 - 2y_{wc}y_w + y_{wc}{}^2\right) = R_w{}^2\,;$$

rearranging yields

$$\left(x_w{}^2 + y_w{}^2\right) - 2(x_{wc}x_w + y_{wc}y_w) + \left(x_{wc}{}^2 + y_{wc}{}^2\right) - R_w{}^2 = 0\,.$$

It is also known that all points on the workpiece circle satisfy

$$x_w = r_w\cos\theta, \;\; y_w = r_w\sin\theta, \;\text{ and }\; x_w{}^2 + y_w{}^2 = r_w{}^2\,,$$

where r_w is shorthand for $r_w(\theta)$. Substituting for x_w, y_w and $x_w{}^2 + y_w{}^2$ yields

$$r_w{}^2 - 2r_w\left(x_{wc}\cos\theta + y_{wc}\sin\theta\right) + \left(x_{wc}{}^2 + y_{wc}{}^2\right) - R_w{}^2 = 0\,.$$

Furthermore, $x_{wo}{}^2 + y_{wo}{}^2 = \varepsilon_{wo}{}^2$, so this becomes

$$r_w{}^2 - 2r_w\left(x_{wc}\cos\theta + y_{wc}\sin\theta\right) + \varepsilon_{wo}{}^2 - R_w{}^2 = 0\,.$$

This is a quadratic in r_w, the solution of which is

$$\begin{aligned} r_w &= \frac{2\left(x_{wc}\cos\theta + y_{wc}\sin\theta\right) \pm \left[4\left(x_{wc}\cos\theta + y_{wc}\sin\theta\right)^2 - 4\left(\varepsilon_{wo}{}^2 - R_w{}^2\right)\right]^{1/2}}{2} \\ &= \left(x_{wc}\cos\theta + y_{wc}\sin\theta\right) \pm \left[\left(x_{wc}\cos\theta + y_{wc}\sin\theta\right)^2 - \left(\varepsilon_{wo}{}^2 - R_w{}^2\right)\right]^{1/2}. \end{aligned}$$

To get this in terms of only the runout parameter ε_{wo} and δ_{wo}, rather than x_{wc} and y_{wc}, the substitution

$$x_{wc} = \varepsilon_{wo}\cos\delta_{wo} \qquad \text{and} \qquad y_{wc} = \varepsilon_{wo}\sin\delta_{wo}$$

is made. This results in the presence of the term $\varepsilon_{wo}(\cos\delta_{wo}\cos\theta + \sin\delta_{wo}\sin\theta)$, which can be replaced by the trigonometric identity

$$\varepsilon_{wo}\left[\cos\delta_{wo}\cos\theta + \sin\delta_{wo}\sin\theta\right] = \varepsilon_{wo}\cos\left(\delta_{wo} - \theta\right) = \varepsilon_{wo}\cos\left(\theta - \delta_{wo}\right).$$

After simplifying, the result is

$$\begin{aligned} r_w &= \varepsilon_{wo}\cos\left(\delta_{wo} - \theta\right) \pm \left[\varepsilon_{wo}{}^2\cos^2\left(\delta_{wo} - \theta\right) - \left(\varepsilon_{wo}{}^2 - R_w{}^2\right)\right]^{1/2} \\ &= \varepsilon_{wo}\cos\left(\delta_{wo} - \theta\right) \pm \left[\varepsilon_{wo}{}^2\left(\cos^2\left(\delta_{wo} - \theta\right) - 1\right) + R_w{}^2\right]^{1/2} \end{aligned}$$

*Making the trigonometric substitution

$$\cos^2\left(\delta_{wo} - \theta\right) - 1 = -\sin^2\left(\delta_{wo} - \theta\right)$$

results in

$$r_w = \varepsilon_{wo}\cos\left(\delta_{wo} - \theta\right) \pm \left[R_w{}^2 - \varepsilon_{wo}{}^2\sin^2\left(\delta_{wo} - \theta\right)\right]^{1/2}.$$

Factoring out ε_{wo}, replacing θ with θ_i for tooth i, using the '+' of the '±',and subtracting from R_t yields the depth of cut result being sought:

$$d(\theta_i) = R_t - r_w = R_t - \varepsilon_{wo}\left\{\cos\left(\delta_{wo} - \theta_i\right) + \left[\left(R_w/\varepsilon_{wo}\right)^2 - \sin^2\left(\delta_{wo} - \theta_i\right)\right]^{1/2}\right\}.$$

* Continue from here in alternate solution below.

The form of this result highlights large terms, in particular $(R_w/\varepsilon_{wo})^2$. Finding the "small" terms to ignore is facilitated by dividing the term in braces by R_w/ε_{wo} to obtain

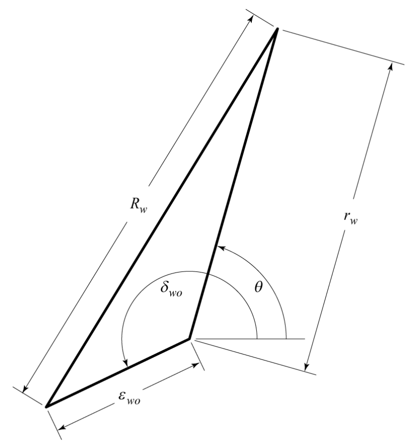
$$\boxed{d(\theta_i) = R_t - R_w \left\{ (\varepsilon_{wo}/R_w)\cos(\delta_{wo}-\theta_i) + \left[1-(\varepsilon_{wo}/R_w)^2 \sin^2(\delta_{wo}-\theta_i)\right]^{1/2} \right\}} .$$

While the term ε_{wo}/R_w is small, setting it to zero would trivialize the solution to $d(\theta_i) = R_t - R_w$. This is not sensible since runout is a small-term phenomena. However, if ε_{wo}/R_w is small, then $(R_w/\varepsilon_{wo})^2$ is very small, and that is what makes sense to ignore for simplification purposes. Doing so results in

$$d(\theta_i) = R_t - R_w - \varepsilon_{wo}\cos(\delta_{wo}-\theta_i) = \boxed{d_{nom} - \varepsilon_{wo}\cos(\delta_{wo}-\theta_i)} .$$

Another approach to the problem is to employ the Law of Cosines to the triangle shown to the right. The Law of Cosines yields

$$r_w^{\,2} + \varepsilon_{wo}^{\,2} - 2 r_w \varepsilon_{wo}\cos(\delta_{wo}-\theta) = R_w^{\,2} .$$



Bringing R_w^2 to the left-hand side provides a quadratic in r_w, the solution of which is

$$\begin{aligned} r_w &= \frac{2\varepsilon_{wo}\cos(\delta_{wo}-\theta) \pm \left[4\varepsilon_{wo}^{\,2}\cos^2(\delta_{wo}-\theta) - 4\left(\varepsilon_{wo}^{\,2} - R_w^{\,2}\right)\right]^{1/2}}{2} \\ &= \varepsilon_{wo}\cos(\delta_{wo}-\theta) \pm \left[\varepsilon_{wo}^{\,2}\cos^2(\delta_{wo}-\theta) - \left(\varepsilon_{wo}^{\,2} - R_w^{\,2}\right)\right]^{1/2} \\ &= \varepsilon_{wo}\cos(\delta_{wo}-\theta) \pm \left[\varepsilon_{wo}^{\,2}\left(\cos^2(\delta_{wo}-\theta) - 1\right) + R_w^{\,2}\right]^{1/2} . \end{aligned}$$

Continue as above from the footnoted point. The Law of Cosines allows skipping the steps involved with representing the center of the workpiece circle in Cartesian coordinates.

d) Graph the x- and y-force components F_x and F_y (in N) as functions of spindle angle θ_s where, by convention, the angle of tooth one is $\theta_1 = \theta_s$.The cutting and thrust force components are

,

where u_C was found earlier to be 3018 N/mm^2 and u_T (in N/mm^2) is

$$u_T = 750\left(0.2\cos(15°)\right)^{-0.5} \mathrm{e}^{-1.25(0)} = 1707 .$$

The x- and y-force components require the tangential and radial force components, which are

$$F_{Tan}(\theta_i) = F_C(\theta_i) = u_C f_t d(\theta_i) \quad \text{and} \quad F_{Rad}(\theta_i) = F_T(\theta_i)\sin\left(\bar{\psi}(\theta_i)\right) = u_T f_t d(\theta_i)\cos\left(\bar{\psi}(\theta_i)\right),$$

where the equivalent lead angle, for this case of zero corner radius, simply equals the lead angle ψ_r. Substituting known values, the result (in N for d(θ_i) in mm) is

$$F_{Tan}(\theta_i) = 3018(0.2)d(\theta_i) = 603.6 d(\theta_i) \quad \text{and} \quad F_{Rad}(\theta_i) = 1707(0.2)d(\theta_i)\sin(15°) = 88.4 d(\theta_i) .$$

The equations for the x- and y-force components are then

$$F_x(\theta_i) = F_{Tan}(\theta_i)\sin\theta_i - F_{Rad}(\theta_i)\cos\theta_i \quad \text{and} \quad F_y(\theta_i) = -F_{Tan}(\theta_i)\cos\theta_i - F_{Rad}(\theta_i)\sin\theta_i ,$$

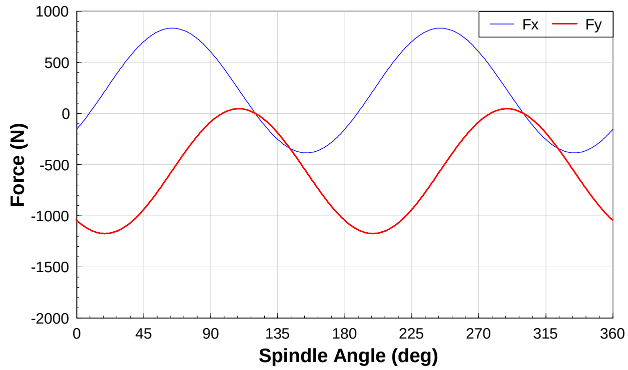
or

$$F_x(\theta_i) = \left(603.6\sin\theta_i - 88.4\cos\theta_i\right)d(\theta_i) \quad \text{and} \quad F_y(\theta_i) = \left(-603.6\cos\theta_i - 88.4\sin\theta_i\right)d(\theta_i) .$$

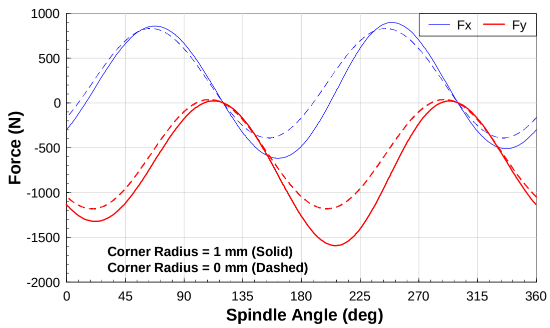
The total forces on the cutter are the sum of those acting on the two teeth, i.e.,

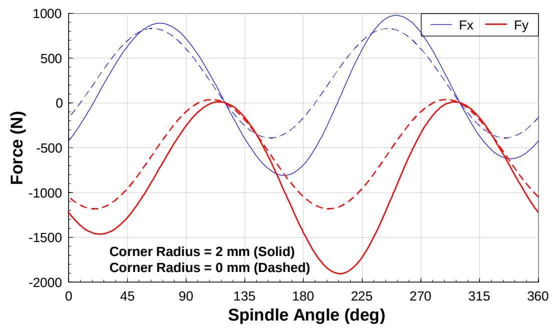
$$F_x = F_{x_1} + F_{x_2} \quad \text{and} \quad F_y = F_{y_1} + F_{y_2}, \quad \text{where} \quad \theta_1 = \theta_s \ \text{and} \ \theta_1 = \theta_s + 180° .$$

Implementing these in a spreadsheet along with the equation for $d(\theta_i)$ and graphing yields the result below.



e) As the corner radius is increased (e.g., from the assumed value of zero above), r_e/d increases for any depth of cut. Likewise, equivalent lead angle increases and average uncut chip thickness decreases. Decreasing average uncut chip thickness, through the size effect, causes all the forces to increase. Increasing equivalent lead angle causes the depth-direction force (F_{Rad} here) to increase. Therefore, since all effects are to increase the radial force, the radial force will increase. However, the tangential force also increases from the change in average uncut chip thickness. As a result, it is difficult to say the what the net effect will be on F_x and F_y given the angle dependence on how these changes in F_{Rad} and F_{Tan}, not to mention one two teeth, will affect the total forces. The plots below provide a graphical assessment of the effect.





f) Even with the runout, since it is <u>not</u> so large that the depth of cut becomes zero at some angles, the machined diameter is simply that of the tool, or 100 mm.

4.1

Given:	
• face milling • number of teeth, $N_t = 8$ • corner radius, $r_\varepsilon = 1.0$mm • lead angle, $\psi_r = 0°$ • back rake angle, $\gamma_p = 0$ • side rake angle, $\gamma_f = 0$ • tool diameter, $D_t = 700$ mm	• feed rate, $f_r = 1.6$ mm/rev • depth of cut, $d = 1.0$ mm • spindle speed, n_s =600 rpm • workpiece width, $W_w = 400$ mm • workpiece offset in y-direction, $\varepsilon = 75$ mm • specific cutting and thrust energy models

a) The angle associated with a point lying on the circle representing the tool is

$$\theta_\bullet = \sin^{-1}\left(\frac{y_\bullet}{R_t}\right), \bullet = en, ex\,,$$

The y coordinate corresponding to the entry and exit points is

$$y_\bullet = \mp\left(\frac{W_w}{2} \mp \varepsilon\right),$$

where the top sign (–) is for tooth entry and the bottom sign (+) is for exit. Therefore, substituting known values, the results (in mm) are

$$y_{en} = -\left(\frac{400}{2} - 75\right) = -125 \text{ and } y_{ex} = \left(\frac{400}{2} + 75\right) = 275\,.$$

Calculating the associated angles yields the final result (in deg) to be

$$\theta_{en} = \sin^{-1}\left(\frac{-125}{700/2}\right) = \boxed{-21°} \text{ and } \theta_{ex} = \sin^{-1}\left(\frac{275}{700/2}\right) = \boxed{52°}\,.$$

b) The tooth spacing $\Delta\theta_t$ is simply 360° divided by the number of teeth, or 360°/8 = 45°. If the total angle of engagement, $\theta_{ex} - \theta_{en}$, is less than the tooth spacing, there will never be more than one tooth engaged at any point in time. Furthermore, if the angle of engagement falls between the tooth spacing and two times then tooth spacing, then at all times there will be either one or two teeth engaged. This caries on to the general relation that the minimum and maximum number of teeth engaged at any time is

$$N_{e_{max}} = \text{ceil}\left(\frac{\theta_{ex} - \theta_{en}}{\Delta\theta_t}\right) \quad \text{and} \quad N_{e_{min}} = \text{floor}\left(\frac{\theta_{ex} - \theta_{en}}{\Delta\theta_t}\right),$$

where the "ceil" and "floor" functions round the argument up and down, respectively, to an integer (note that when the argument is an integer, the ciel and floor functions return the same value, the integer argument). Substituting know values,

$$\frac{52° - (-21°)}{45°} = 1.62 \rightarrow N_{e_{max}} = \text{ceil}(1.62) = 2 \quad \text{and} \quad N_{e_{min}} = \text{floor}(1.62) = 1\,.$$

Therefore, single-tooth cutting does occur in this case for some period of time, i.e., the answer is not zero.

When tooth i is entering the cut, its angle is $\theta_i = \theta_{en} = -21°$ and the angle of the tooth it is following, tooth i+1, is $\theta_{i+1} = \theta_{en} + 45° = 24°$. When tooth i+1 reaches the exit point at $\theta_{i+1} = \theta_{ex} = 52°$, $\theta_i = \theta_{ex} - 45° = 7°$. Tooth i will then cut by itself from 7° until the tooth before it (tooth i–1) enters the cut, at which point $\theta_i = 24°$. Therefore, the range over which a single tooth (tooth i this illustration) is cutting is from 7° to 24° or 17°.

c) For a corner-radiused tool as is the case here, the average uncut chip thickness is the chip area divided by the equivalent width of cut, which is the length of the cutting tooth profile that is engaged with the workpiece. The chip area as a function of angle is

$$a(\theta_i) = (f_t \cos\theta_i)d \ .$$

The width of cut is simplified in this case since the corner radius equals the depth of cut and the lead angle is zero, so that by inspection the depth of cut equals the transition depth of cut. Therefore, noting that the feed per tooth is $f_t = f_r/N_t$,

$$w(\theta_i) = \left[\frac{\pi}{2} - \psi_r + \sin^{-1}\left(\frac{f_t \cos\theta_i}{2r_\varepsilon}\right)\right] r_\varepsilon \ .$$

Substituting known values, the result (in mm) as a function of angle is

$$w(\theta_i) = \left[\frac{\pi}{2} - 0 + \sin^{-1}\left(\frac{(1.6/8)\cos\theta_i}{2(1.0)}\right)\right](1.0) = \left[\frac{\pi}{2} + \sin^{-1}\left((0.1)\cos\theta_i\right)\right].$$

Substituting the values of θ_{en} and θ_{ex} from above, the final results are

$$\bar{h}_{en} = \frac{\left((1.6/8)\cos(-21°)\right)(1.0)}{\frac{\pi}{2} + \sin^{-1}\left((0.1)\cos(-21°)\right)} = \boxed{0.112}$$

and

$$\bar{h}_{ex} = \frac{\left((1.6/8)\cos(52°)\right)(1.0)}{\frac{\pi}{2} + \sin^{-1}\left((0.1)\cos(52°)\right)} = \boxed{0.120} \ .$$

d) Fu’s method is equivalent to connecting the tip of the tool (not the intersection of the two profiles at the cusp as for Colwell’s method) to the intersection of the tooth profile with the un-machined surface. By inspection, for this special case as noted above, the equivalent lead angle will be 45°. As such, $F_{Rad}(\theta_i) = F_{Lon}(\theta_i)$.