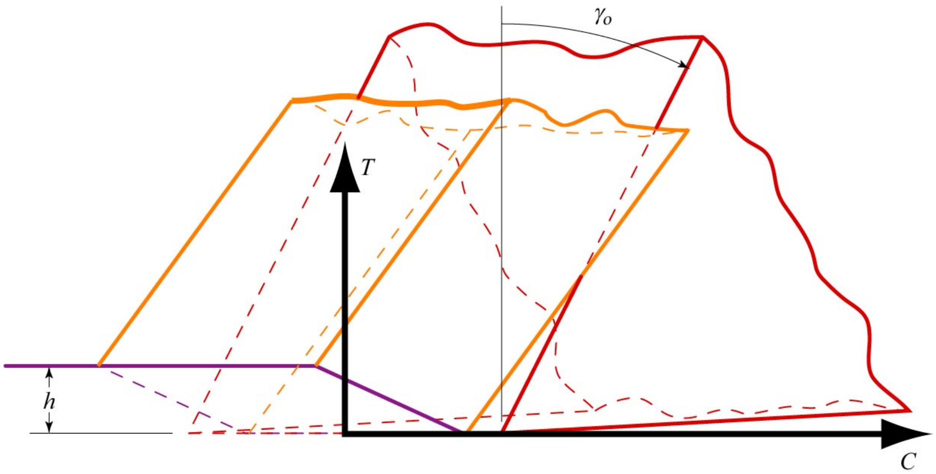
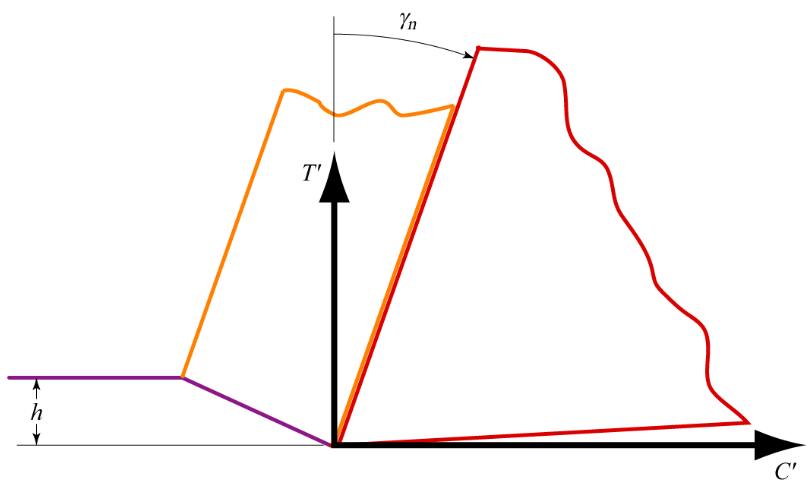
3.1

Given:	
• oblique cutting • orthogonal rake angle, γ_o = 10° • inclination angle, λ = 30°	• uncut chip thickness, h = 0.25 mm • width of cut, w = 2.5 mm • cutting speed, V = 200 m/min • Stabler's Rule applies.

a) The orthogonal rake angle is measured in the orthogonal plane, i.e., the *C*-*T* plane.



b) The normal rake angle is measured in the normal plane, i.e., the *C'*-*T'* plane, which is the *C*-*T* plane rotated by the inclination angle about the *T* axis.



c) The expression for normal rake angle, as a function of orthogonal rake angle and inclination angle, is

$$\gamma_n = \tan^{-1}\left(\tan\gamma_o \cos\lambda\right).$$

Substituting known values, the final result is

$$\gamma_n = \tan^{-1}\left(\tan(10°)\cos(30°)\right) = \boxed{8.7°}.$$

For the remainder of the problem, assume the answer to part (c) is γ_n = 8.5°.

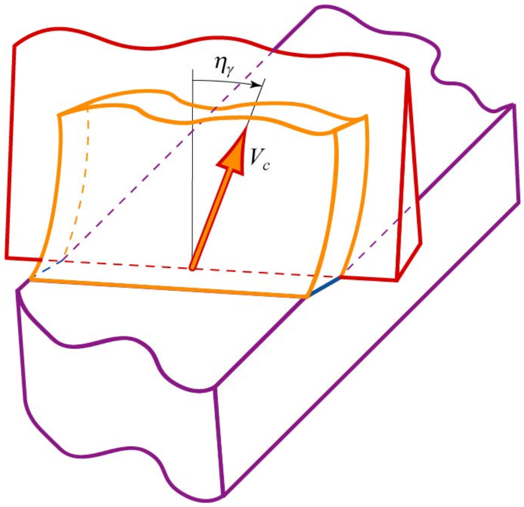
d) The expression for effective rake angle, as a function of normal rake angle, inclination angle and chip-flow angle, is

$$\gamma_e = \sin^{-1}\left(\sin\lambda \sin\eta_\gamma + \cos\lambda \cos\eta_\gamma \sin\gamma_n\right).$$

Substituting known values, including the substitution $\eta_\gamma \rightarrow \lambda$ per Stabler's Rule, the final result is

$$\gamma_e = \sin^{-1}\left(\sin(30°)\sin(30°) + \cos(30°)\cos(30°)\sin(8.5°)\right) = \boxed{21.2°}.$$

e) This view is looking normal to the rake face, as if your line of sight is the normal rake-face force vector.



3.2

Given:	
• oblique cutting • normal rake angle, $\gamma_n = 10°$ • uncut chip thickness, $h = 0.25$ mm • inclination angle, $\lambda = 20°$	• width of cut, $w = 2.5$ mm • cutting speed, $V = 200$ m/min • orthogonal shear angle, $\phi_o = 30°$ • coefficient of friction, $\mu = 0.5$ • shear yield strength, $S_{sy} = 375$ MPa

a) The expression for Armarego's chip-flow angle solution is

$$\eta_\gamma = \tan^{-1}\left[\frac{\tan\lambda\cos\gamma_n}{\tan\left(\phi_n + \tan^{-1}(\mu\cos\eta_\gamma)\right)} + \sin\gamma_n \tan\lambda\right].$$

The expression is transcendental, requiring iteration to solve it. Substituting known values

$$\eta_\gamma = \tan^{-1}\left[\frac{\tan(20°)\cos(10°)}{\tan\left(30° + \tan^{-1}(0.5\cos\eta_\gamma)\right)} + \sin(10°)\tan(30°)\right].$$

Simplifying as much as possible,

$$\eta_\gamma = \tan^{-1}\left[\frac{0.358441}{\tan\left(30° + \tan^{-1}(0.5\cos\eta_\gamma)\right)} + 0.063203\right].$$

A good initial guess is that of Stabler's Rule, i.e., $\eta_\gamma = \lambda = 20°$. The first iteration yields

$$\eta_\gamma = \tan^{-1}\left[\frac{0.358441}{\tan\left(30° + \tan^{-1}(0.5\cos(20°))\right)} + 0.063203\right] = 17.4° .$$

Substituting this result, the second iteration yields

$$\eta_\gamma = \tan^{-1}\left[\frac{0.358441}{\tan\left(30° + \tan^{-1}(0.5\cos(17.4°))\right)} + 0.063203\right] = \boxed{17.2°} .$$

This seems to be relatively converged, indicating a final result.

For the remainder of the problem, assume the answer to part (a) is η_γ = 17.2°.

b) The specific shear energy is the product of the shear-yield strength and shear strain, where the shear strain in oblique cutting is

$$\gamma = \frac{\cot\phi_n + \tan(\phi_n - \gamma_n)}{\cos\eta_\phi} .$$

The expression for shear-flow angle is

$$\eta_\phi = \tan^{-1}\left[\frac{\tan\lambda\cos(\phi_n - \gamma_n) - \tan\eta_\gamma\sin\phi_n}{\cos\gamma_n}\right].$$

Substituting known values,

$$\eta_\phi = \tan^{-1}\left[\frac{\tan(20°)\cos(30° - 10°) - \tan(17.2°)\sin(30°)}{\cos(10°)}\right] = 10.8° .$$

This allows the shear strain to be computed as

$$\gamma = \frac{\cot(30°) + \tan(30° - 10°)}{\cos(10.8°)} = 2.13 .$$

Now, substituting this along with the given shear-yield strength, the final result (in N/mm^2) is

$$u_s = S_{sy}\gamma = 375(2.13) = \boxed{800} .$$

Also of interest is the in-plane shear force, which is simply the product of the shear yield strength and the shear-plane area:

$$P_\phi = S_{sy}a_\phi ,$$

where

$$a_\phi = \frac{hw}{\cos\lambda\sin\phi_n} .$$

Substituting known values, the final result (in N) is

$$P_\phi = 375\frac{(0.25)(2.5)}{\cos(20°)\sin(30°)} = \boxed{499} .$$

For the remainder of the problem, assume the answer to part (b) is P_ϕ = 500 N.

c) The FCD can be used to draw a direct relation between the in-plane shear force known from above and the in-plane rake-face force, but doing so only in the normal plane. Relations of use include

$$R_n = \frac{P_{\phi n}}{\cos(\beta_n - \gamma_n + \phi_n)} \quad \text{and} \quad R_n = \frac{P_{\gamma n}}{\sin \beta_n} .$$

Setting these two expressions yields

$$P_{\gamma n} = \frac{\sin \beta_n}{\cos(\beta_n - \gamma_n + \phi_n)} P_{\phi n} .$$

The normal-plane components $P_{\phi n}$ and $P_{\gamma n}$ are related to their resultant counterparts as

$$P_{\phi n} = P_\phi \cos \eta_\phi \quad \text{and} \quad P_{\gamma n} = P_\gamma \cos \eta_\gamma .$$

Combining with the expressions above,

$$P_\gamma = \frac{\sin \beta_n \cos \eta_\phi}{\cos(\beta_n - \gamma_n + \phi_n) \cos \eta_\gamma} P_\phi .$$

This calculation requires the normal friction angle

$$\beta_n = \tan^{-1}\left(\mu \cos \eta_\gamma\right).$$

Substituting known values, the normal friction angle is computed to be

$$\beta_n = \tan^{-1}\left(0.5 \cos(17.2°)\right) = 25.5° .$$

Substituting this and other known values, the final result (in N) is

$$P_\gamma = \frac{\sin(25.5°)\cos(10.8°)}{\cos(25.5° - 10° + 30°)\cos(17.2°)} 500 = \boxed{316} .$$

Another approach that is very direct but perhaps not as obvious is to note that the force component acting out of the normal plane must be equal on the rake face and shear plane, i.e., $P_{\phi s} = P_{\gamma s}$. therefore,

$$P_{\phi s} = P_\phi \sin \eta_\phi \quad \text{and} \quad P_{\gamma s} = P_\gamma \sin \eta_\gamma \quad \Rightarrow \quad P_\gamma = \frac{\sin \eta_\phi}{\sin \eta_\gamma} P_\phi .$$

Rearranging and substituting known values for the two flow angles, the final result (in N) is

$$P_\gamma = \frac{\sin(10.8°)}{\sin(17.2°)} 500 = \boxed{317} .$$

d) By definition, the specific energies are

$$u_C = F_C / a, \quad u_T = F_T / a, \quad u_L = F_L / a .$$

The forces required to compute the specific energies, F_C, F_T and F_L, can be determined from N_ϕ, $P_{\phi n}$, and $P_{\phi s}$, or N_γ, $P_{\gamma n}$, and $P_{\gamma s}$. The rake-face force components will be used here; using the shear-plane force components follows in an identical fashion.

The in-plane rake-face force was found above to be P_γ = 316 N. The rake-face coefficient of friction/shear is given to be μ = 0.5, so the normal rake-face force is

$$N_\gamma = \frac{P_\gamma}{\mu} = \frac{316}{0.5} = 632 .$$

The transformation from F_C, F_T and F_L to N_γ, $P_{\gamma n}$ and $P_{\gamma s}$ are given in the text to be

$$\begin{Bmatrix} N_\gamma \\ P_{\gamma n} \\ P_{\gamma s} \end{Bmatrix} = \begin{bmatrix} \cos\gamma_n \cos\lambda & -\sin\gamma_n & -\cos\gamma_n \sin\lambda \\ \sin\gamma_n \cos\lambda & \cos\gamma_n & -\sin\gamma_n \sin\lambda \\ \sin\lambda & 0 & \cos\lambda \end{bmatrix} \begin{Bmatrix} F_C \\ F_T \\ F_L \end{Bmatrix} = \left[R_{\lambda\gamma} \right]^{-1} \begin{Bmatrix} F_C \\ F_T \\ F_L \end{Bmatrix},$$

where

$$P_{\gamma n} = P_\gamma \cos \eta_\gamma = 316 \cos(17.2°) = 302 \text{ N}$$

and

$$P_{\gamma s} = P_\gamma \sin \eta_\gamma = 316 \sin(17.2°) = 935 \text{ N} .$$

To invert the force relation matrix equation, $[R_{\lambda\gamma}]$ is needed. The general expression for the inverse of a matrix is

$$[A]^{-1} = \frac{\text{Adj}([A])}{\det([A])} .$$

Since $[R_{\lambda\gamma}]^{-1}$ is a rotational transformation, it is known that $\det([R_{\lambda\gamma}]^{-1}) = 1$. The adjoint of $[R_{\lambda\gamma}]^{-1}$ is

$$\begin{bmatrix} \cos\gamma_n \cos\lambda + \cancel{0 \cdot \sin\gamma_n \sin\lambda} & -\left(\sin\gamma_n \cos^2\lambda + \sin\gamma_n \sin^2\lambda\right) & \cancel{0 \cdot \sin\gamma_n \cos\lambda} - \sin\lambda\cos\gamma_n \\ -\left(-\sin\gamma_n \cos\lambda + \cancel{0 \cdot \cos\gamma_n \sin\lambda}\right) & \cos\gamma_n \cos^2\lambda + \cos\gamma_n \sin^2\lambda & -\left(\cancel{0 \cdot \cos\gamma_n \cos\lambda} + \sin\lambda \sin\gamma_n\right) \\ \sin^2\gamma_n \sin\lambda + \cos^2\gamma_n \sin\lambda & \begin{array}{c} -\left(-\cos\gamma_n \cos\lambda \sin\gamma_n \sin\lambda \right. \\ \left. + \cos\gamma_n \cos\lambda \sin\gamma_n \sin\lambda\right) \end{array} & \cos^2\gamma_n \cos\lambda + \sin^2\gamma_n \cos\lambda \end{bmatrix}^T ,$$

or

$$\begin{bmatrix} \cos\gamma_n \cos\lambda & \sin\gamma_n \cos\lambda & \sin\lambda \\ -\sin\gamma_n & \cos\gamma_n & 0 \\ -\cos\gamma_n \sin\lambda & -\sin\gamma_n \sin\lambda & \cos\lambda \end{bmatrix} .$$

The cutting, thrust and lateral forces could be computed through matrix multiplication; they are computed here in a more illustrated fashion by writing each force as a function of the rake-face force components, i.e., writing the matrix equation as three separate scalar equations. Doing so,

$$F_C = N_\gamma \cos\gamma_n \cos\lambda + P_{\gamma n} \sin\gamma_n \cos\lambda + P_{\gamma s} \sin\lambda ,$$

$$F_T = -N_\gamma \sin\gamma_n + P_{\gamma n} \cos\gamma_n$$

and

$$F_L = -N_\gamma \cos\gamma_n \sin\lambda - P_{\gamma n} \sin\gamma_n \sin\lambda + P_{\gamma s} \cos\lambda .$$

Substituting known values, the forces (in N) are

$$F_C = 632 \cos(10°) \cos(20°) + 302 \sin(10°) \cos(20°) + 93.5 \sin(20°) = 666 ,$$

$$F_T = -632 \sin(10°) + 302 \cos(10°) = 188$$

and

$$F_L = -632 \cos(10°) \sin(20°) - 302 \sin(10°) \sin(20°) + 93.5 \cos(20°) = -143 .$$

Dividing each force value by the chip area, $a = (0.25)(2.5) = 0.625$ (in mm^2), the final results (in N/mm^2) are

$$u_C = F_C / a = 666/0.625 = \boxed{1066} ,$$

$$u_T = F_T / a = 188/0.625 = \boxed{301}$$

and

$$u_L = F_L / a = |-143|/0.625 = \boxed{229} .$$

e) The cutting power is simply the product of the cutting force and the cutting speed. Substituting known values, and noting that cutting speed must be converted from 200 m/min to 3.33 m/s, the final result (in kW) is

$$\mathscr{P}_C = F_C V = 666(3.33)/1000 = \boxed{2.22} .$$

The material removal rate is simply the product of the uncut chip thickness, the width of cut and the cutting speed. Substituting known values, and noting that cutting speed must be converted from 200 m/min to 200,000 mm/min, the final result (in mm^3/min) is

$$\dot{v}_r = hwV = (0.25)(2.5)(200,000) = \boxed{125,000}\ .$$