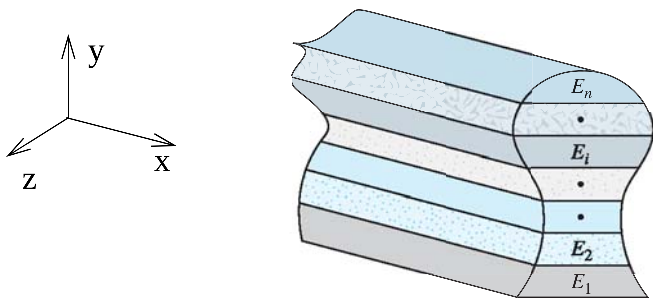
Composite Structural Members

- Assumption 8 on material homogenity across the cross-section is not valid.

The learning objective of this chapter is:

Understand the incorporation and implications of material in-homogeneity across the cross-section in the theories for axial members, circular shafts in torsion, and symmetric bending of beams.

Composite Axial Members



$$\sigma_{xx} = E\frac{du}{dx}(x)$$

Internal Forces and Moments

$$N = \int_A \sigma_{xx} dA \qquad M_z = -\int_A y\sigma_{xx} dA = 0 \quad \text{or}$$

$$N = \frac{du}{dx}(x)\int_A E dA$$

Location of origin: $$\int_A yE dA = 0$$

Formulas for composite axial rods

$$N = \frac{du}{dx}\int_A E\,dA = \frac{du}{dx}\left[\int_{A_1} E_1 dA + \int_{A_2} E_2 dA + \cdot + \cdot + \int_{A_n} E_n dA\right]$$

$$N = \frac{du}{dx}\left[\sum_{j=1}^{n} E_j A_j\right]$$

$$(\sigma_{xx})_i = \frac{N E_i}{\sum_{j=1}^{n} E_j A_j}$$

$$u_2 - u_1 = \frac{N(x_2 - x_1)}{\sum_{j=1}^{n} E_j A_j}$$

Location of axial force application (origin)

$$\eta_c = \frac{\sum_{i=1}^{n} \eta_i E_i A_i}{\sum_{i=1}^{n} E_i A_i}$$

4.3 A wooden rod (E_W = 2000 ksi) and steel strip (E_s = 30,000 ksi) are fastened securely to each other and to the rigid plates as shown in Figure P4.3. Determine (a) the location h of the line along which the external forces must act to produce no bending. (b) the maximum axial stress in steel and wood.

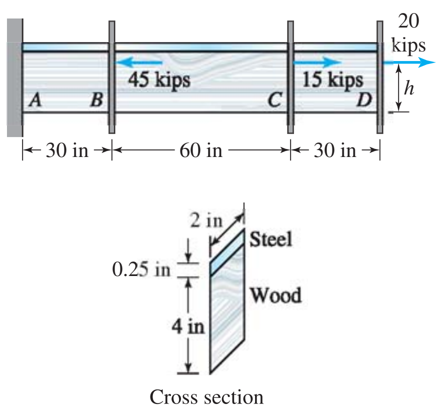


Figure P4.3

4.8 A for use in a building is modeled as shown in Figure P4.8. The column is constructed by reinforcing concrete with nine steel circular bars of diameter 1 inch. The modulus of elasticity for concrete and iron are E_c= 4,500 ksi and E_i = 25,000 ksi. Determine the maximum axial stress in concrete and steel.

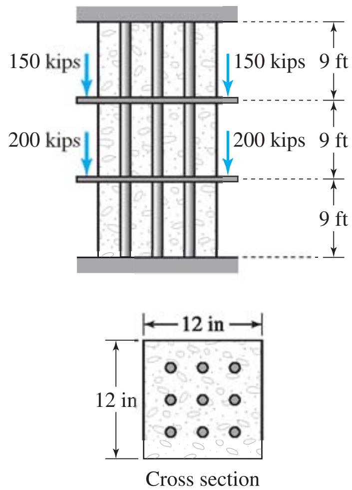
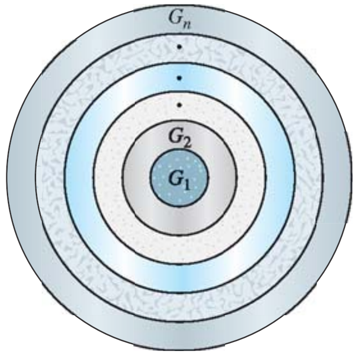


Figure P4.8

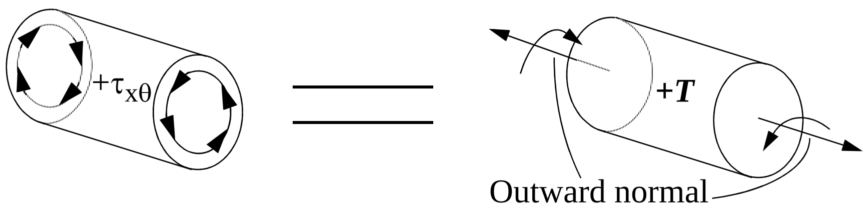
Composite Shafts



$$\tau_{x\theta} = G\rho\frac{d\phi}{dx}(x)$$

Internal Forces and Moments

$$T = \int_A \rho\tau_{x\theta}\,dA \quad \text{or} \quad \boldsymbol{T} = \frac{d\phi}{dx}\int_A G\rho^2\,dA$$



Formulas for composite shafts

$$\boldsymbol{T} = \frac{d\phi}{dx}\left[\int_{A_1} G_1\rho^2\,dA + \int_{A_2} G_2\rho^2\,dA + \cdot + \cdot + \int_{A_n} G_n\rho^2\,dA\right]$$

$$\boldsymbol{T} = \frac{d\phi}{dx}\left[\sum_{j=1}^{n} G_j J_j\right]$$

$$(\tau_{x\theta})_i = \frac{G_i\rho\boldsymbol{T}}{\left[\sum_{j=1}^{n} G_j J_j\right]}$$

$$\phi_2 - \phi_1 = \frac{\boldsymbol{T}(x_2 - x_1)}{\left[\sum_{j=1}^{n} G_j J_j\right]}$$

4.12 A solid steel (G = 80 GPa) shaft 3 m long is securely fastened to a hollow bronze (G = 40 GPa) shaft that is 2 m long as shown Figure P4.12. Determine (a) the magnitude of maximum shear stress in the shaft. (b) the rotation of section at 1 m from the left wall.

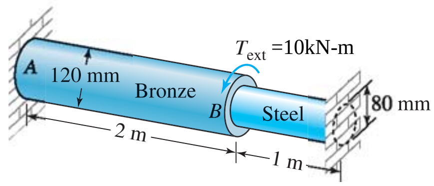
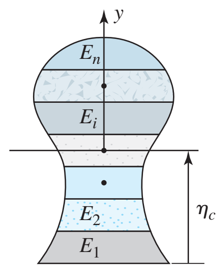


Figure P4.12

Composite Beams



$$\sigma_{xx} = -Ey\frac{d^2v}{dx^2}(x)$$

Internal Forces and Moments

$$N = \int_A \sigma_{xx} dA = 0 \qquad M_z = -\int_A y\sigma_{xx} dA \quad \text{or}$$

$$\mathbf{M}_z = \frac{d^2v}{dx^2}\int_A Ey^2 dA$$

Location of origin: $\int_A yE dA = 0$

Formulas for composite beams

$$\mathbf{M_z} = \frac{d^2v}{dx^2}\left[\int_{A_1} E_1 y^2 dA + \int_{A_2} E_2 y^2 dA + \cdot + \cdot + \int_{A_n} E_n y^2 dA\right]$$

$$\boxed{\mathbf{M_z} = \frac{d^2v}{dx^2}\left[\sum_{j=1}^{n} E_j (I_{zz})_j\right]} \qquad \boxed{(\sigma_{xx})_i = -\left[\frac{E_i y \mathbf{M_z}}{\sum_{j=1}^{n} E_j (I_{zz})_j}\right]}$$

Location of neutral axis (origin) : $\eta_c = \dfrac{\sum_{i=1}^{n} \eta_i E_i A_i}{\sum_{i=1}^{n} E_i A_i}$

Bending shear stress in composite beams

Equilibrium equation

$$\tau_{sx}t = -\frac{d\boldsymbol{N}_s}{dx} = -\frac{d}{dx}\int_{A_s}\sigma_{xx}dA$$

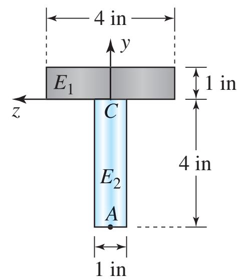
$$\tau_{sx}t = -\frac{d}{dx}\int_{A_s}\frac{-Ey\boldsymbol{M_z}}{\sum_{j=1}^{n}E_j(I_{zz})_j}dA = \frac{d}{dx}\left|\frac{\boldsymbol{M_z}}{\sum_{j=1}^{n}E_j(I_{zz})_j}\int_{A_s}EydA\right| = \frac{d}{dx}\left|\frac{\boldsymbol{M_z}Q_{comp}}{\sum_{j=1}^{n}E_j(I_{zz})_j}\right|$$

$$Q_{comp} = \int_{A_s}EydA$$

$$\boxed{\tau_{sx} = \tau_{xs} = -\frac{Q_{comp}\boldsymbol{V_y}}{\left[\sum_{j=1}^{n}E_j(I_{zz})_j\right]t}}$$

$$\boxed{Q_{comp} = \sum_{j=1}^{n_s}E_j(Q_z)_j}$$

4.16 The cross-section of a composite beam with a coordinate system that has an origin at C is shown . The normal strain at point A due to bending about the z-axis is $\varepsilon_{xx} = -200\ \mu$, and the modulus of elasticity of the materials are $E_1 = 8000\ ksi$ and $E_2 = 2000\ ksi$ (a) Plot the stress distribution across the cross-section. (b) Determine the maximum bending normal stress in each material. (c) Determine the equivalent internal bending moment M_z.



4.20 A wooden rod (E_W = 2000 ksi) and steel strip (E_s = 30,000 ksi) are fastened securely to rigid plates as shown. Determine (a) the maximum intensity of the load w, if the allowable bending normal stresses in steel and wood are 20 ksi, and 4 ksi.respectively. (b) the magnitude of the maximum shear stress in the beam corresponding to the load in part (a).