2.13

Given:	
• orthogonal cutting • orthogonal rake angle, $\gamma_o = 20°$ • uncut chip thickness, $h = 0.3$ mm • width of cut, $w = 1.0$ mm	• cutting force, $F_C = 900$ N • cutting force, $F_T = 300$ N • Ernst and Merchant shear angle model is valid

a) The average effective coefficient of friction μ is the ratio of the in-plane rake-face force P_γ to the in-plane normal force N_γ. The force-circle diagram provides a simple relation, which introduces the friction angle $\beta = \tan^{-1}\mu$, between the thrust and cutting force components, i.e.,

$$F_T = \tan(\beta - \gamma_o)F_C \quad \rightarrow \quad \beta = \gamma_o + \tan^{-1}\left(\frac{F_T}{F_C}\right).$$

Substituting known values, the friction angle is

$$\beta = 20° + \tan^{-1}\left(\frac{300}{900}\right) = 20° + 18.4° = 38.4° .$$

Since $\beta = \tan^{-1}\mu$, $\mu = \tan\beta$. Substituting the result for β yields the final result of $\mu = \boxed{0.793}$.

b) The formula for shear strain is

$$\gamma = \cot\phi_o + \tan(\phi_o - \gamma_o) ,$$

where the orthogonal rake angle γ_o is given and the orthogonal shear angle ϕ_o can be computed using the Ernst and Merchant model, which is given to be valid, as

$$\phi_o = 45° + \frac{\gamma_o}{2} - \frac{\beta}{2} .$$

Substituting known values, the shear angle result and subsequent final result for shear strain are

$$\phi_o = 45° + \frac{20°}{2} - \frac{38.4°}{2} = 35.8° \quad \rightarrow \quad \gamma = \cot(35.8°) + \tan(35.8° - 20°) = \boxed{1.67}$$

For the remainder of the problem, assume the answer to part (b) is $\gamma = 1.70$ and that the corresponding shear angle is $\phi_o = 35°$.

c) The specific cutting energy is simply the cutting force divided by the chip area $a = hw$. Substituting known values, the final result (in N/mm^2) is

$$u_C = \frac{F_C}{hw} = \frac{900}{(0.3)(1.0)} = \boxed{3000} .$$

d) The specific shear and friction energies can be found two ways as follows

By definition, the specific shear energy is the shear power divided by the material removal rate, i.e.,

$$u_s = \frac{\mathscr{P}_s}{\dot{v}_r} = \frac{P_\phi V_s}{hwV} .$$

The specific friction energy can then be found, knowing the specific cutting energy above, as

$$u_f = u_C - u_s .$$

The above approach requires values for the in-

By definition, the specific friction energy is the friction power divided by the material removal rate, i.e.,

$$u_f = \frac{\mathscr{P}_f}{\dot{v}_r} = \frac{P_\gamma V_c}{hwV} .$$

The specific shear energy can then be found, knowing the specific cutting energy above, as

$$u_s = u_C - u_f .$$

The above approach requires values for the plane shear force P_ϕ and the shear-velocity ratio V_s/V, each of which requires the orthogonal shear angle, which is now given to be 35°.

The in-plane shear force is

$$P_\phi = F_C \cos\phi_o - F_T \sin\phi_o \, .$$

Substituting known values for the cutting and thrust forces, the result (in N) is

$$P_\phi = 900\cos(35°) - 300\sin(35°) = 565 \, .$$

The shear velocity ratio is

$$\frac{V_s}{V} = \frac{\cos\gamma_o}{\cos(\phi_o - \gamma_o)} \, .$$

Substituting known values for γ_o and ϕ_o,

$$\frac{V_s}{V} = \frac{\cos(20°)}{\cos(35° - 20°)} = 0.973 \, .$$

Finally, substituting known values computed above, the final results (in N/mm^2) are

$$u_s = \frac{565}{(0.3)(1.0)}(0.973) = \boxed{1832}$$

and

$$u_f = 3000 - 1832 = \boxed{1168} \, .$$

in-plane rake-face force P_γ and the chip-velocity ratio V_c/V, each of which requires the orthogonal shear angle, which is now given to be 35°.

The in-plane friction force is

$$P_\gamma = F_C \sin\gamma_o + F_T \cos\gamma_o \, .$$

Substituting known values for the cutting and thrust forces, the result (in N) is

$$P_\gamma = 900\sin(20°) - 300\cos(20°) = 590 \, .$$

The chip velocity ratio is

$$\frac{V_c}{V} = \frac{\sin\phi_o}{\cos(\phi_o - \gamma_o)} \, .$$

Substituting known values for γ_o and ϕ_o,

$$\frac{V_c}{V} = \frac{\sin(35°)}{\cos(35° - 20°)} = 0.594 \, .$$

Finally, substituting known values computed above, the final results (in N/mm^2) are

$$u_f = \frac{590}{(0.3)(1.0)}(0.594) = \boxed{1168}$$

and

$$u_c = 3000 - 1168 = \boxed{1832} \, .$$

e) Since the specific shear energy is the shear strain energy and the shear stress on the shear plane is the shear-yield strength, the latter equals the specific shear energy divided by the shear strain, given to be 1.70, as

$$S_{sy} = \frac{u_s}{\gamma} \, .$$

Substituting known values, the final result (in MPa, which is equivalent to N/mm^2) is

$$S_{sy} = \frac{1832}{1.70} = \boxed{1078} \, .$$

2.15

Given:

- orthogonal rake angle, $\gamma_o = 10°$
- effective coefficient of friction, $\mu = 0.75$
- shear yield, $S_{sy} = 400 l_\phi^{-0.25}$ MPa, l_ϕ in mm
- Lee and Shaffer shear angle model is valid

a) The following are the four contributors to size effect:

- Dislocation availability is dominant at very low levels of h.
- Apparent negative rake angle, which is caused by the edge radius creating a continuously more negative rake angle, contributes as h/r_n becomes small.
- Shear zone dilation, which causes strain rate reduction as h decreases, contributes at all levels of h.
- Edge rubbing is present for all levels of h and becomes noticeable at moderate to low h/r_n.

b) The specific shear energy is the shear strain energy where the shear stress on the shear plane is the shear-yield strength. Therefore,

$$u_s = S_{sy}\gamma .$$

where the shear-yield strength is given as a function of shear-plane length, and shear strain is

$$\gamma = \cot\phi_o + \tan(\phi_o - \gamma_o) ,$$

and the orthogonal rake angle γ_o is given and the orthogonal shear angle ϕ_o can be computed using the Lee and Shaffer model, which is given to be valid, as

$$\phi_o = 45° + \gamma_o - \beta .$$

Noting that the friction angle is $\beta = \tan^{-1}\mu = 36.9°$, substituting this and other known values, the shear angle result and subsequent result for shear strain are

$$\phi_o = 45° + 20° - 36.9° = 18.1° \quad \rightarrow \quad \gamma = \cot(18.1°) + \tan(18.1° - 10°) = 3.20 .$$

As a result, the specific shear energy, u_s, is $3.20 S_{sy}$.

The final step to achieve the end result of S_{sy} as a function of uncut chip thickness h is to introduce the shear-plane length l_ϕ as a function of h, which is

$$l_\phi = \frac{h}{\sin\phi_o} .$$

Substituting the known value for shear angle yields the replacement for $l_\phi^{-0.25}$ as

$$l_\phi^{-0.25} = \frac{h^{-0.25}}{\left[\sin(18.1°)\right]^{-0.25}} = 0.747 h^{-0.25} .$$

The final result (in N/mm^2 with h in mm) is

$$u_s = (3.20)(400)(0.747) h^{-0.25} = \boxed{956 h^{-0.25}} .$$

For the remainder of the problem, assume the answer to part (c) is
$u_s = 950 \cdot h^{-0.25}$ N/mm^2 for h in mm.

c) The shear power, based on the definition specific energy being the shear power divided by the material removal rate, is equal to the product of the specific shear energy and material removal rate, i.e.,

$$\mathcal{P}_s = u_s V a .$$

where the material removal rate, $Va = Vhw$, can be computed with the experimental conditions given. Substituting $V = 50$ m/min, $h = 0.20$ mm, $w = 2.0$ mm and using $u_s = 950 h^{-0.25}$ N/mm^2, the material removal rate (in m·mm^2/s) is

$$Va = \left(50\frac{1}{60}\right)(0.20 \cdot 2.0) = 0.333$$

and the specific shear energy (in N/mm^2) is

$$u_s = 950(0.20)^{-0.25} = 1421\,.$$

Substituting these values, the final result for shearing power (in Nm/s = W) is obtained as

$$\mathscr{P}_s = (1421)(0.333) = 473\,.$$

d) Yes, enough information (four, including at least two magnitudes or directions) exists to draw the FCD, as follows:

- The in-plane shear force magnitude, $P_\phi = S_{sy}a_\phi = S_{sy}a/\sin\phi_o$ is known.
- The shear angle ϕ_o provides the direction of P_ϕ.
- The rake angle γ_o provides the directions of N_γ and P_γ.
- The relative magnitudes (or the direction of the resultant force R) of N_γ and P_γ are known given μ (or $\beta - \gamma_o$ as the direction of R).