

INTERMEDIATE MECHANICS OF MATERIALS

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PREFACE

Plastics engineering is becoming a popular new minor, and biomedical engineering is a growing new discipline; in both fields, stress analysis requires an understanding of the mechanics of inelastic and nonlinear material behavior. Applications of inelastic and nonlinear material are also growing as metals compete with new materials in engineering design by operating in the plastic region through prestressing. The growing use of metal matrix composites, polymer composites, reinforced concrete, and wooden beams stiffened with steel strips and other laminated structures emphasizes the need to expose students to the analysis of one-dimensional composite structural members. The ubiquity of the finite element method in engineering analysis and design emphasizes the educational importance of energy principles and the concepts of the finite element method. Equally significantly, the evaluation and use the results produced by commercial finite element computer software requires an understanding of stress and strain transformation in three dimensions and of failure theories. Capstone senior design projects have added to the importance of understanding unsymmetric bending and the concept and use of shear centers in design. Beam and shaft vibrations, beams on elastic foundations, Timoshenko beams, and so on are among the many topics in existing aerospace, civil, and mechanical engineering courses that use the principles of mechanics of materials. If a student is to be taught the mechanics of the topics described without being overwhelmed by all their inherent complexities, then the presentation of the material must have coherence and compactness that consolidates what he or she has already learned in the introductory course and builds on it. This is the underlying design of this book, and the Note to the Instructor elaborates on it.

An introductory course on the mechanics of materials is required in most engineering disciplines. The educational philosophy behind the requirement is to teach common mechanics concepts and principles in a single course and to present the extensions and applications of the mechanics concepts and principles in individual disciplines. This educational philosophy addresses the need of interdisciplinary education while realizing curriculum efficiency by reducing duplication of course content—an important consideration as educators attempt to modernize engineering education by incorporating research into a burgeoning curriculum. The introductory course on the mechanics of materials has served the engineering community well, but the tremendous growth in the applications of mechanics of materials is bringing added importance to a second course, Intermediate Mechanics of Materials. The intermediate course is often taught by instructors who use their own notes in conjunction with either optional topics from a textbook designed primarily to teach the introductory course or selected topics from a book primarily designed for a graduate course on advanced mechanics of materials. The pedagogical needs of the students with respect to theoretical details, numerical examples, and posttext problems are difficult to meet and put undue burden on the instructor teaching the course—such has been my experience in the past twenty years. This book is designed to provide educational material for a second course on the mechanics of materials taught to juniors or seniors.

Although consistent in its design and notation with my introductory mechanics of materials book, this book does not depend upon the book used for an introductory course.^x

There are many pedagogical features to help students meet the educational objectives. The Note to the Student describes some of the features that address their pedagogical needs.

I welcome any comments, suggestions, concerns, or corrections that will help me improve the book. Readers may relay their input to the publisher or to me. My e-mail address is mavable@mtu.edu.

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To my children, Anusha and Adhiraj, who are such a joy, I am sorry I could not incorporate your suggestion for the title of this book ‘An exciting sequel.’

NOTE TO THE STUDENT

The following features, in particular, should help you meet the learning objectives of the book.

- Appendix A briefly reviews some of the concepts from the prerequisite course on introductory mechanics of materials. Other brief reviews, particularly in the first three chapters, are introduced in the text before new concepts are built on the introductory material.
- Appendix B reviews basic matrix algebra that is needed in the book.
- All internal forces and moments are in ***bold italics***, emphasizing their difference from external forces and moments.
- Every chapter starts with the section titled *Overview*, which describes the motivation for studying the chapter and the major learning objective(s) in the chapter.
- Every chapter ends with the section titled *Closure*, which highlights the important points and concepts studied in the chapter.
- Every example statement is followed by a section called *Plan* and ends with *Comments*. Developing a *plan* before solving a problem is essential for the development of analysis skills. The *comments* are observations deduced from the example, highlighting concepts discussed in the text that precedes the example.
- On the inside back cover of the textbook is a *formula sheet* for easy reference. To give your instructor the option of permitting the use of the formula sheet in an exam, there is no explanation of the variables or the equations.

NOTE TO THE INSTRUCTOR

The best way of showing how the presentation in this book meets the objectives stated in the Preface is to draw your attention to specific features. This note also gives my own instructor's perspective on topics covered in each chapter.

In the introductory course on the mechanics of materials, the students learn the theories for axial rods, torsion of circular shafts, and symmetric bending of beams. The derivation of all three theories is presented in a consolidated form as a synopsis in Table 3.3 (on pages 113–115), which highlights the commonality in the three theories and the modular character that is depicted in Figure 3.2 (on page 104). The four links connecting the five variables shown in Figure 3.2 are kinematic equations relating displacements and strains, the constitutive equations relating strains and stresses, the equivalencies between stresses and internal forces, and the equilibrium equations relating internal and external forces. Any changes to the assumptions in one module affect only the equations in that module; the equations in other modules remain unchanged. With this view, the beam vibration equations are a simple modification of equilibrium equations as demonstrated in Example 3.6 on page 122. Similarly, inclusion of dynamic terms in axial members (Problem 3.25) and the torsion of shafts (Problem 3.28) and the foundation effects in beams on elastic foundations (Problem 3.23) are simple modifications of equilibrium equations and are given as posttext problems. In Example 3.7 the derivation of equations governing the deformation of the Timoshenko beam is demonstrated as a change in kine-

matics, while all other equations remain the same; although we carry a new set of variables, the process of moving from one step to the next remains the same as in the derivation of the elementary theories highlighted in Table 3.3. Having demonstrated the modularity in the derivation of theories and how complexities are incorporated, the theories on composite structural members (Chapter 4), inelastic structural behavior (Chapter 5), and thin-walled structural members (Chapter 6) can be obtained by modifications as described in the Overview of each chapter.

Based on student performance and feedback, I believe that the foregoing presentation consolidates what the student learned in the introductory course, and that the repetitive and the compact character of the derivations both helps in the understanding and retention of the key ideas and exposes the students to a vast array of complexities in the derivation of theories of one-dimensional structural members. It is not my intention to convey the impression that students understand all the implications of all the complexities they see in this book. For greater understanding of complexities, time has to be spent with the application of the solution to the equations, as is done in the courses that will use these theories. I believe, however, that students learn to appreciate the mechanics of incorporating complexities into the elementary theory of one-dimensional structures they learned in the introductory course of mechanics of materials.

Chapters 1 through 3 briefly review introductory mechanics of materials, introduce notation, and then introduce new concepts that build on what the student already knows. Students in the introductory course have seen stress and strain transformation in two dimensions. Familiar conclusions from two dimensions are derived by using the matrix method, and the matrix method approach is generalized for use in stress and strain transformation in three dimensions. The familiar generalized Hooke's law is discussed as a subclass of linear material models in Chapter 2. One of the conclusions that is observed is that the principal direction of stresses and strains is the same only for isotropic materials; for materials of other types, such as orthotropic materials, the principal directions for stresses and strains are different. Stress concentration factors and stress intensity factors are introduced as a means of extrapolating nominal stress results from elementary structural theories into regions of stress concentration and vicinity of cracks. Failure theories and fatigue are introduced, and are used along with stress concentration factors and stress intensity factors during a review of axial, torsion, and bending problems in Chapter 3. Shear stresses in bending are covered in more detail than is needed in a review, as this topic is critical in the determination of shear centers and because many students in the introductory course struggle with it. Discontinuity functions, introduced in Chapter 3, are used throughout the book for statically determinate and indeterminate axial, torsion, and bending problems.

Chapter 7 covers the classical energy methods in detail. Once more, concepts are elaborated by using one-dimensional structural members. Chapter 8 introduces the basic equations in elasticity and its relationship to mechanics of materials. Elasticity is also used to systematically obtain results for thick cylinders, thin rotating disks, and torsion of noncircular shafts, all within the capability of an undergraduate student. In Chapter 9, the Rayleigh-Ritz method introduced in Chapter 7 is used to formulate a procedure for applying the finite element method. The terminology used in the finite element method is introduced, and the method is applied to solve simple axial, torsion, and bending problems for which analytical solutions can be obtained.

The book requires elementary knowledge of partial derivatives, some matrix algebra, and simple calculus, which most undergraduate students have been exposed to in their cur-

riculum. Though the mathematics is not involved, the algebra can be tedious. Upper-end calculators and symbolic manipulators can reduce the tedium, but each instructor will have to define what is permissible.

The book has more material than can be covered in a fifteen-week, three-credit course to accommodate the pace and choice of topics of individual instructors. A sample syllabus, lecture slides, and sample exams that I use with this book are posted as pdf files for downloading on my personal web page. All these, along with a solution manual, will also be available to the instructors through the publisher.