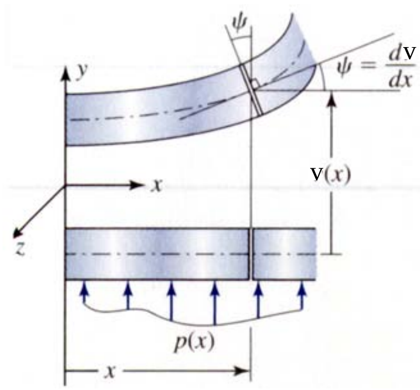
Deflection of Symmetric Beams

Learning objective

- Learn to formulate and solve the boundary-value problem for the deflection of a beam at any point.

Second-Order Boundary Value Problem

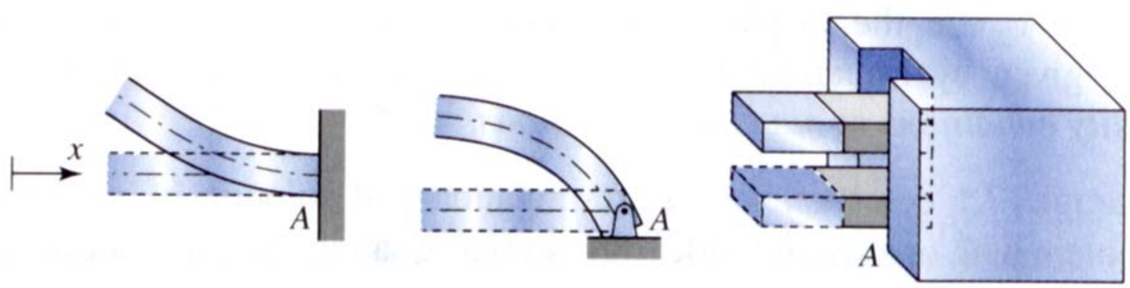


- The deflected curve represented by v(x) is called the Elastic Curve.

Differential equation: $\boldsymbol{M}_z = EI_{zz}\dfrac{d^2\text{v}}{dx^2}$

- The mathematical statement listing all the differential equations and all the conditions necessary for solving for v(x) is called the Boundary Value Problem for the beam deflection.

Boundary Conditions



$\text{v}(x_A) = 0$ $\qquad$ $\text{v}(x_A) = 0$ $\qquad$ $\dfrac{d\text{v}}{dx}(x_A) = 0$

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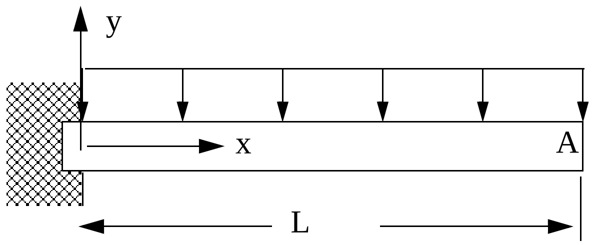
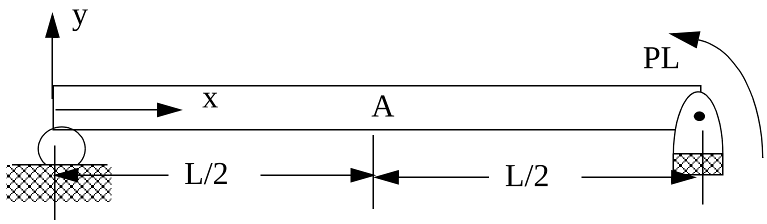
C7.1 In terms of w, P, L, E, and I determine (a) equation of the elastic curve. (b) the deflection of the beam at point A.



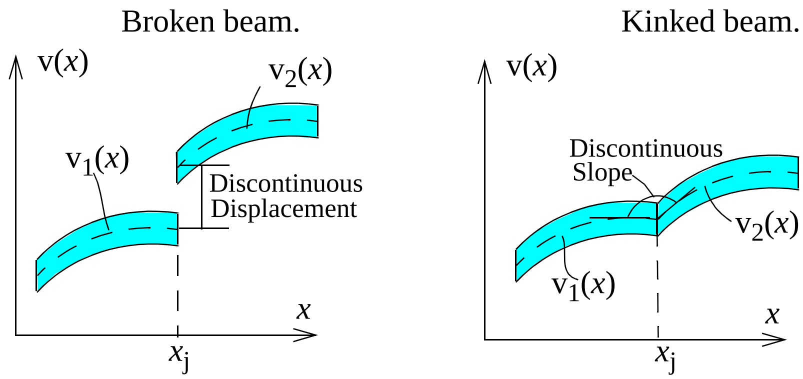
Class Problem 1

Write the boundary value problem to determine the elastic curve.



Continuity Conditions

- The internal moment $\boldsymbol{M_z}$ will change with change in applied loading.
- Each change in $\boldsymbol{M_z}$ represents a new differential equation, hence new integration constants.



$$v_1(x_j) = v_2(x_j)$$

$$\frac{dv_1}{dx}(x_j) = \frac{dv_2}{dx}(x_j)$$

- 'continuity conditions', also known as 'compatibility conditions' or 'matching conditions'.

C7.2 In terms of w, L, E, and I, determine (a) the equation of the elastic curve. (b) the deflection at x = L.

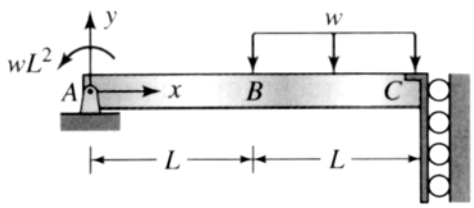
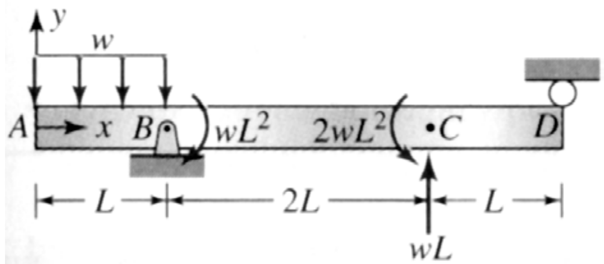


Fig. C1.2

Class Problem 2

C7.3 Write the boundary value problem for determining the deflection of the beam at any point x. Assume EI is constant. Do not integrate or solve.



The internal moments are:

In AB: $M_1 = -\frac{wx^2}{2}$

In BC: $M_2 = \frac{wLx}{6} + \frac{wL^2}{3}$

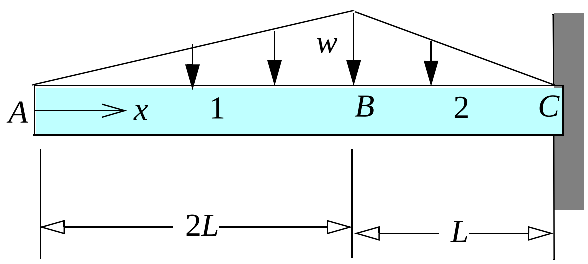
In CD: $M_3 = \frac{7wLx}{6} - \frac{14wL^2}{3}$

Class Problem 3

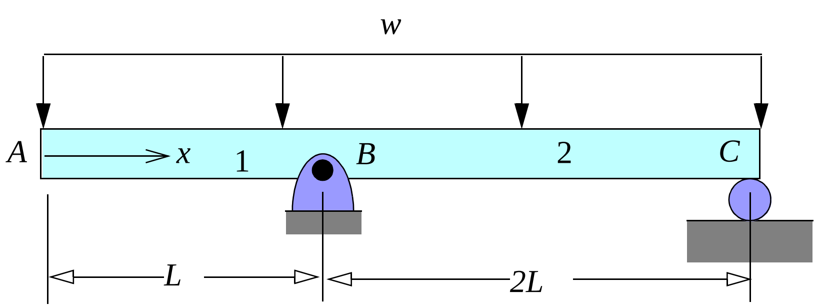
v_1 and v_2 represents the deflection in segment *AB* and *BC*. For the beams shown, identify all the conditions from the table needed to solve for the deflection v(x) at any point on the beam.

(a) $v_1(0) = 0$	(e) $v_2(2L) = 0$	(i) $v_1(L) = v_2(L)$
(b) $v_1(L) = 0$	(f) $v_2(3L) = 0$	(j) $v_1(2L) = v_2(2L)$
(c) $v_2(L) = 0$	(g) $\frac{dv_1}{dx}(0) = 0$	(k) $\frac{dv_1}{dx}(L) = \frac{dv_2}{dx}(L)$
(d) $v_1(2L) = 0$	(h) $\frac{dv_2}{dx}(3L) = 0$	(l) $\frac{dv_1}{dx}(2L) = \frac{dv_2}{dx}(2L)$

Beam 1



Beam 2



Beam 3