

This book is under preparation with an expected publication date of January, 2007. The table of content is described below. Followed by the preface of the book and note to the instructor. The slides for use in lectures are posted on my main web page under the section of Educational Material. If you have suggestions for supplementary material in addition to slides and solution manual, then please send me an e-mail at mavable@mtu.edu

Thank you

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Preface

The introductory course of mechanics of materials is a required course in most engineering disciplines. The educational philosophy behind the requirement is to teach common mechanics concepts and principles in a single course and teach the extensions and applications of the mechanics concepts and principles in individual disciplines. This educational philosophy addresses the need of interdisciplinary education while realizing curriculum efficiency by reducing duplication of course content—an important consideration as educators attempt to modernize engineering education by incorporating research into a burgeoning curriculum. The introductory course of mechanics of materials has served the engineering community well, but the tremendous growth in the applications of mechanics of materials is bringing added importance to a second course ‘Intermediate mechanics of materials’. The course is often taught by instructors using their own notes in conjunction with either the optional topics from a textbook designed primarily to teach the introductory course or using selected topics from a book primarily designed for graduate course on advanced mechanics of materials. The pedagogical needs of the students with respect to the theoretical details, numerical examples, and post-text problems are difficult to meet and put undue burden on the instructor teaching the course—such has been my experience in teaching this course in the past twenty years. This book is designed to provide educational material for a second course of mechanics of materials taught to juniors or seniors.

Plastics engineering is becoming a popular new minor, bio-medical engineering is a growing new discipline — both require understanding of the mechanics of inelastic and non-linear material behavior in their applications of stress analysis. Applications of inelastic and non-linear material is also growing as metals compete with new materials in engineering design by operating in the plastic region or through pre-stressing. The growing usage of metal matrix composites, polymer composites, reinforced concrete, wooden beams stiffened with steel strips, and other laminated structures emphasize the growing importance of composite structural members. The ubiquity of finite element method in engineering analysis and design not only emphasizes the educational importance of energy principles and concepts of finite element method, but as significantly requires the understanding of stress and strain transformation in three dimensions and failure theories in order to evaluate and use the results produced by commercial finite element computer software. The capstone senior design projects have added to the importance of under-

standing the unsymmetric bending of beams and the concept and use of shear centers in design. Beam and shaft vibrations, beams on elastic foundations, Timoshenko beam, etc. are among the many topics in existing aerospace, civil, and mechanical engineering courses that use the principles of mechanics of materials. If a student is to be taught the mechanics of the topics described and not be overwhelmed with all the complexities inherent in these topics, then the presentation of the material must have coherence and compactness that consolidates what the students have already learned in the introductory course and builds on it —this is the underlying design of this book and is elaborated in the ‘Note to the instructor’.

This book is consistent in its design and notation with my introductory mechanics of materials book. However, this book is a stand alone book that does not depend upon the book used in introductory course of mechanics of materials. There are many pedagogical features to help students meet the educational objectives. Some of these pedagogical features are listed below.

- Appendix A briefly reviews some of the concepts from the pre-requisite course of introductory mechanics of materials. Brief reviews are also introduced in the text before new concepts are build on the introductory material.
- All internal forces and moments are in ***bold italics*** emphasizing the difference from external forces and moments.
- Every chapter starts with the section of *Overview* which describes the motivation for studying the chapter and the major learning objective(s) in the chapter.
- Every *Example* problem starts with a *Plan* and ends with *Comments*, both of which are indented to emphasize the importance of these two features. Developing a *Plan* before solving a problem is essential for the development of analysis skills. *Comments* are, observations deduced from the example highlighting concepts discussed in the text before the example, or, observations that suggests the direction of development of concepts in the text after the example.

I welcome any comments, suggestions, concerns, or corrections you may have that will help me improve the book. You could either relay your input to the publisher or to me. My e-mail address is: mavable@mtu.edu.

Note to the Instructor

The best way by which I can show you how the presentation in this book meets the objectives stated in the ‘Preface’ is by drawing your attention to specific features. This note also gives my perspective on topics covered in each chapter from an instructor’s view point.

In the introductory course of mechanics of materials the students learn the theories for axial rods, torsion of circular shaft, and symmetric bending of beams. The derivation of all three theories is presented in a consolidated form as a synopsis in Table 3.3 (on page 115) which highlights the commonality in the three theories and the modular character that is depicted in Figure 3.1 (on page 104). The four links connecting the five variables shown in Figure 3.1 are: kinematic equations between displacements and strains; the constitutive equations between strains and stresses; the equivalency equations between stresses and internal forces; and the equilibrium equations between internal and external forces. It is emphasized that if the assumptions in one module are changed then these changes only affect the equations in that module while the equations in other modules remain unchanged—Examples 3.1, 3.2, and 3.3 elaborate this observation. With this view, the beam vibration equations are simple modification of equilibrium equations as demonstrated in Example 3.6 on page 121. Similarly inclusion of dynamic terms in axial members (problem 3.25) and torsion of shafts (problem 3.28) and the foundation effects in beams on elastic foundations (problem 3.23) are simple modifications of equilibrium equations and are given as post-text problems. In Example 3.7 the derivation of equations governing the deformation of Timoshenko beam is demonstrated as a change in kinematics while all other equations remain the same, and though we carry a new set of variables, the process of moving from one step to next remains the same as in the derivation of the elementary theories highlighted in Table 3.3. Having demonstrated the modular character in the derivation of theories and how complexities are incorporated, the theories on ‘Composite Structural Members’ in Chapter 4 and on ‘Inelastic Structural Behavior’ in Chapter 5 can be obtained by modifications as described in the ‘Overview’ of each chapter. ‘Thin Walled Structural Members’ in Chapter 6, the starting displacement equation is different but the steps and assumptions in the derivation of the theory on unsymmetric bending of beam follow the same sequence as symmetric bending of beams in Table 3.3. I believe (student performance and feed back bolster this belief) this presentation consolidates what the student learned in the introductory course, the repetitive and the compact character of derivation helps in the understanding and retention of the key ideas, and exposes the students to a vast array of com-

plexities in the derivation of theories of one-dimensional structural elements. It is not my intention to convey the impression that students understand all the implications of all the complexities they see in this book. For greater understanding of complexities, time has to be spent with the application of the solution to the equations as is done in the courses that will use these theories, but I believe the students do learn to appreciate the mechanics of incorporating complexities into the elementary theory of one-dimensional structures they learned in the introductory course of mechanics of materials.

Chapters 1 through 3 briefly review introductory mechanics of materials, introduce notation, and then introduce new concepts that build on what the student already knows. The students in the introductory course have seen stress and strain transformation in two-dimensions. Familiar conclusions from two dimensions are derived using the matrix method and the matrix method approach is generalized for use in stress and strain transformation in three dimensions. The familiar generalized Hooke's law is discussed as a sub-class of linear material models in Chapter 2. One of the conclusions that is observed is that the principal directions of stresses and strains are only the same for isotropic materials, but for other types of materials such as orthotropic material, the principal directions for stresses and strains are different. Stress concentration factor and stress intensity factor are introduced as means of extrapolating nominal stress results from elementary structural theories into regions of stress concentration and vicinity of cracks. Failure theories and fatigue are introduced, and along with stress concentration factor and stress intensity factor used during review of axial, torsion, and bending problems in Chapter 3. Shear stresses in bending are covered in more detail than needed in a review as it is critical in the determination of shear centers and because many students in the introductory course struggle with it. Discontinuity functions introduced in Chapter 3 are used throughout the book for statically determinate and indeterminate axial, torsion, and bending problems.

Chapter 7 covers the classical energy methods in detail and provides the necessary concepts for the next chapter. In Chapter 8, Rayleigh-Ritz's method is used for deriving conclusions and equations that affect the effective use of the finite element method without excessive algebra. The terminology used in the finite element method is introduced and simple problems of axial, torsion, and bending for which analytical solutions can be obtained are solved by the finite element method. Chapter 9 is the introduction to elasticity and the purpose is to provide a link to the graduate level course of 'Advanced mechanics of materials'.

The book has more material than can be covered in a fifteen week three credit course to account for instructor pace and choice of topics. Sample syllabus, lecture slides, and sample exams which I use with this book are posted as pdf files for downloading on my personal web page¹. All of these along with a solution manual will also be available to the instructors through the publisher.

1. University network changes sometimes change web addresses. I will maintain a personal web-page with a link for the educational material for Intermediate Mechanics of Materials or Mechanics of Materials II. Current web address is: <http://www.me.mtu.edu/%7Emavable/>