Problem: A 100x100 square with a central crack of 2 unit length under plane stress and uniaxial tension in the y-direction.

Analytical Solution: For a crack length of (2a), at a radial distance r from the crack tip, the analytical solution for non-dimensionalised normal stress in the y-direction is given by:

$$\frac{\sigma_{yy}}{\sigma_\infty} = \sqrt{\frac{2a}{r}}\left[1 + \left(\frac{3}{4}\right)\left(\frac{r}{a}\right) - \left(\frac{5}{32}\right)\left(\frac{r}{a}\right)^2 + \left(\frac{7}{128}\right)\left(\frac{r}{a}\right)^3 + \cdots \right]$$

Numerical Model: The square was modeled by Direct BEM. The crack was modelled using displacement discontinuity. The density function was modelled using Linear Lagrange elements, Quadratic Lagrange elements, Cubic Lagrange elements, and Cubic Hermite polynomials. Starting with a uniform mesh one iteration of hr-mesh refinement was permitted.

Results: The numerical results were compared to the above analytical series and percentage difference was plotted. The closest point to the crack tip was r/a=0.005. Results are shown below.
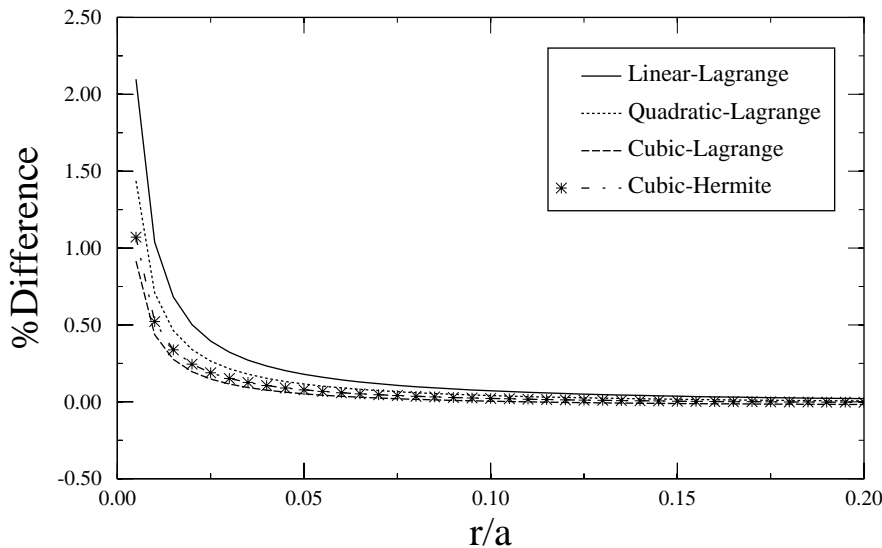

Table 1: Results.

	Linear-Lagrange	Quadratic-Lagrange	Cubic-Lagrange	Cubic-Hermite
Error at r/a=0.005	2.1%	1.44%	0.92%	1.07%
Number of unknowns	1393	381	295	263