3.3

Given:	
• turning • corner radius, r_ε = 0 mm • lead angle, ψ_r = 15° • back rake angle, γ_p = –5° • side rake angle, γ_f = –5°	• initial workpiece diameter, D_w = 100 mm • specific cutting and thrust energy models • feed rate, f_r = 0.20 mm/rev • depth of cut, d = 0.75 mm • spindle speed, n_s =1000 rpm

a) The chip area is approximated as the product of the feed and depth, ignoring the actual absence of the triangular cusp left on the machined surface. Substituting known values, the final result (in mm^2) is

$$a = (0.020)(0.75) = \boxed{0.15}\ .$$

The material removal rate, by definition, is the product of the chip area and the cutting speed. The cutting speed is the product of the spindle speed and the workpiece circumference. The average cutting speed (averaged across the tooth profile contact), (in m/min) is

$$\bar{V} = n_s\left[\pi\left(D_w - d\right)\right] = 1000\left[\pi(100 - 0.75)\right] = 311.8\ .$$

Of course, since the depth of cut is very small compared to the workpiece diameter, the surface speed at the outer diameter (in m/min),

$$V \approx n_s\left[\pi D_w\right] = 1000\left[\pi(100)\right] = 314.2\ ,$$

could just as soon be used as a good approximation.

Converting cutting speed from 311.8 m/min to 311,800 mm/min, the final result (in mm^3/min) is

$$\dot{v}_r = (0.15)(311{,}800) = \boxed{46{,}770} \quad \approx \quad (0.15)(314{,}200) = 47{,}130\ .$$

b) The cutting and thrust forces are each the product of the respective specific energy and the chip area found above. Computing the specific energies from the empirical models given requires uncut chip thickness, cutting speed (found above for material removal rate calculation) and normal rake angle.

The uncut chip thickness for a zero corner radius is

$$h = f\cos\psi_r,\ f = f_r \cdot 1\text{rev}\ .$$

Substituting known values, the result (in mm) is

$$h = 0.15\cos(15°) = 0.1932\ .$$

The normal rake angle is

$$\gamma_n = \tan^{-1}\left[\tan\gamma_o\cos\lambda\right],$$

where the orthogonal rake angle is

$$\gamma_o = \tan^{-1}\left[\tan\gamma_f\cos\psi_r + \tan\gamma_p\sin\psi_r\right]$$

and the inclination angle is

$$\lambda = \tan^{-1}\left[\tan\gamma_p\cos\psi_r - \tan\gamma_f\sin\psi_r\right].$$

Substituting known values, the inclination angle is

$$\lambda = \tan^{-1}\left[\tan(-5°)\cos(15°) - \tan(-5°)\sin(15°)\right] = -3.54°$$

and, subsequently, the orthogonal rake angle is

$$\gamma_o = \tan^{-1}\left[\tan(-5°)\cos(15°) + \tan(-5°)\sin(15°)\right] = -6.12\ .$$

Therefore, the normal rake angle is

$$\gamma_n = \tan^{-1}\left[\tan(-6.12°)\cos(-3.54°)\right] = -6.11°\ .$$

Substituting known values (h in mm, V in m/min and γ_n in radians (–0.1066 rad)) into the specific energy models, the specific energies (in N/mm^2) are

$$u_C = 3150\,(0.1932)^{-0.233}\,(311.8)^{-0.122}\,e^{-1.318(-0.1066)} = 2638$$

and

$$u_T = 1175\,(0.1932)^{-0.615}\,(311.8)^{-0.148}\,e^{-1.115(-0.1066)} = 1555\,.$$

Using these and the chip area computed earlier, the final results (in N) are

$$F_C = u_C a = (2638)(0.15) = \boxed{396} \quad \text{and} \quad F_T = u_T a = (1555)(0.15) = \boxed{233}\,.$$

c) The transformations from the edge-local cutting and thrust forces to the tooth-local tangential, longitudinal and radial are given in the text as

$$F_{Tan} = F_C\,, \quad F_{Lon} = F_T \cos\psi_r \quad \text{and} \quad F_{Rad} = F_T \sin\psi_r\,.$$

Substituting known values, the final results (in N) are

$$F_{Tan} = \boxed{396}\,, \quad F_{Lon} = 233\cos(15°) = \boxed{225} \quad \text{and} \quad F_{Rad} = 233\sin(15°) = \boxed{60.3}\,.$$

d) Introducing a non-zero corner radius would decrease the uncut chip thickness used in the specific energy models, which would increase the specific energies, more so in the thrust direction. A non-zero corner radius would also dramatically change how the thrust force is oriented between the radial and longitudinal directions. Therefore, introducing a non-zero corner radius would be expected to affect the force components would be affected in the following ways:

Force Component	**Magnitude**	**Direction**
Cutting	Increase very slightly	None, direction is by definition
Thrust	Increase slightly more than F_C	Much more towards the radial direction
Tangential	Identical to F_C	None, direction is by definition
Longitudinal	Likely decrease since the direction of F_T is more strongly moved away from the longitudinal direction than the magnitude of F_T is decreased	None, direction is by definition
Radial	Increases since both the increase in magnitude of F_T and the change in direction of F_T both serve to increase in the radial force.	None, direction is by definition

e) The final workpiece diameter is the original diameter less twice the depth of cut. Substituting known values, the final result (in mm) is

$$D_{wf} = 100 - 2(0.75) = \boxed{98.5}\,.$$

3.4

Given:	
• turning • corner radius, r_ε = 1.0 mm • lead angle, ψ_r = 15° • back rake angle, γ_p = –5° • side rake angle, γ_f = –5°	• initial workpiece diameter, D_w = 100 mm • specific cutting and thrust energy models • feed rate, f_r = 0.20 mm/rev • depth of cut, d = 0.75 mm • spindle speed, n_s =1000 rpm

a) The exact chip area is the product of the feed and depth less the cusp left on the machined surface. When the corner radius is not zero, the exact chip area is

$$a = f \cdot d - 2\left\{ \frac{f}{4}\left[2r_\varepsilon - \left(r_\varepsilon^{\,2} - \frac{f^2}{4} \right)^{1/2} \right] - \frac{r_\varepsilon^{\,2}}{2}\sin^{-1}\left(\frac{f}{2r_\varepsilon} \right) \right\}.$$

Substituting known values, the final result (in mm^2) is

$$a = (0.020)(0.75) - 2\left\{ \frac{0.15}{4}\left[2(1.0) - \left((1.0)^2 - \frac{(0.15)^2}{4} \right)^{1/2} \right] - \frac{(1.0)^2}{2}\sin^{-1}\left(\frac{0.15}{2(1.0)} \right) \right\} = \boxed{0.149666}\ .$$

The material removal rate, by definition, is the product of the chip area and the cutting speed. The cutting speed is the product of the spindle speed and the workpiece circumference. The average cutting speed (averaged across the tooth profile contact), (in m/min) is

$$\bar{V} = n_s\left[\pi\left(D_w - d \right) \right] = 1000\left[\pi(100 - 0.75) \right] = 311.8\,.$$

Of course, since the depth of cut is very small compared to the workpiece diameter, the surface speed at the outer diameter (in m/min),

$$V \approx n_s\left[\pi D_w \right] = 1000\left[\pi(100) \right] = 314.2\,,$$

could just as soon be used as a good approximation.

Converting cutting speed from 311.8 m/min to 311,800 mm/min, the final result (in mm^3/min) is

$$\dot{v}_r = (0.149666)(311{,}800) = \boxed{46{,}666} \quad \approx \quad (0.149666)(314{,}200) = 47{,}025\,.$$

b) The expression for equivalent lead angle under Colwell's method is

$$\bar{\psi} = \tan^{-1}\left(\frac{y_2 - y_1}{x_1 - x_2} \right).$$

The depth of cut is greater than the transition depth of cut

$$d_t = r_\varepsilon\left(1 - \sin\psi_r \right) = (1.0)\left(1 - \sin(15°) \right) = 0.74\,,$$

barely! Therefore, the expressions for the ***x.*** and ***y.*** terms are

$$x_1 = +\left(r_\varepsilon^{\,2} - f^2/4 \right)^{1/2}, \qquad y_1 = -f/2\,,$$

$$x_2 = r_\varepsilon - d\,, \qquad y_2 = \frac{r_\varepsilon}{\cos\psi_r} + (d - r_\varepsilon)\tan\psi_r\,.$$

Substituting known values,

$$x_1 = +\left((1.0)^2 - (0.15)^2/4 \right)^{1/2} = 0.995\,, \qquad y_1 = -0.15/2 = -0.1\,,$$

$$x_2 = 1.0 - 0.75 = 0.25\,, \qquad y_2 = \frac{1.0}{\cos(15°)} + (0.75 - 1.0)\tan(15°) = 0.968\,.$$

Substituting these into the equivalent lead angle expression, the final result is

$$\bar{\psi} = \tan^{-1}\left(\frac{0.968 - (-0.1)}{0.995 - 0.25} \right) = \boxed{55.1°}\,.$$

c) By definition of equivalency, the average uncut chip thickness is the chip area divided by the equivalent width of cut. The chip area was found above; the equivalent width of cut, noting again that the depth of cut is greater than the transition depth, is

$$w = r_\varepsilon \left[\sin^{-1}\left(f/2r_\varepsilon \right) + \pi/2 - \psi_r \right] + \frac{d - r_\varepsilon \left(1 - \sin\psi_r\right)}{\cos\psi_r} .$$

Substituting known values, the result is

$$w = (1.0)\left[\sin^{-1}\left(0.15/2(1.0)\right) + \pi/2 - (15°)\left(\frac{2\pi}{360°}\right) \right] + \frac{0.75 - (1.0)\left(1 - \sin(15°)\right)}{\cos(15°)}$$

$$= 1.4092 + 0.0091 = 1.4183.$$

Subsequently, the final result (in mm) is

$$\bar{h} = \frac{0.149666}{1.4183} = 0.1055 .$$

d) The cutting and thrust forces are each the product of the respective specific energy and the chip area found above. Computing the specific energies from the empirical models given requires average uncut chip thickness (found above), cutting speed (found above for material removal rate calculation) and normal rake angle, where it is computed using the equivalent lead angle.

The normal rake angle is

$$\gamma_n = \tan^{-1}\left[\tan\gamma_o \cos\lambda \right],$$

where the orthogonal rake angle is

$$\gamma_o = \tan^{-1}\left[\tan\gamma_f \cos\bar{\psi} + \tan\gamma_p \sin\bar{\psi} \right]$$

and the inclination angle is

$$\lambda = \tan^{-1}\left[\tan\gamma_p \cos\bar{\psi} - \tan\gamma_f \sin\bar{\psi} \right].$$

Substituting known values, the inclination angle is

$$\lambda = \tan^{-1}\left[\tan(-5°)\cos(55.1°) - \tan(-5°)\sin(55.1°) \right] = 1.24°$$

and, subsequently, the orthogonal rake angle is

$$\gamma_o = \tan^{-1}\left[\tan(-5°)\cos(55.1°) + \tan(-5°)\sin(55.1°) \right] = -6.95 .$$

Therefore, the normal rake angle is

$$\gamma_n = \tan^{-1}\left[\tan(-6.95°)\cos(1.24°) \right] = -6.95° .$$

Substituting known values (h in mm, V in m/min and γ_n in radians (–0.1213 rad)) into the specific energy models, the specific energies (in N/mm^2) are

$$u_C = 3150\,(0.1055)^{-0.233}\,(311.8)^{-0.122}\,e^{-1.318(-0.1213)} = 3092$$

and

$$u_T = 1175\,(0.1055)^{-0.615}\,(311.8)^{-0.148}\,e^{-1.115(-0.1213)} = 2289 .$$

Using these and the chip area computed earlier, the final results (in N) are

$$F_C = u_C a = (3092)(0.149666) = 463 \quad \text{and} \quad F_T = u_T a = (2289)(0.149666) = 343 .$$

The transformations from the edge-local cutting and thrust forces to the tooth-local tangential, longitudinal and radial are given in the text as

$$F_{Tan} = F_C , \quad F_{Lon} = F_T \cos\bar{\psi} \quad \text{and} \quad F_{Rad} = F_T \sin\bar{\psi} .$$

Substituting known values, the final results (in N) are

$$F_{Tan} = \boxed{463}, \quad F_{Lon} = 343\cos(15°) = \boxed{196} \quad \text{and} \quad F_{Rad} = 343\sin(55.1°) = \boxed{281} .$$

e) The change in chip area is negligible; but, since the average uncut chip thickness is substantially (45%) less than $f\cos\psi_r$ in the previous problem, the specific energies are substantially higher (17% for the cutting direction, 47% for the thrust direction) here; therefore, F_C and F_T are higher by the same percentages. In addition, since the equivalent lead angle is much (267%) greater than the lead angle, F_{Rad} increases enormously (366%) while F_{Lon} decreases slightly (13%), even though F_T has increased.