

HOMEWORK #1

For the first 3 problems, you may need to use EXCEL to calculate the probabilities. To create the plots, feel free to use EXCEL or whatever plotting software you wish.

1. t-Distribution

$$f(t) = \frac{\Gamma\left(\frac{v+1}{2}\right)}{\sqrt{v\pi} \times \Gamma\left(\frac{v}{2}\right)} \left(1 + \frac{t^2}{v}\right)^{-(v+1)/2}$$

- Plot the probability density function for t-distributions with 2, 4, 10, and 25 degrees of freedom.
- For 4 degrees of freedom, what t value has an associated cumulative probability of 0.90?

2. Chi-square Distribution

$$f(x) = \frac{1}{\Gamma(v/2)2^{v/2}} x^{(v/2)-1} e^{(-x)/2}$$

where $\Gamma(\cdot)$ is the gamma function, and $x = \chi^2$

- Plot the probability density function for chi-square distributions with $v=2, 4, 10,$ and 25 degrees of freedom.
- For 9 degrees of freedom, what χ^2 value has an associated cumulative probability of 0.75?

3. F distribution

$$f(y) = \frac{\Gamma\left(\frac{v_1 + v_2}{2}\right)}{\Gamma\left(\frac{v_1}{2}\right)\Gamma\left(\frac{v_2}{2}\right)} v_1^{v_1/2} v_2^{v_2/2} y^{(v_1/2)-1} (v_2 + v_1 y)^{-(v_1 + v_2)/2}$$

where $y = F$

- Plot the probability density functions for the following F distributions:
Plot 1: $v_1=2$ and $v_2= 3, 6$ and 8 degrees of freedom.
Plot 2: $v_2=5$ and $v_1= 3, 6$ and 8 degrees of freedom.
- For $(v_1, v_2) = (4,5)$ degrees of freedom, what F value has an associated cumulative probability of 0.975? What F value has an associated cumulative probability of 0.025?

4. A lathe is being used to manufacture shafts. The diameters being produced by the process are normally distributed. The mean of the shaft diameter distribution is 10 cm. and the standard deviation is 0.05 cm.

- a. Find the probability that a single shaft selected randomly from the process would have a diameter less than 9.93 cm.
- b. Find the probability of selecting a single shaft between 9.97 cm. and 10.08 cm.
- c. A sample of 3 shafts are selected from process. Find the probability that the sample mean is between 9.97 and 10.08 cm.
- d. For a sample size of 5 find the probability of obtaining a sample variance less than 0.002 cm^2 .
- e. For a sample size of 5 find the probability of obtaining a sample variance greater than 0.008 cm^2 .

5. A manufacturer produces small extruded polymeric sheets. The manufacturer claims that the mean thickness of the sheets is 2 mm. The sheet thicknesses are known to be normally distributed. Use an α value of 0.05 in the following hypothesis tests.

- a. $\sigma_x = 0.1 \text{ mm}$. A single sheet drawn at random has a thickness of 2.1 mm -- evaluate the claim.
- b. $\sigma_x = 0.1 \text{ mm}$. A sample of 4 sheets is collected from the process and the sample mean is calculated to be 1.9 mm -- evaluate the claim.
- c. The standard deviation is unknown. A sample of 4 sheets is collected from the process and the sample mean is calculated to be 1.88 mm and the sample variance is calculated to be 0.009. Evaluate the claim.

6. A manufacturer is producing bolts, and the length of the bolts is of interest. Previous investigations have shown the bolt lengths to be normally distributed. The manufacturer claims that the bolt length-to-bolt length variability is characterized by $\sigma_x = 0.05 \text{ mm}$. To evaluate the claim, a sample of 5 bolts are selected at random from the process -- the sample variance is calculated to be 0.005.

- a. Evaluate the manufacturer's claim using an α value of 0.10.
- b. A sample of 7 bolts is collected from a competitor's process. For this group of bolts, the sample variance is calculated to be 0.001. Using an α value of 0.05, evaluate the hypothesis that the two processes have equal variability.