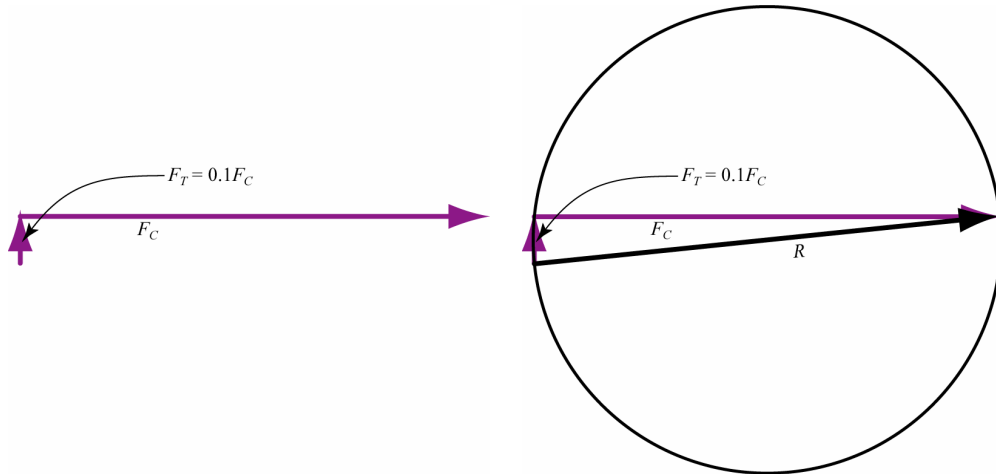


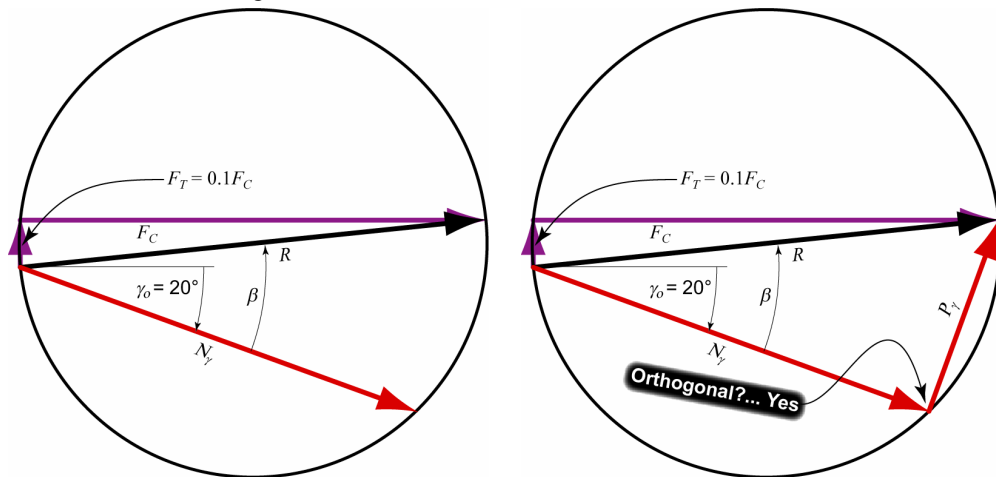
**Given:**

- orthogonal cutting
- orthogonal rake angle,  $\gamma_o = 20^\circ$
- cutting force,  $F_C = 1000$  N
- thrust force,  $F_T = 100$  N

- a) First, construct vectors representing  $F_C$  and  $F_T$  in their default vertical and horizontal orientations, respectively, as shown below (left). Next, draw a circle that passes through the coincident start points and the end points of the two vectors. Construct a vector with its start coincident with the others such that it falls on the diameter of the circle. This is the vector representing the resultant force,  $R$ . The result is shown below (right).



- Draw  $N_\gamma$  with its start coincident with those of the others and oriented  $20^\circ$  clockwise from the horizontal and pointing toward the right, as shown below (left). Finally, draw  $P_\gamma$  with its start coincident with the end of  $N_\gamma$  and its end coincident with those of the others ( $F_C$ ,  $F_T$  and  $R$ ). It should be orthogonal to  $N_\gamma$ . The result is shown below (right).



By measuring  $P_\gamma$  and  $N_\gamma$  vectors relative to  $F_C$  (or  $F_T$ ), the rake-face force components are estimated to be (in N)  $N_\gamma = 904$  and  $P_\gamma = 438$ . One could confirm the results using the equations given in the text for  $N_\gamma$  and  $P_\gamma$  in terms of  $F_C$ ,  $F_T$  and  $\gamma_o$ .

- b) The Ernst and Merchant model is

$$\phi_o = 45^\circ + \frac{\gamma_o}{2} - \frac{\beta}{2}$$

The friction angle can be either measured from the FCD to be  $26^\circ$  or more easily and precisely calculated using the values for  $F_C$  and  $F_T$  as

$$\tan(\beta - \gamma_o) = \frac{F_T}{F_C} \Rightarrow \beta = \tan^{-1}\left(\frac{F_T}{F_C}\right) + \gamma_o.$$

The result for friction angle is

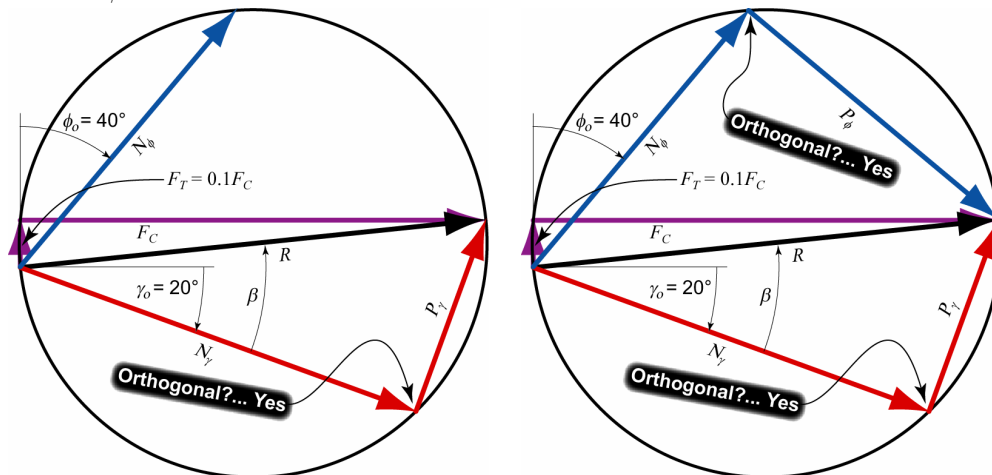
$$\beta = \tan^{-1}\left(\frac{100}{1000}\right) + 20^\circ = 5.7^\circ + 20^\circ = 25.7^\circ.$$

Substituting known values, the final result is

$$\phi_o = 45^\circ + \frac{20^\circ}{2} - \frac{25.7^\circ}{2} = \boxed{42.3^\circ, \text{ or } 42^\circ}.$$

**For the remainder of the problem, assume the answer to part (b) is  $\phi_o = 40^\circ$ .**

- c) One piece of information (shear angle) about the shear-plane forces is now known, which is required to construct the shear-plane forces. Continuing with the FCD from above the shear-plane force components may be drawn. First, draw  $N_\phi$  with its start coincident with those of the others and oriented  $40^\circ$  clockwise from the vertical and pointing upward, as shown below (left). Then, draw  $P_\phi$  with its start coincident with the end of  $N_\phi$  and its end coincident with those of the others ( $F_C$ ,  $F_T$  and  $R$ ). It should be orthogonal to  $N_\phi$ . The result is shown below (right).



The shear-plane force components are estimated via measurement to be (in N)  $N_\phi = \boxed{720}$  and  $P_\phi = \boxed{700}$ . One could confirm the results using the equations given in the text for  $N_\phi$  and  $P_\phi$  in terms of  $F_C$ ,  $F_T$  and  $\phi_o$ .

- d) The relations for the rake-face force components as functions of the cutting and thrust force components and rake angle are given in the text to be

$$N_\gamma = F_C \cos \gamma_o - F_T \sin \gamma_o \quad \text{and} \quad P_\gamma = F_C \sin \gamma_o + F_T \cos \gamma_o.$$

Substituting known values, the final result (in N) is

$$N_\gamma = 1000 \cos(20^\circ) - 100 \sin 20^\circ = 905.5 \approx 904$$

and

$$P_\gamma = 1000 \sin(20^\circ) + 100 \cos(20^\circ) = 436.0 \approx 438.$$

The relations for the shear-plane force components as functions of the cutting and thrust force components and shear angle are given in the text to be

$$P_{\phi} = F_C \cos \phi_o - F_T \sin \phi_o \quad \text{and} \quad N_{\phi} = F_C \sin \phi_o + F_T \cos \phi_o .$$

Substituting known values, the final result (in N) is

$$P_{\phi} = 1000 \cos(40^{\circ}) - 100 \sin(40^{\circ}) = \boxed{701.8 \approx 700}$$

and

$$N_{\phi} = 1000 \sin(40^{\circ}) + 100 \cos(40^{\circ}) = \boxed{719.4 \approx 720} .$$

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**Given:**

- orthogonal cutting
- orthogonal rake angle,  $\gamma_o = 10^\circ$
- uncut chip thickness,  $h = 0.25$  mm
- width of cut,  $w = 2.5$  mm
- cutting speed,  $V = 200$  m/min
- orthogonal shear angle,  $\phi_o = 30^\circ$
- coefficient of friction,  $\mu = 0.5$
- tensile yield strength,  $S_y = 750$  MPa

- a) The specific shear energy is the shear-strain energy, which is equivalent to the area under the shear-stress vs. shear-strain curve. Since the material is assumed to exhibit elastic-perfectly plastic behavior, the stress on all material that is yielding is equal to the shear-yield strength  $S_{sy}$ . The shear-strain energy is the area of a rectangle with height of  $S_{sy}$  and width equal to the shear strain  $\gamma$ . Therefore,

$$u_s = S_{sy} \gamma .$$

To be exact, the triangle associated with the elastic rise to yield should be subtracted, or equivalently, only the plastic strain ( $\gamma - S_{sy}/G$ ,  $G$  being the shear modulus) should be used, not the total strain  $\gamma$  as the width of the parallelogram formed by the rise to yield, straining to  $\gamma$  and unloading along slope  $G$  back to zero stress. However, since the plastic strain in machining is quite large ( $> 100\%$ ) compared to the yield strain ( $\sim 0.2\%$ ), the simple product of  $S_{sy}$  and  $\gamma$  is a sufficient approximation.

The shear yield strength can be estimated from the tensile yield strength. The most simple estimation is based on the shear-yield criterion that dictates that  $S_{sy} = 0.5S_y$ . Another is based on the Mises criterion that dictates  $S_{sy} = 0.577S_y$ . Therefore, the shear yield strength is (in MPa)

$$S_{sy} = 0.5(750) = 375 \quad \text{or} \quad S_{sy} = 0.577(750) = 433 .$$

The shear strain is

$$\gamma = \cot \phi_o + \tan(\phi_o - \gamma_o) ,$$

or

$$\gamma = \cot(30^\circ) + \tan(30^\circ - 10^\circ) = 2.1 .$$

The final result (in  $\text{N}/\text{mm}^2$ ) is

$$u_s = 375(2.1) = \boxed{787.5} \quad \text{or} \quad u_s = 433(2.1) = \boxed{909.3} .$$

- b) The in-plane shear force,  $P_\phi$ , can be calculated in two ways as follows

The in-plane shear force is the product of the shear-yield strength and the area of the shear plane, i.e.,

$$P_\phi = S_{sy} a_\phi = S_{sy} \frac{hw}{\sin \phi_o} .$$

Substituting known values (using  $S_{sy} = 375$  MPa), the final result (in N) is

$$P_\phi = 375 \frac{(0.25)(2.5)}{\sin(30^\circ)} = \boxed{468.8} .$$

Using  $S_{sy} = 433$ , the final result (in N) is

$$P_\phi = \boxed{541.3} .$$

By definition, the specific shear energy is the shear yield power divided by the material removal rate, i.e.,

$$u_s = \frac{\dot{P}_s}{\dot{V}_r} = \frac{P_\phi V_s}{hwV} .$$

Rearranging shows the in-plane shear force to be the product of the specific shear energy  $u_s$ , the (uncut) chip area  $a = hw$ , and the inverse of the shear-velocity ratio  $V_s/V$ , i.e.,

$$P_\phi = u_s a \frac{V}{V_s} = u_s hw \frac{\cos(\phi_o - \gamma_o)}{\cos \gamma_o} .$$

Substituting known values (using  $S_{sy} = 375$  MPa), the final result (in N) is

$$P_\phi = 787.5(0.25)(2.5) \frac{\cos(30^\circ - 10^\circ)}{\cos(10^\circ)} = \boxed{469.6} .$$

Using  $S_{sy} = 433$ , the final result (in N) is

$$P_\phi = \boxed{542.3}.$$

**For the remainder of the problem, assume the answer to part (b) is  $P_\phi = 470$  N.**

- c) Using the given value of  $P_\phi$  of 470 N to proceed, the in-plane rake-face force,  $P_\gamma$ , can be found in two ways as follows:

Based on equations provided to relate shear-plane and rake-face forces to the cutting and thrust forces  $F_C$  and  $F_T$ ,  $F_C$  and  $F_T$  can be determined as functions of  $P_\phi$  and then used to determine  $P_\gamma$  using coefficient of friction/shear,  $\mu$ .

The equations relating the shear-plane forces to  $F_C$  and  $F_T$  are

$$P_\phi = F_C \cos \phi_o - F_T \sin \phi_o$$

and

$$N_\phi = F_C \sin \phi_o + F_T \cos \phi_o,$$

or

$$\begin{Bmatrix} P_\phi \\ N_\phi \end{Bmatrix} = \begin{bmatrix} \cos \phi_o & -\sin \phi_o \\ \sin \phi_o & \cos \phi_o \end{bmatrix} \begin{Bmatrix} F_C \\ F_T \end{Bmatrix}.$$

Solving these simultaneously (i.e., inverting the rotational transformation matrix of sines and cosines of  $\phi_o$ ),

$$\begin{Bmatrix} F_C \\ F_T \end{Bmatrix} = \begin{bmatrix} \cos \phi_o & \sin \phi_o \\ -\sin \phi_o & \cos \phi_o \end{bmatrix} \begin{Bmatrix} P_\phi \\ N_\phi \end{Bmatrix}.$$

The equations relating the rake-face forces to  $F_C$  and  $F_T$  are

$$N_\gamma = F_C \cos \gamma_o - F_T \sin \gamma_o$$

and

$$P_\gamma = F_C \sin \gamma_o + F_T \cos \gamma_o,$$

which can be used directly without inverting. Substituting the expressions for  $F_C$  and  $F_T$  in terms of  $N_\phi$  and  $P_\phi$  yields

$$\begin{aligned} N_\gamma &= (P_\phi \cos \phi_o + N_\phi \sin \phi_o) \cos \gamma_o \\ &\quad - (-P_\phi \sin \phi_o + N_\phi \cos \phi_o) \sin \gamma_o \end{aligned}$$

and

$$\begin{aligned} P_\gamma &= (P_\phi \cos \phi_o + N_\phi \sin \phi_o) \sin \gamma_o \\ &\quad + (-P_\phi \sin \phi_o + N_\phi \cos \phi_o) \cos \gamma_o. \end{aligned}$$

The expression for  $P_\gamma$  can be divided by that for  $N_\gamma$  and set equal to  $\mu$ . Rearranging and solving for  $N_\phi$  in terms of  $P_\phi$  and  $\mu$  yields

Using the force-circle diagram, given all the angles (friction angle  $\beta$  coming from  $\mu$ ), the rake-face forces may be quickly related to the in-plane shear force  $P_\phi$ .

The most direct way is to note that the resultant force is

$$R = \frac{P_\phi}{\cos(\phi_o + \beta - \gamma_o)}, \quad \beta = \tan^{-1} \mu.$$

Substituting values,

$$\beta = \tan^{-1}(0.5) = 26.6^\circ$$

and the result for  $R$  (in N) is

$$R = \frac{470}{\cos(30^\circ + 26.6^\circ - 10^\circ)} = 684.$$

The in-plane rake-face force  $P_\gamma$  is then easily related to the resultant force as

$$P_\gamma = R \sin \beta.$$

Substituting known values, the final result (in N) is

$$P_\gamma = 684 \sin(26.6^\circ) = 306.$$

Using  $S_{sy} = 433$ , the final result (in N) is

$$P_\phi = \boxed{542.3}.$$

A less direct way is to project  $N_\phi$  and  $P_\phi$  into  $N_\gamma$  and  $P_\gamma$ , which are related through  $\mu$ .

The projection is through the angle  $\phi_o - \gamma_o$ , specifically

$$N_\gamma = P_\phi \cos(\phi_o - \gamma_o) + N_\phi \sin(\phi_o - \gamma_o)$$

and

$$P_\gamma = -P_\phi \sin(\phi_o - \gamma_o) + N_\phi \cos(\phi_o - \gamma_o).$$

As in the solution to the left, the expression for  $P_\gamma$  can be divided by that for  $N_\gamma$  and set equal to  $\mu$ . Rearranging and solving for  $N_\phi$  in terms of  $P_\phi$  and  $\mu$  yields

$$N_\phi = -P_\phi \frac{\cos(\phi_o - \gamma_o)\mu + \sin(\phi_o - \gamma_o)}{\sin(\phi_o - \gamma_o)\mu - \cos(\phi_o - \gamma_o)},$$

$$N_\phi = -P_\phi \frac{(\sin \phi_o \sin \gamma_o + \cos \phi_o \cos \gamma_o)\mu - (-\sin \phi_o \cos \gamma_o + \cos \phi_o \sin \gamma_o)\mu}{(-\sin \phi_o \cos \gamma_o + \cos \phi_o \sin \gamma_o)\mu + (\sin \phi_o \sin \gamma_o + \cos \phi_o \cos \gamma_o)\mu}$$

For convenience in computing and comparison the subsequent solution to the right, trigonometric identities for angle summations can be introduced to obtain

$$N_\phi = -P_\phi \frac{\cos(\phi_o - \gamma_o)\mu + \sin(\phi_o - \gamma_o)}{\sin(\phi_o - \gamma_o)\mu - \cos(\phi_o - \gamma_o)} \quad (*)$$

Substituting known values, noting that  $\phi_o - \gamma_o$  is  $20^\circ$ , the result for  $N_\phi$  (in N) is

$$N_\phi = -470 \frac{\cos(20^\circ)(0.5) + \sin(20^\circ)}{\sin(20^\circ)(0.5) - \cos(20^\circ)} = 496.$$

Back-substituting the value for  $N_\phi$  into the  $P_\gamma$  equation yields the final result (in N) as

$$\begin{aligned} P_\gamma &= (470 \cos(30^\circ) + 496 \sin(30^\circ)) \sin(10^\circ) \\ &\quad + (-470 \sin(30^\circ) + 496 \cos(30^\circ)) \cos(10^\circ) \\ &= \boxed{305} \end{aligned}$$

which is identical to result (\*) in the solution to the left.

Proceeding in the same way as to the left, the same final result (in N) of  $P_\gamma = \boxed{305}$ .