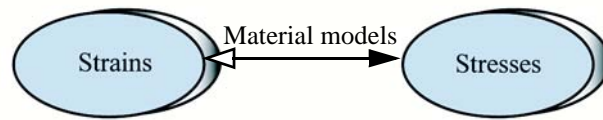


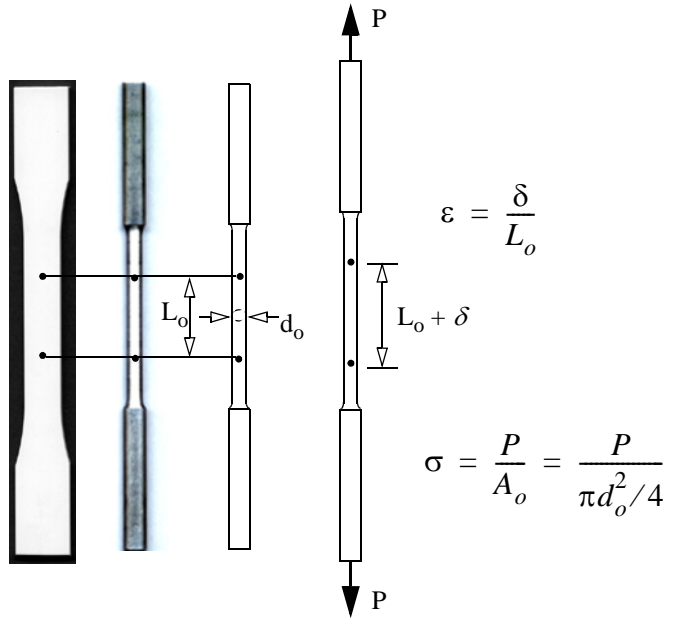
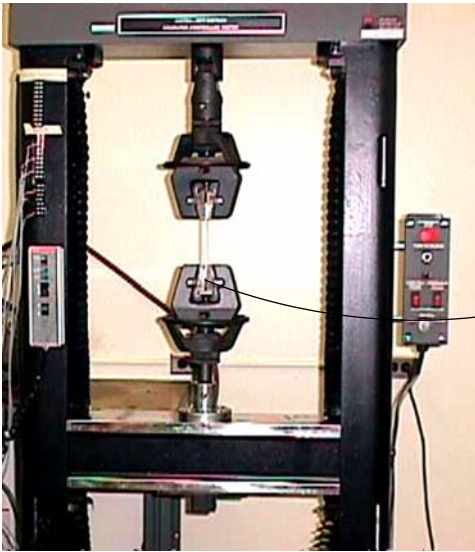
Mechanical Properties of Materials



Learning objectives

- Understand the qualitative and quantitative description of mechanical properties of materials.
- Learn the logic of relating deformation to external forces.

Tension Test



$$\epsilon = \frac{\delta}{L_0} \quad \sigma = \frac{P}{A_0} = \frac{P}{\pi d_0^2 / 4}$$

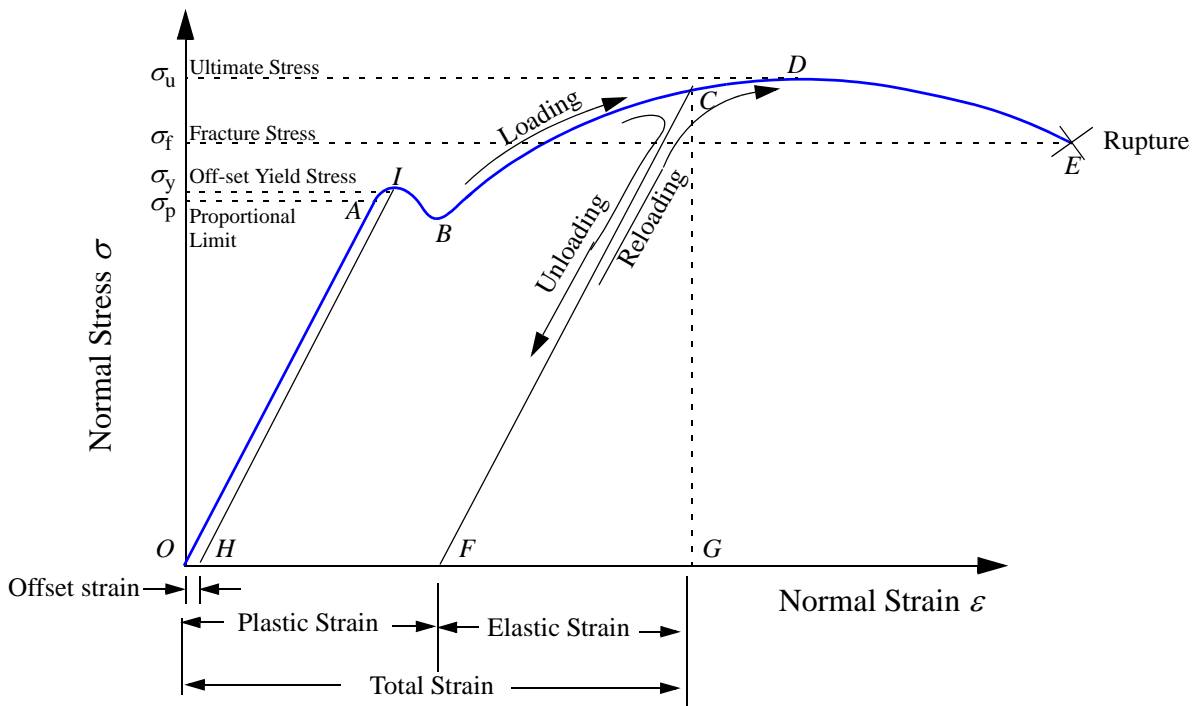


Figure 3.1 Stress-strain curve.

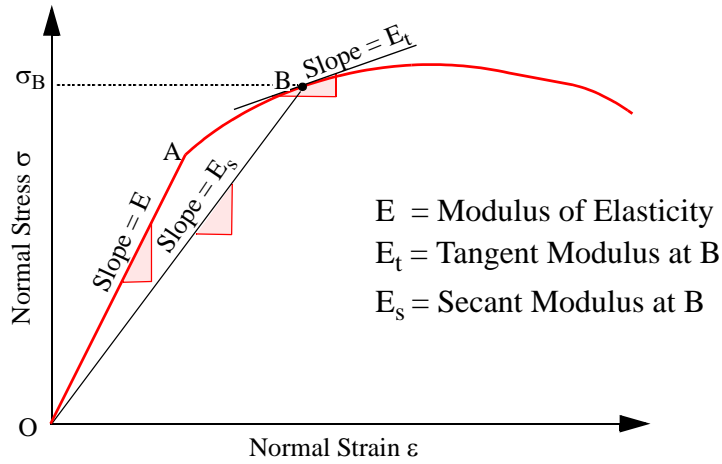
Printed from: <http://www.mech.mtu.edu/~mavable/MoM2nd.htm>

Definitions

- The point up to which the stress and strain are linearly related is called the **proportional limit**.
- The largest stress in the stress strain curve is called the **ultimate stress**.
- The stress at the point of rupture is called the **fracture or rupture stress**.
- The region of the stress-strain curve in which the material returns to the undeformed state when applied forces are removed is called the **elastic region**.
- The region in which the material deforms permanently is called the **plastic region**.
- The point demarcating the elastic from the plastic region is called the **yield point**. The stress at yield point is called the **yield stress**.
- The permanent strain when stresses are zero is called the **plastic strain**.
- The **off-set yield** stress is a stress that would produce a plastic strain corresponding to the specified off-set strain.
- A material that can undergo large plastic deformation before fracture is called a **ductile material**.
- A material that exhibits little or no plastic deformation at failure is called a **brittle material**.
- **Hardness** is the resistance to indentation.
- The raising of the yield point with increasing strain is called **strain hardening**.
- The sudden decrease in the area of cross-section after ultimate stress is called **necking**.

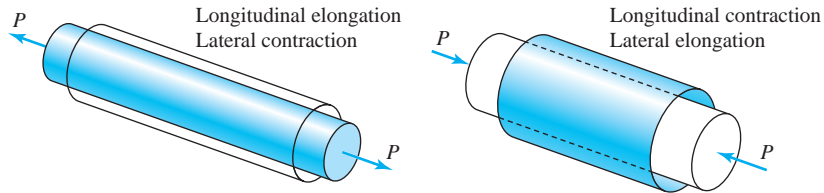


Material Constants

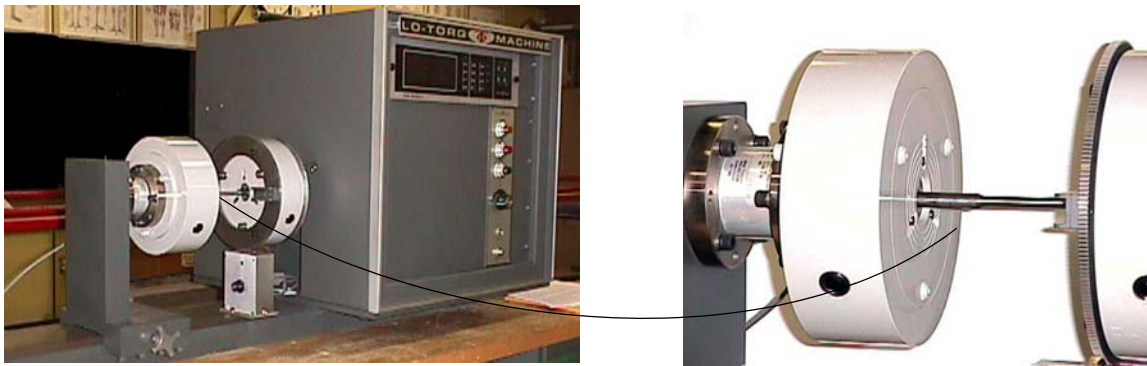


$\sigma = E\epsilon$ ----- Hooke's Law

- **E** Young's Modulus or Modulus of Elasticity



- Poisson's ratio:
$$\nu = -\left(\frac{\epsilon_{lateral}}{\epsilon_{longitudnal}}\right)$$



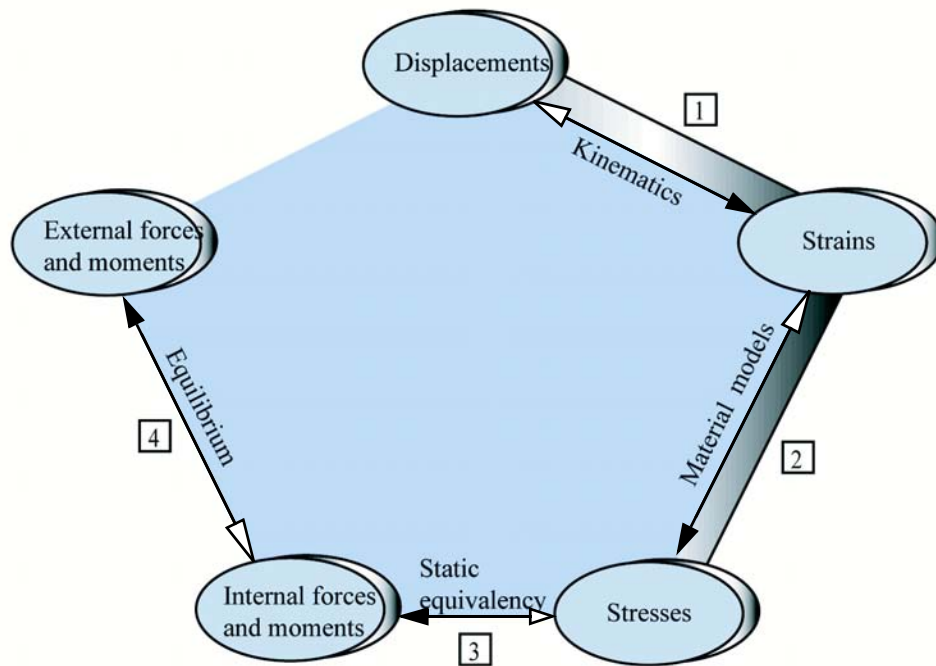
$\tau = G\gamma$

G is called the **Shear Modulus of Elasticity** or the **Modulus of Rigidity**

C3.1 A circular bar of length 6 inch and diameter of 1 inch is made from a material with a Modulus of Elasticity of $E=30,000$ ksi and a Poisson's ratio of $\nu=1/3$. Determine the change in length and diameter of the bar when a force of 20 kips is applied to the bar.

C3.2 An aluminum rectangular bar has a cross-section of 25 mm x 50 mm and a length of 500 mm. The Modulus of Elasticity of $E = 70$ GPa and a Poisson's ratio of $\nu = 0.25$. Determine the percentage change in the volume of the bar when an axial force of 300 kN is applied to the bar.

Logic in structural analysis



C3.3 A roller slides in a slot by the amount $\delta_P = 0.25$ mm in the direction of the force F . Both bars have an area of cross-section of $A = 100 \text{ mm}^2$ and a Modulus of Elasticity $E = 200 \text{ GPa}$. Bar AP and BP have lengths of $L_{AP} = 200 \text{ mm}$ and $L_{BP} = 250 \text{ mm}$ respectively. Determine the applied force F .

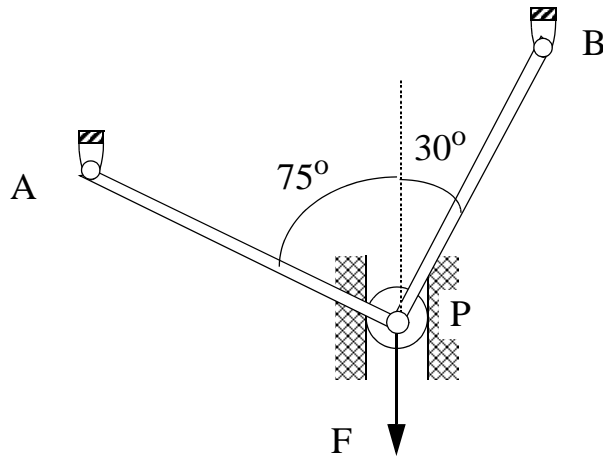
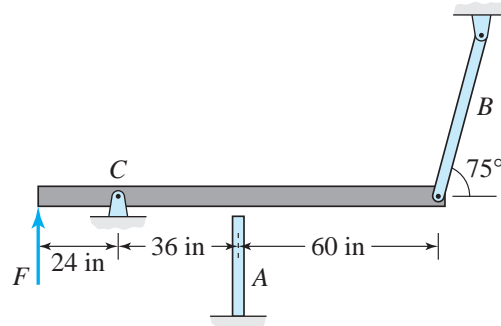


Fig. C3.3

C3.4 A gap of 0.004 in. exists between a rigid bar and bar *A* before a force *F* is applied (Figure P3.4). The rigid bar is hinged at point *C*. Due to force *F* the strain in bar *A* was found to be $-500 \mu\text{in/in}$. The lengths of bars *A* and *B* are 30 in. and 50 in., respectively. Both bars have cross-sectional areas $A = 1 \text{ in.}^2$ and a modulus of elasticity $E = 30,000 \text{ ksi}$. Determine the applied force *F*.

Fig. C3.4



C3.5 The pins in the truss shown in Fig. C3.5 are displaced by u and v in the x and y direction respectively, as given. All rods in the truss have an area of cross-section $A= 100 \text{ mm}^2$ and a Modulus of Elasticity $E= 200 \text{ GPa}$.

$$\begin{aligned}
 u_A &= -4.6765 \text{ mm} & v_A &= 0 \\
 u_B &= -3.3775 \text{ mm} & v_B &= -8.8793 \text{ mm} \\
 u_C &= -2.0785 \text{ mm} & v_C &= -9.7657 \text{ mm} \\
 u_D &= -1.0392 \text{ mm} & v_D &= -8.4118 \text{ mm} \\
 u_E &= 0.0000 \text{ mm} & v_E &= 0.0000 \text{ mm} \\
 u_F &= -3.2600 \text{ mm} & v_F &= -8.4118 \text{ mm} \\
 u_G &= -2.5382 \text{ mm} & v_G &= -9.2461 \text{ mm} \\
 u_H &= -1.5500 \text{ mm} & v_H &= -8.8793 \text{ mm}
 \end{aligned}$$

Determine the external force P_4 and P_5 in the truss shown in Fig. C3.5

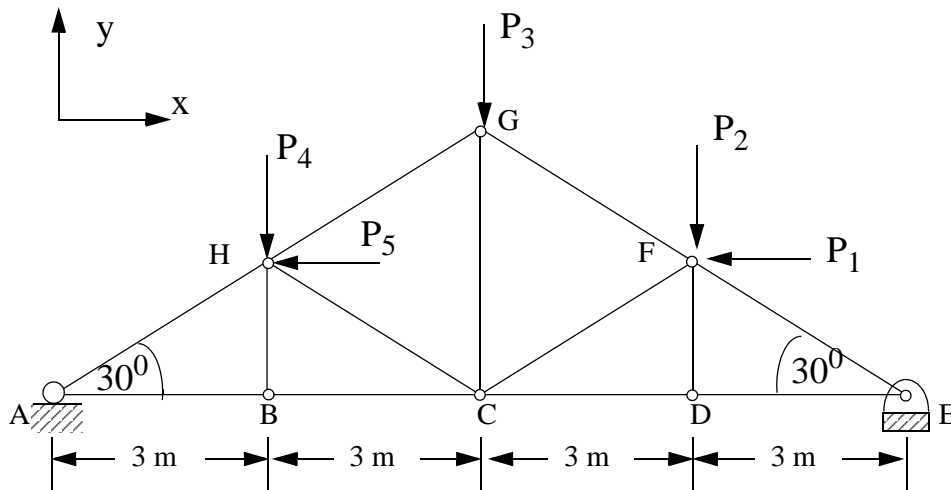


Fig. C3.5

Isotropy and Homogeneity

Linear relationship between stress and strain components:

$$\varepsilon_{xx} = C_{11}\sigma_{xx} + C_{12}\sigma_{yy} + C_{13}\sigma_{zz} + C_{14}\tau_{yz} + C_{15}\tau_{zx} + C_{16}\tau_{xy}$$

$$\varepsilon_{yy} = C_{21}\sigma_{xx} + C_{22}\sigma_{yy} + C_{23}\sigma_{zz} + C_{24}\tau_{yz} + C_{25}\tau_{zx} + C_{26}\tau_{xy}$$

$$\varepsilon_{zz} = C_{31}\sigma_{xx} + C_{32}\sigma_{yy} + C_{33}\sigma_{zz} + C_{34}\tau_{yz} + C_{35}\tau_{zx} + C_{36}\tau_{xy}$$

$$\gamma_{yz} = C_{41}\sigma_{xx} + C_{42}\sigma_{yy} + C_{43}\sigma_{zz} + C_{44}\tau_{yz} + C_{45}\tau_{zx} + C_{46}\tau_{xy}$$

$$\gamma_{zx} = C_{51}\sigma_{xx} + C_{52}\sigma_{yy} + C_{53}\sigma_{zz} + C_{54}\tau_{yz} + C_{55}\tau_{zx} + C_{56}\tau_{xy}$$

$$\gamma_{xy} = C_{61}\sigma_{xx} + C_{62}\sigma_{yy} + C_{63}\sigma_{zz} + C_{64}\tau_{yz} + C_{65}\tau_{zx} + C_{66}\tau_{xy}$$

- An **isotropic material** has a stress-strain relationships that are independent of the orientation of the coordinate system at a point.
- A material is said to be **homogenous** if the material properties are the same at all points in the body. Alternatively, if the material constants C_{ij} are functions of the coordinates x , y , or z , then the material is called non-homogenous.

For Isotropic Materials:
$$G = \frac{E}{2(1 + \nu)}$$

Generalized Hooke's Law for Isotropic Materials

- The relationship between stresses and strains in three-dimensions is called the **Generalized Hooke's Law**.

$$\varepsilon_{xx} = [\sigma_{xx} - \nu(\sigma_{yy} + \sigma_{zz})]/E$$

$$\varepsilon_{yy} = [\sigma_{yy} - \nu(\sigma_{zz} + \sigma_{xx})]/E$$

$$\varepsilon_{zz} = [\sigma_{zz} - \nu(\sigma_{xx} + \sigma_{yy})]/E$$

$$\gamma_{xy} = \tau_{xy}/G$$

$$\gamma_{yz} = \tau_{yz}/G$$

$$\gamma_{zx} = \tau_{zx}/G$$

$$G = \frac{E}{2(1 + \nu)}$$

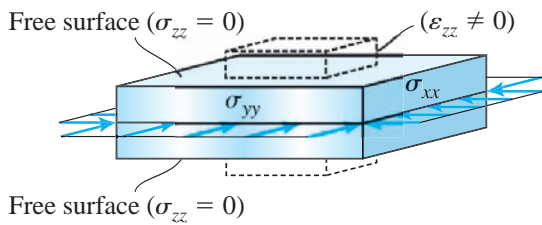
$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \end{Bmatrix} = \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \end{Bmatrix}$$

Plane Stress and Plane Strain

Plane Stress \rightarrow
$$\begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Generalized Hooke's Law}} \begin{bmatrix} \epsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} = -\frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) \end{bmatrix}$$

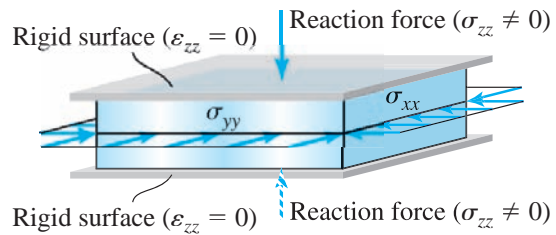
Plane Strain \rightarrow
$$\begin{bmatrix} \epsilon_{xx} & \gamma_{xy} & 0 \\ \gamma_{yx} & \epsilon_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{\text{Generalized Hooke's Law}} \begin{bmatrix} \sigma_{xx} & \tau_{xy} & 0 \\ \tau_{yx} & \sigma_{yy} & 0 \\ 0 & 0 & \sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) \end{bmatrix}$$

Plane Stress



(a)

Plane Strain



(b)

C3.6 A 2 in x 2 in square with a circle inscribed is stressed as shown Fig. C3.6. The plate material has a Modulus of Elasticity of $E = 10,000$ ksi and a Poisson's ratio $\nu = 0.25$. Assuming plane stress, determine the major and minor axis of the ellipse formed due to deformation.

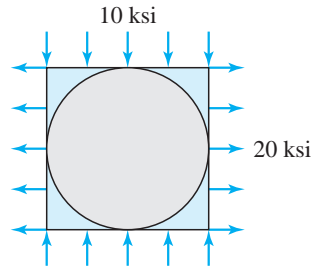


Fig. C3.6

Class Problem 1

The stress components at a point are as given.

Determine ϵ_{xx} assuming (a) Plane stress (b) Plane strain

$$\sigma_{xx} = 100 \text{ MPa}(T)$$

$$\sigma_{yy} = 200 \text{ MPa}(C)$$

$$\tau_{xy} = -125 \text{ MPa}$$

$$E = 200 \text{ GPa}$$

$$\nu = 0.25$$

Failure and factor of safety

- Failure implies that a component or a structure does not perform the function it was designed for.

$$K_{safety} = \frac{\textit{Failure producing value}}{\textit{Computed(allowable)value}} \quad 3.1$$

C3.7 An adhesively bonded joint in wood is fabricated as shown. For a factor of safety of 1.25, determine the minimum overlap length L and dimension h to the nearest 1/8th inch. The shear strength of adhesive is 400 psi and the wood strength is 6 ksi in tension.

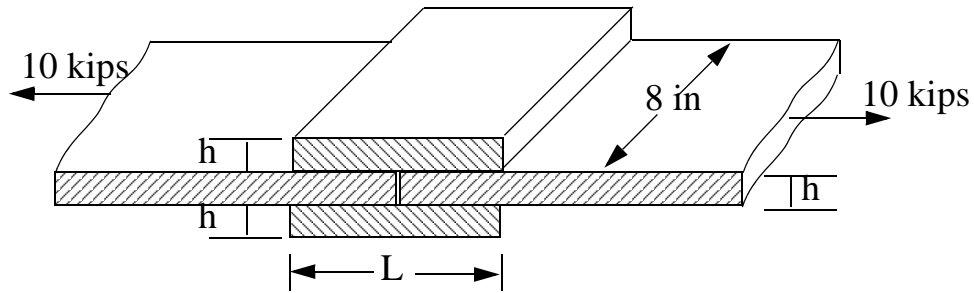


Fig. C3.7

Common Limitations to Theories in Chapters 4-7

- The length of the member is significantly greater (approximately 10 times) than the greatest dimension in the cross-section.
- We are away from regions of stress concentration, where displacements and stresses can be three-dimensional.
- The variation of external loads or changes in the cross-sectional area is gradual except in regions of stress concentration.
- The external loads are such that the axial, torsion and bending problems can be studied individually.