Axial Members

- Members with length significantly greater than the largest cross-sectional dimension and with loads applied along the longitudinal axis.

Cables of Mackinaw bridge

(a)

Hydraulic cylinders in a dump truck

(b)

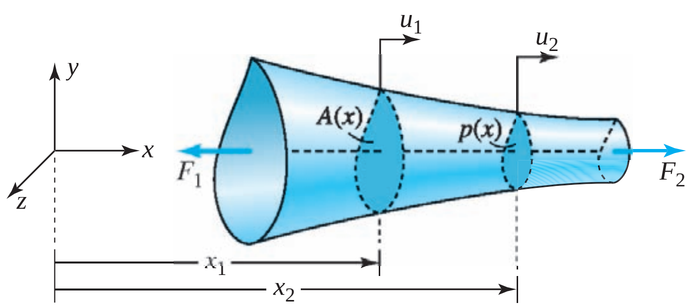
Learning objectives are:

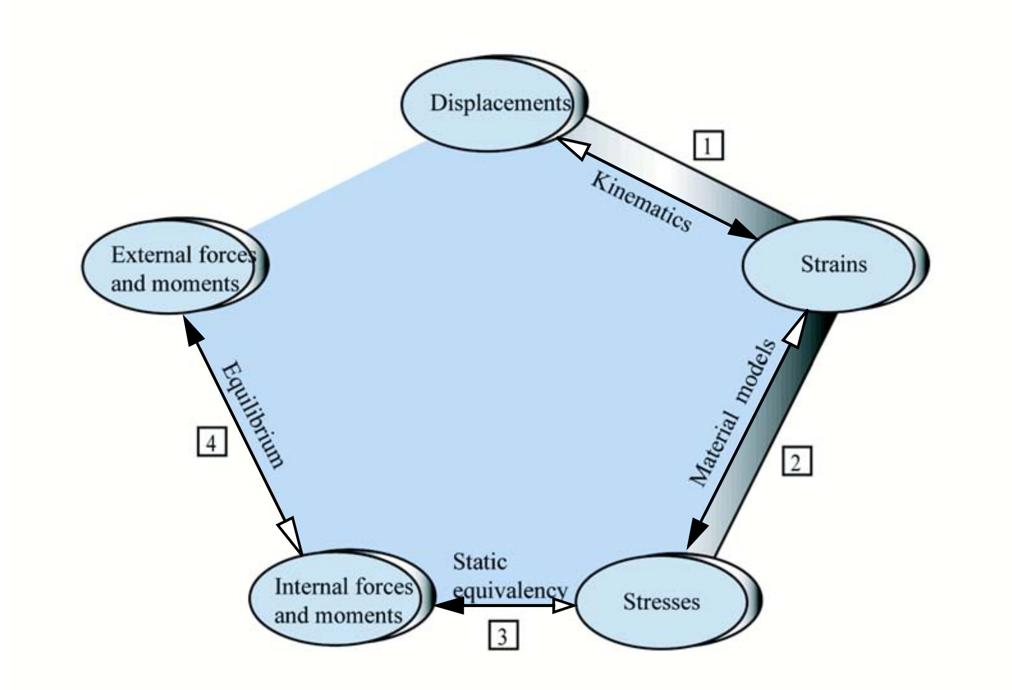
- Understand the theory, its limitations, and its applications for design and analysis of axial members.
- Develop the discipline to draw free body diagrams and approximate deformed shapes in the design and analysis of structures.

Theory

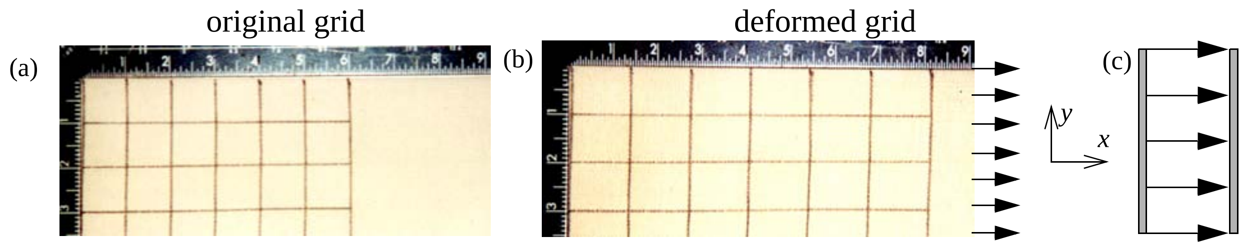
Theory Objective

- to obtain a formula for the relative displacements (u_2-u_1) in terms of the internal axial force ***N***.
- to obtain a formula for the axial stress σ_{xx} in terms of the internal axial force ***N***.





Kinematics



Assumption 1 Plane sections remain plane and parallel. $u = u(x)$

- The displacement u is considered positive in the positive x-direction.

Assumption 2 Strains are small. $\varepsilon_{xx} = \dfrac{du(x)}{dx}$

Material Model

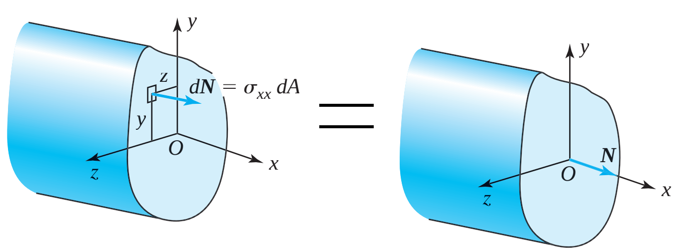
Assumption 3 Material is isotropic.

Assumption 4 Material is linearly elastic.

Assumption 5 There are no inelastic strains.

From Hooke's Law: $\sigma_{xx} = E\varepsilon_{xx}$, we obtain $\sigma_{xx} = E\dfrac{du}{dx}$

Internal Axial Force



$$N = \int_A \sigma_{xx} dA$$

$$N = \int_A E\frac{du}{dx}dA = \frac{du}{dx}\int_A E dA$$

- For pure axial problems the internal moments (bending) $\boldsymbol{M}_y$ and $\boldsymbol{M}_z$ must be zero.
- For homogenous materials all external and internal axial forces must pass through the centroids of the cross-section and all centroids must lie on a straight line.

Axial Formulas

Assumption 6 Material is homogenous across the cross-section.

$$\boldsymbol{N} = E\frac{du}{dx}\int_A dA = EA\frac{du}{dx} \qquad or \qquad \frac{du}{dx} = \frac{\boldsymbol{N}}{EA}$$

$$\sigma_{xx} = E\frac{du}{dx} = E\left(\frac{\boldsymbol{N}}{EA}\right) \qquad or \qquad \sigma_{xx} = \frac{\boldsymbol{N}}{A}$$

- The quantity EA is called the Axial rigidity.

Assumption 7 Material is homogenous between x_1 and x_2.

Assumption 8 The bar is not tapered between x_1 and x_2.

Assumption 9 The external (hence internal) axial force does not change with x between x_1 and x_2.

$$u_2 - u_1 = \frac{\boldsymbol{N}(x_2 - x_1)}{EA}$$

Two options for determining internal axial force ***N***

- ***N*** is always drawn in tension at the imaginary cut on the free body diagram.
  - Positive value of σ_{xx} will be tension.
  - Positive u_2-u_1 is extension.
  - Positive u is in the positive x-direction.
- ***N*** is drawn at the imaginary cut in a direction to equilibrate the external forces on the free body diagram.
  - Tension or compression for σ_{xx} has to be determined by inspection.
  - Extension or contraction for $\delta=u_2-u_1$ has to be determined by inspection.
  - Direction of displacement u has to be determined by inspection.

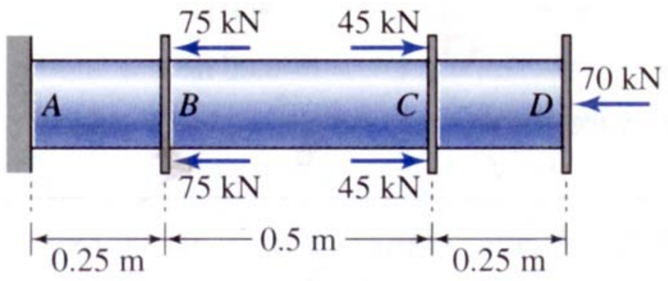
Axial stresses and strains

- all stress components except σ_{xx} can be assumed zero.

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E}$$

$$\varepsilon_{yy} = -\left(\frac{\nu\sigma_{xx}}{E}\right) = -\nu\varepsilon_{xx} \qquad \varepsilon_{zz} = -\left(\frac{\nu\sigma_{xx}}{E}\right) = -\nu\varepsilon_{xx}$$

C4.1 Determine the internal axial forces in segments AB, BC, and CD by making imaginary cuts and drawing free body diagrams.



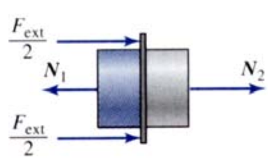
Axial Force Diagrams

- An axial force diagram is a plot of internal axial force ***N*** vs. x
- Internal axial force jumps by the value of the external force as one crosses the external force from left to right.
- An axial template is used to determine the direction of the jump in ***N.***
- A template is a free body diagram of a small segment of an axial bar created by making an imaginary cut just before and just after the section where the external force is applied.

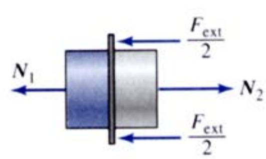
Template 1 Equation

$$N_2 = N_1 - F_{ext}$$

Template 1



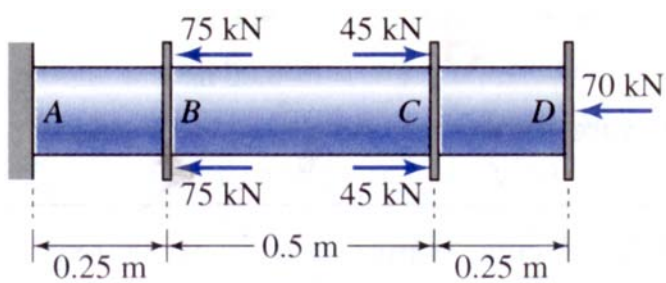
Template 2



Template 2 Equation

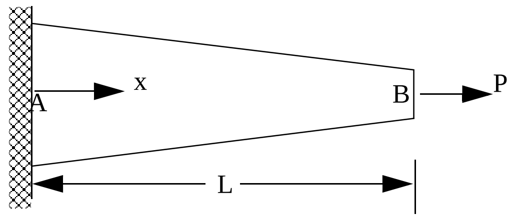
$$N_2 = N_1 + F_{ext}$$

C4.2 Determine the internal axial forces in segments AB, BC, and CD by drawing axial force diagram.



C4.3 The axial rigidity of the bar in problem 4.8 is EA = 80,000 kN. Determine the movement of section at C.

C4.4 The tapered bar shown in Fig. C4.4 has a cross-sectional area that varies with x as given. Determine the elongation of the bar in terms of P, L, E and K.



$$A = K(4L - 3x)$$

Fig. C4.4

C4.5 The columns shown has a length L, modulus of elasticity E, specific weight γ, and length a as the side of an equilateral triangle. Determine the contraction of the column in terms of L, E, γ, and a.

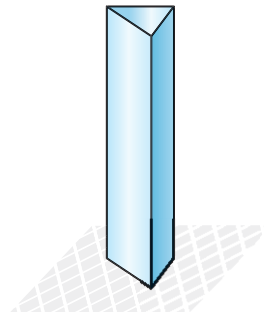
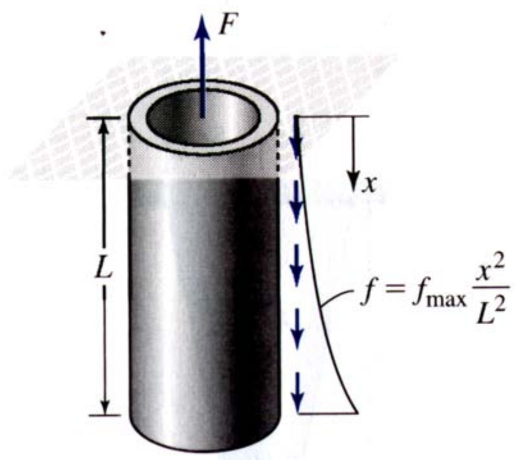


Fig. C4.5

C4.6 The frictional force per unit length on a cast iron pipe being pulled from the ground varies as a quadratic function as shown. Determine the force F needed to pull the pipe out of ground and the elongation of the pipe before the pipe slips in terms of the modulus of elasticity E, area of cross-section A, length L and the maximum value of frictional force f_{max}.



C4.7 A hitch for an automobile is to be designed for pulling a maximum load of 3,600 lbs. A solid-square-bar fits into a square-tube, and is held in place by a pin as shown. The allowable axial stress in the bar is 6 ksi, the allowable shear stress in the pin is 10 ksi, and the allowable axial stress in the steel tube is 12 ksi. To the nearest 1/16th of an inch, determine the minimum cross-sectional dimensions of the pin, the bar and the tube. Neglect stress concentration. (Note: Pin is in double shear)

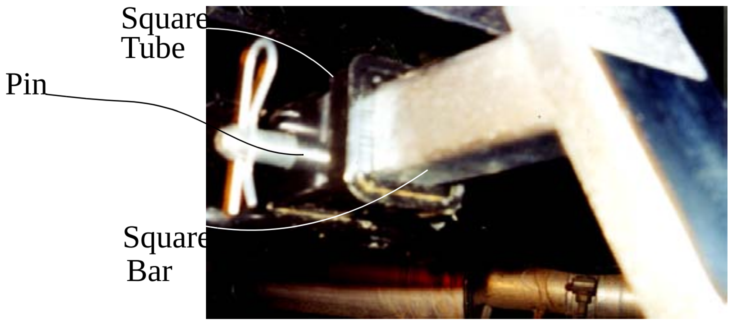


Fig. C4.7

Structural analysis

$$\delta = \frac{NL}{EA}$$

- δ is the deformation of the bar in the undeformed direction.
- If N is a tensile force then δ is elongation.
- If N is a compressive force then δ is contraction.
- Deformation of a member shown in the drawing of approximate deformed geometry must be consistent with the internal force in the member that is shown on the free body diagram.
- In statically indeterminate structures number of unknowns exceed the number of static equilibrium equations. The extra equations needed to solve the problem are relationships between deformations obtained from the deformed geometry.
- Force method----Internal forces or reaction forces are unknowns.
- Displacement method---Displacements of points are unknowns.

General Procedure for analysis of indeterminate structures.

- If there is a gap, assume it will close at equilibrium.
- Draw Free Body Diagrams, write equilibrium equations.
- Draw an exaggerated approximate deformed shape. Write compatibility equations.
- Write internal forces in terms of deformations for each member.
- Solve equations.
- Check if the assumption of gap closure is correct.

C4.8 A force F= 20 kN is applied to the roller that slides inside a slot. Both bars have an area of cross-section of A = 100 mm^2 and a Modulus of Elasticity E = 200 GPa. Bar AP and BP have lengths of L_{AP}= 200 mm and L_{BP}= 250 mm respectively. Determine the displacement of the roller and axial stress in bar A.

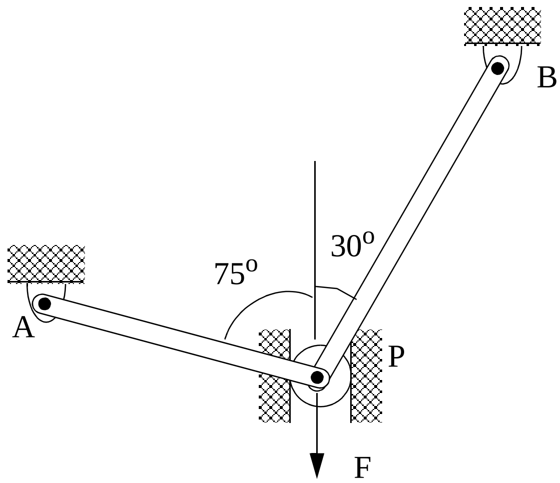


Fig. C4.8

C4.9 In Fig. C4.9, a gap exists between the rigid bar and rod A before the force F=75 kN is applied. The rigid bar is hinged at point C. The lengths of bar A and B are 1 m and 1.5 m respectively and the diameters are 50 mm and 30 mm respectively. The bars are made of steel with a modulus of elasticity E = 200 GPa and Poisson's ratio is 0.28. Determine (a) the deformation of the two bars. (b) the change in the diameters of the two bars.

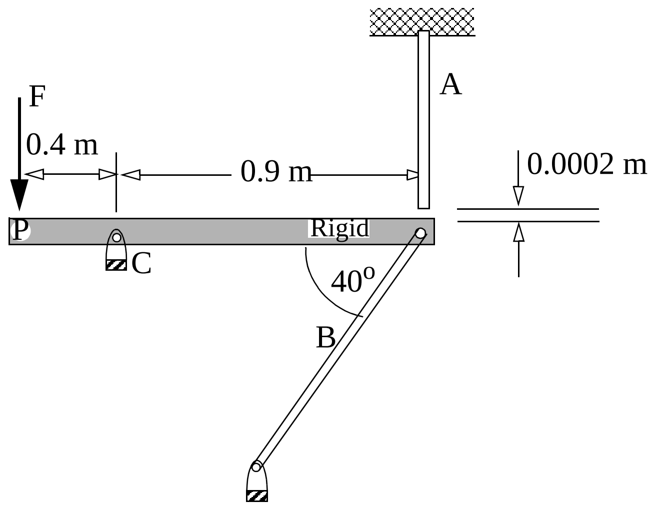
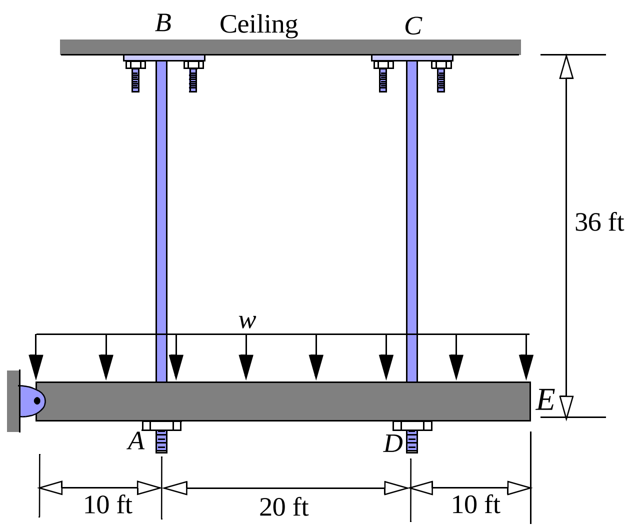


Fig. C4.9

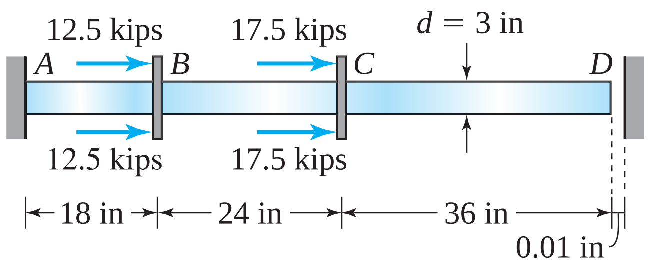
Class Problem 1

Write equilibrium and compatibility equations for the following problems.

Use displacement of point E δ_E as unknown.



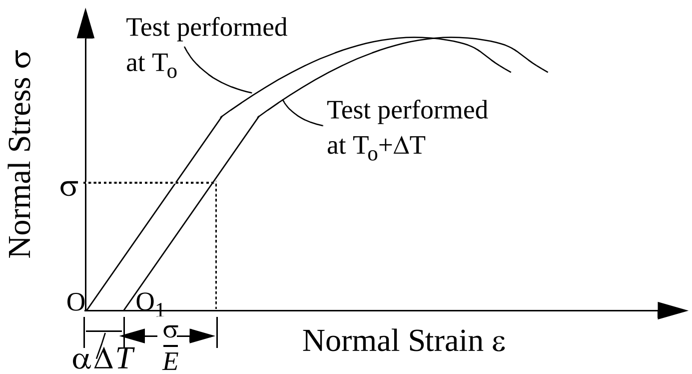
Use reaction force at A (R_A) as unknown.



Initial Stress/Strain and Temperature Effects

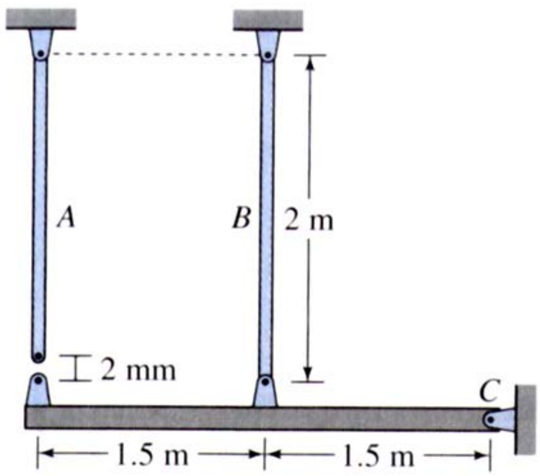
$$\delta = \frac{NL}{EA} + \varepsilon_o L$$

ε_o=Initial strain

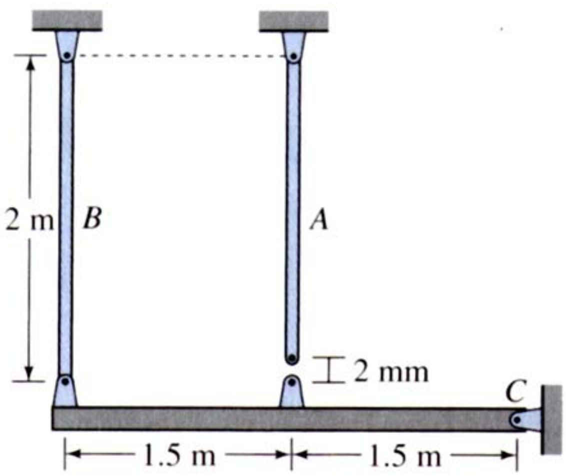


- $\varepsilon_o = \alpha\Delta T$ Thermal Strains.
- No thermal stresses are produced in a homogenous, isotropic, unconstrained body due to uniform temperature changes.
- Increase of temperature ---extension.
- Decrease of temperature---contraction.
- Sign of $\varepsilon_o L$ must be consistent with ***N*** shown on free body diagrams.

C4.10 Bar A was manufactured 2 mm less than bar B due to an error. The attachment of these bars to the rigid bar would cause a misfit of 2 mm. Calculate the initial stress in each assembly. Which of the two assembly configuration you would recommend? Use modulus of elasticity of E = 70 GPa and diameter of the circular bars as 25 mm.



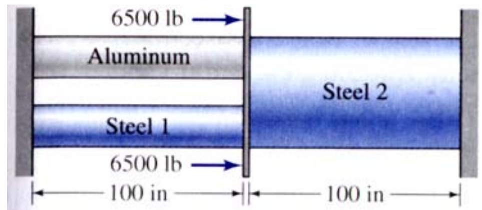
Assembly 1



Assembly 2

C4.11 Three metallic rods are attached to a rigid plate as shown. The temperature of the rods is lowered by 100 ^oF after the forces are applied. Assuming the rigid plate does not rotate, determine the movement of the rigid plate

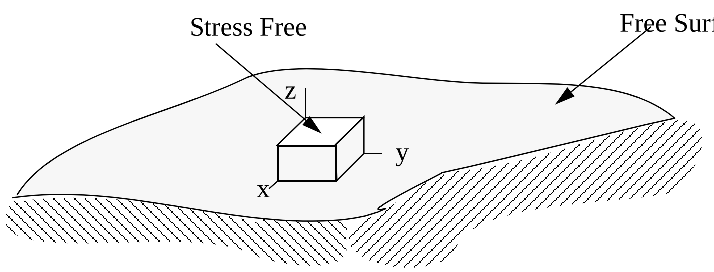
	Area in^2	E ksi	α $10^{-6}/ {}^oF$
Aluminum	4	10,000	12.5
Steel-1	4	30,000	6.6
Steel-2	12	30,000	6.6

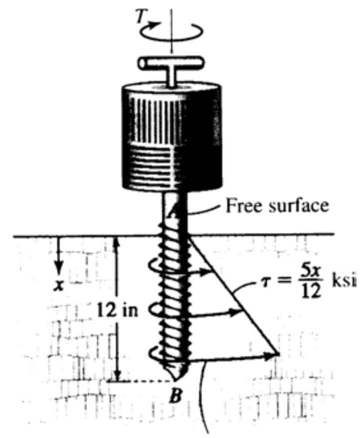


Stress Approximation

Free Surface

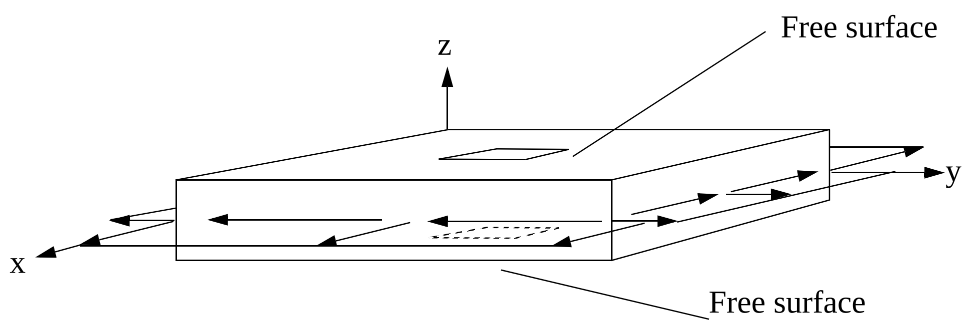
- A surface on which no external forces or moments are acting is called a *free surface*.





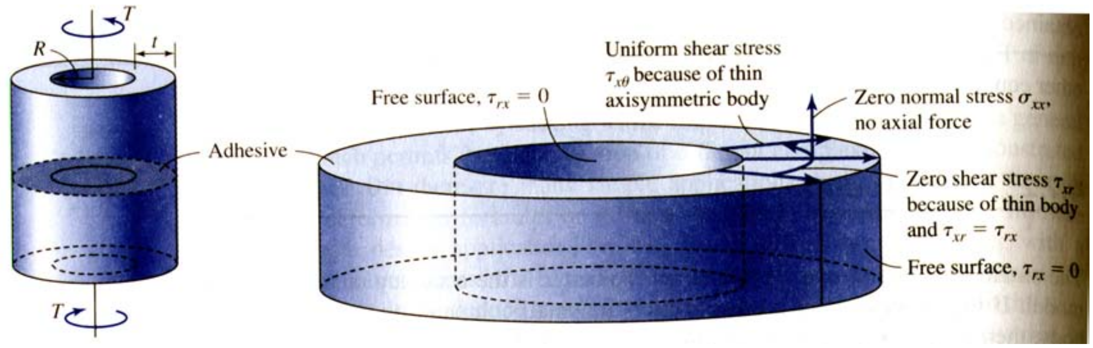
Thin Bodies

- The smaller the region of approximation, the better is the accuracy of the analytical model.



Axi-symmetric Bodies

- If a body has a cross-section that is symmetric about an axis and if the applied external forces or moments are also symmetric about the same axis, then the stresses cannot depend upon the *angular* location of the point.



Thin Walled Pressure Vessels

- The *"thin wall"* limitation implies that the ratio of inner radius R to the wall thickness t is greater than 10.

Cylindrical Vessels

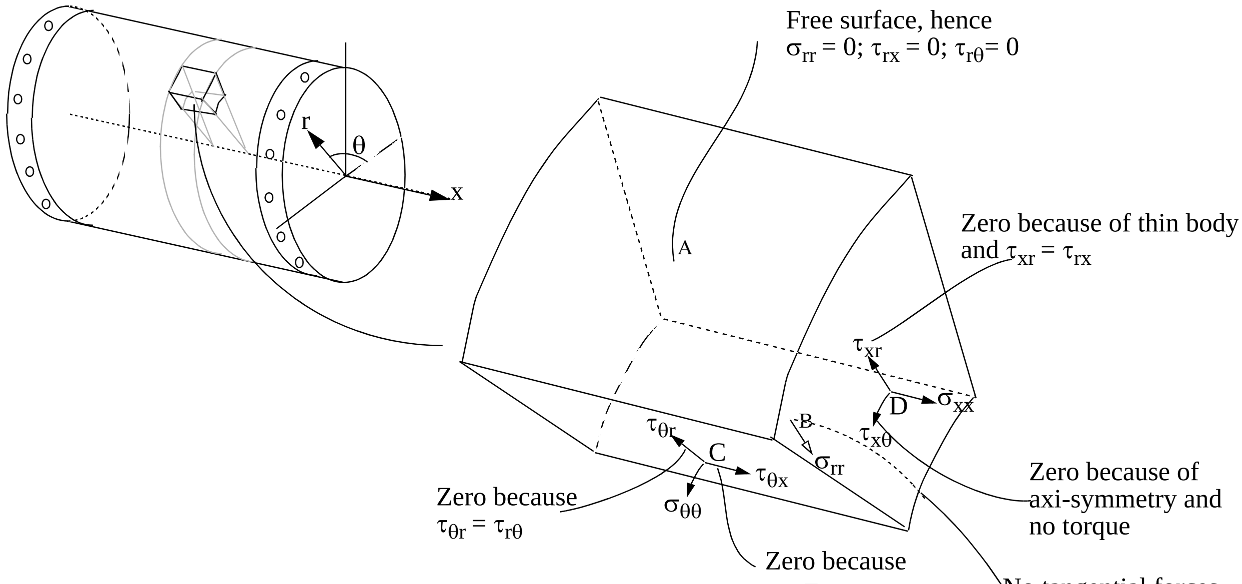
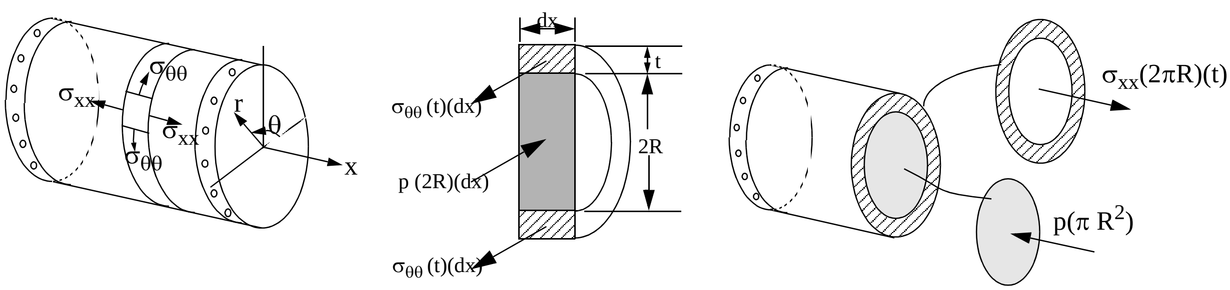


Figure 4.1. Stress element in cylindrical coordinates.

- All shear stresses are zero, the radial normal stress is neglected, the *axial stress* σ_{xx} and the *hoop stress* $\sigma_{\theta\theta}$ are assumed uniform across the thickness and across the circumference.



$$\sigma_{\theta\theta} = \frac{pR}{t} \qquad \sigma_{xx} = \frac{pR}{2t}$$

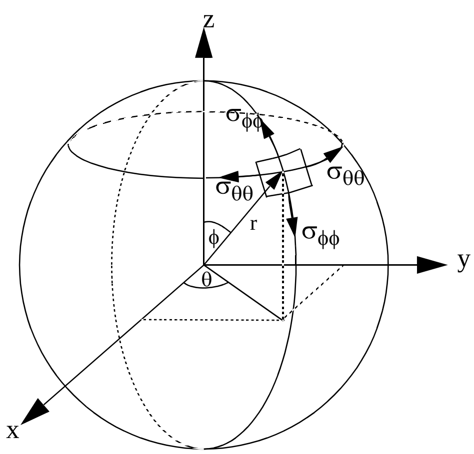
- With R/t > 10 the stresses σ_{xx} and $\sigma_{\theta\theta}$ are greater than the maximum value of radial stress σ_{rr} (=p) by a factor of at least 5 and 10, respectively.

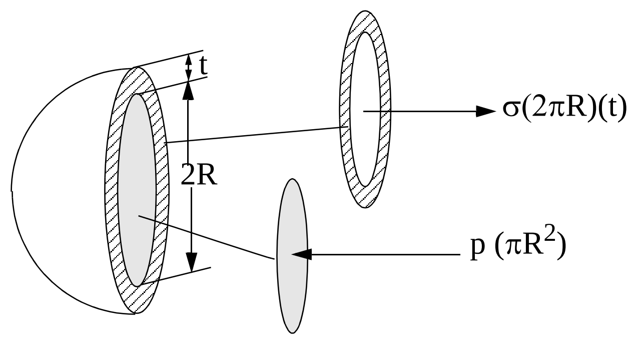
Spherical vessels

- *(i)* All shear stresses are zero. i.e.

$$\tau_{r\phi} = \tau_{\phi r} = 0 \qquad \tau_{r\theta} = \tau_{\theta r} = 0 \qquad \tau_{\theta\phi} = \tau_{\phi\theta} = 0$$

- *(ii)* Normal radial stress σ_{rr} varies from a zero value on the outside to the value of the pressure on the inside. We will once more neglect the radial stress in our analysis and justify it post-priori.
- *(iii)* The normal stresses $\sigma_{\theta\theta}$ and $\sigma_{\phi\phi}$ are equal and are constant over the *entire vessel*. We set $\sigma_{\theta\theta} = \sigma_{\phi\phi} = \sigma$.





$$\sigma = \frac{pR}{2t}$$