**MEEM 5990 - Design of Experiments**
**Final Take-home Exam**

Assigned: April 19, 2005 Due: April 29, 2005

**Problem #1**

The results of a replicated $2^4$ factorial experiment are shown in the table below:

| Test | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $Y_1$ | $Y_2$ |
|---|---|---|---|---|---|---|
| 1 | -1 | -1 | -1 | -1 | 21.0 | 14.6 |
| 2 | +1 | -1 | -1 | -1 | 6.8 | 5.0 |
| 3 | -1 | +1 | -1 | -1 | 21.2 | 24.5 |
| 4 | +1 | +1 | -1 | -1 | 12.3 | 6.1 |
| 5 | -1 | -1 | +1 | -1 | 44.0 | 37.2 |
| 6 | +1 | -1 | +1 | -1 | 25.7 | 29.4 |
| 7 | -1 | +1 | +1 | -1 | 25.5 | 22.4 |
| 8 | +1 | +1 | +1 | -1 | 10.2 | 16.5 |
| 9 | -1 | -1 | -1 | +1 | 7.7 | 13.6 |
| 10 | +1 | -1 | -1 | +1 | 14.3 | 10.6 |
| 11 | -1 | +1 | -1 | +1 | 15.1 | 18.0 |
| 12 | +1 | +1 | -1 | +1 | 17.3 | 23.8 |
| 13 | -1 | -1 | +1 | +1 | 28.2 | 36.9 |
| 14 | +1 | -1 | +1 | +1 | 36.9 | 34.6 |
| 15 | -1 | +1 | +1 | +1 | 9.5 | 14.1 |
| 16 | +1 | +1 | +1 | +1 | 23.9 | 15.1 |

a. Analyze the experiment results as appropriate. Suggestions: calculate effects, prepare a normal probability plot of the effects, develop confidence intervals for the true effect values, etc.
b. Prepare sketches to illustrate the meaning of significant effects.
c. Draw suitable conclusions.

**Problem #2**

Consider the fractional factorial ($2^{k-p}$) design shown in the table below.
a. Identify the values for k and p and the generators associated with the design.
b. What is the defining relationship for this design? What is the resolution of the design?

c. Write out the complete confounding pattern for the design. In other words, what linear combinations of confounded effects may be estimated from the design -- you may assume that FOUR-factor and higher order interactions are negligible.

| $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $X_6$ |
|---|---|---|---|---|---|
| - | - | + | + | + | - |
| + | + | - | + | + | + |
| + | - | - | - | - | - |
| + | + | + | - | + | - |
| - | - | - | - | + | + |
| + | - | + | + | - | + |
| - | + | + | - | - | + |
| - | + | - | + | - | - |

d. The design provided in the table above was performed. The results were examined, and it was decided to follow up the design with a second fractional factorial design. The results from the two designs should provide unconfounded estimates of the main effects. Write out the recipe matrix for the second design.
e. Find the generators associated with the design and determine the defining relationship for the design.
f. Write out the complete confounding pattern for the second design. In other words, find the linear combinations of confounded effects that may be estimated from the second design -- as with part (c), you may assume that FOUR-factor and higher order interactions are negligible.
g. Combine the results from parts (c) and (f) to reveal the effects and groups of confounded effects that may be obtained from the two designs (by adding and subtracting).

**Problem #3**

The results of a $2^5$ factorial experiment are shown in the table below.

a. Analyze the experiment results as appropriate. Suggestions: calculate effects, prepare a normal probability plot of the effects, develop confidence intervals for the true effect values, etc.
b. Prepare sketches to illustrate the meaning of significant effects.
c. Draw suitable conclusions.

| Test | $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | $Y_2$ |
|---|---|---|---|---|---|---|
| 1 | -1 | -1 | -1 | -1 | -1 | 5.0 |
| 2 | 1 | -1 | -1 | -1 | -1 | 6.9 |
| 3 | -1 | 1 | -1 | -1 | -1 | -7.0 |
| 4 | 1 | 1 | -1 | -1 | -1 | 0.3 |
| 5 | -1 | -1 | 1 | -1 | -1 | -20.5 |
| 6 | 1 | -1 | 1 | -1 | -1 | -13.0 |
| 7 | -1 | 1 | 1 | -1 | -1 | -6.3 |
| 8 | 1 | 1 | 1 | -1 | -1 | 1.6 |
| 9 | -1 | -1 | -1 | 1 | -1 | 5.9 |
| 10 | 1 | -1 | -1 | 1 | -1 | 3.4 |
| 11 | -1 | 1 | -1 | 1 | -1 | -3.0 |
| 12 | 1 | 1 | -1 | 1 | -1 | -9.7 |
| 13 | -1 | -1 | 1 | 1 | -1 | -15.4 |
| 14 | 1 | -1 | 1 | 1 | -1 | -19.4 |
| 15 | -1 | 1 | 1 | 1 | -1 | -1.2 |
| 16 | 1 | 1 | 1 | 1 | -1 | -9.1 |
| 17 | -1 | -1 | -1 | -1 | 1 | 12.4 |
| 18 | 1 | -1 | -1 | -1 | 1 | 20.5 |
| 19 | -1 | 1 | -1 | -1 | 1 | 1.9 |
| 20 | 1 | 1 | -1 | -1 | 1 | 4.8 |
| 21 | -1 | -1 | 1 | -1 | 1 | -11.0 |
| 22 | 1 | -1 | 1 | -1 | 1 | -3.9 |
| 23 | -1 | 1 | 1 | -1 | 1 | 1.3 |
| 24 | 1 | 1 | 1 | -1 | 1 | 8.4 |
| 25 | -1 | -1 | -1 | 1 | 1 | 16.5 |
| 26 | 1 | -1 | -1 | 1 | 1 | 12.7 |
| 27 | -1 | 1 | -1 | 1 | 1 | 5.7 |
| 28 | 1 | 1 | -1 | 1 | 1 | 1.1 |
| 29 | -1 | -1 | 1 | 1 | 1 | -2.6 |
| 30 | 1 | -1 | 1 | 1 | 1 | -8.2 |
| 31 | -1 | 1 | 1 | 1 | 1 | 8.3 |
| 32 | 1 | 1 | 1 | 1 | 1 | 2.3 |

**Problem #4**

In the TURN5DOE workshop the roles of 5 variables were considered in an examination of a turning process. In this problem, we will consider the roles of 9 variables in a similar process. As a reminder, the turning process is focused on producing the outside diameter of a bushing, with the finish of the outside surface serving as the quality characteristic of interest; the specification on the finish is $70 \pm 15 \mu$ inches. Your assignment is to determine how the 9 factors of interest influence the surface roughness, either independently, or in concert with each other, or both. The potentially important factors together with the feasible ranges for these factors are shown in the table below. Your job is to recommend levels for the 9 variables that will center the two machines of interest.

**Potentially Important Process Factors**

| **Factor** | **Feasible Range for Experiments** | **Current Operating Condition** |
|---|---|---|
| 1. Tool Material | Two grades (A and B) of cemented carbide tools were considered applicable | Grade A is being used |
| 2. Use of Cutting Fluid | Either present or absent | No cutting fluid is being used |
| 3. Machine Used | Only the American and Leblond lathes can be used for experimentation | American, LeBlond, Monarch |
| 4. Tool Nose Radius | 0.0156” to 0.0625” in 0.0156” increments | fixed at 0.0312” |
| 5. Tool Rake Angle (Back) | 0 degrees to 30 degrees | Varies between 5 and 10 degrees positive |
| 6. Tool side Cutting Edge Angle | 5 degrees to 30 degrees | Fixed at 10 degrees |
| 7. Depth of Cut for Final Pass | 0.005” to 0.030” | Varies between 0.005” and 0.015” |
| 8. Feedrate | 0.0070 - 0.0095 ipr | 0.008” - 0.0089” ipr |
| 9. Cutting speed | 900 - 1150 sfpm | 1000 - 1100 sfpm |

A computer simulation program called TURN9DOE has been developed to simulate the act of running experiments with the nine (9) process variables previously discussed. The program simply “produces” a surface roughness value as a test result for any given combination of process settings for the nine variables. Test results are obtained one-by-one, given the user selected processing conditions and are printed as they are generated. If you are experimenting with only a subset of the full set of nine variables, default values will be used for the process conditions you do not specifically set.

Use the general classes of $2^k$ and/or $2^{k-p}$ factorial and fractional factorial designs to study the process. Your experimental effort should be directed at establishing a model for predicting the surface finish. The following guidance may be useful:

1. You should be concerned only with process centering.
2. You may experiment with all (9) factors or with any subset that you choose - it's all up to you.
3. You should be able to complete your investigation in roughly 40-50 experiments.
4. You should probably think in terms of running several smaller and perhaps related experiments as opposed to one big experiment.
5. You are free to choose the high and low levels for the experiments anywhere within the feasible range - again, it's all up to you.

Your activities should include: planning your experiments, running experiments using TURN9DOE, analysis of experiment results, and then drawing conclusions about how to improve the process, i.e., recommend new operating conditions for the poorly centered machines. Of course, you should run a few validation/confirmatory tests for your recommended variable settings for each machine to see if your recommendations can be reproduced at the process.

Describe in detail the results of your investigation. Be sure to list all experiments you conducted, the data obtained, and the associated analysis. Don't forget to indicate the logic used in your evolving design of experiments strategy. Include graphs of the relevant response surface(s). Carefully explain your recommendations and the logic associated with them. Show the results of confirmatory tests.

**Problem #5**

Consider the data shown in the table below

| t | 0 | 2 | 3 | 5 | 6 | 8 | 10 |
|---|---|---|---|---|---|---|---|
| y | 5 | 1 | 3 | 3 | 7 | 11 | 24 |

It is suggested that the following model be applied to the data in the table

$$y = B_0 + B_1 t + B_2 t^2 + \varepsilon$$

a. Find the least squares estimates of the parameters in the equation above.
b. Calculate the model residuals and estimate the experimental error variance, $\sigma_y^2$.
b. Develop 95% marginal confidence intervals for the true parameter values.
c. Plot the 95% confidence interval for the mean response as a function of t.
d. Plot the 95% prediction interval for a single response as a function of t.

**Problem #6**

It is desired to find the settings of Temperature (degrees F), Catalyst Concentration (percent), and Time (min.) that produce maximum yield (in grams) for a chemical process subject to the following conditions:

1. Past experience has led to the conclusion that the temperature must be a least 100 degrees in order to obtain an appreciable yield. Also, at temperatures above 300 degrees, the reaction becomes highly unstable.
2. The catalyst employed is expensive and available only in limited quantities. It is therefore desirable to restrict the amount used to less than 25 percent.
3. The production schedule for the resultant chemical requires that the reaction time be less than 140 minutes per batch.

At present, the settings for the three variables are

| | |
|---|---|
| Temperature | 180 degrees F |
| Catalyst Concentration: | 15 percent |
| Time | 30 minutes |

which gives rise to an average yield of 60 grams.

Your Job

1. Use the Response Surface Methodology (RSM) procedure to optimize the process.
2. You may use the software, RSMGAME, to run the tests (obtain the yield/response) you decide to conduct as part of the RSM.
3. To fit models to the data it may be necessary to employ MATLAB or EXCEL.
4. Plot the response surface using whatever software you wish.
5. Be sure to validate your recommended operating conditions with confirmatory tests.

Document all the steps taken in your investigation.