Chapter 7

SOLAR RADIATION

Solar radiation has important effects on both the heat gain and heat loss of a building.

The designer should distinguish between the maximum solar load on a surface which is important for load calculations against an average value that the surface experiences.

Key issues to be learned:

a. Thermal Radiation
b. Earth-Sun geometry
c. Solar Time, Local Standard Time
d. Solar Angles
e. Solar Irradiation, Mean Solar Constant
   - ASHRAE Clear Sky Model
f. Heat Gain Through Fenestrations
   - Solar Heat Gain Coefficients
   - Simplified Solar Heat Gain Calculations
   - Shading Coefficient
g. Energy Calculations

QUESTION

Is there any usable solar energy available in Houghton during the month of March? If yes, where is it?

ANSWER

Yes. Behind the clouds. Very obvious in a clear day!

7-1 Thermal Radiation

The perfect radiant emitter is also given a name blackbody. For a given temperature T in R (K), a black emitter exhibits a maximum monochromatic emissive power at wavelength lmax, given by

$$\lambda_{max} = \frac{5215.6(2897.6)}{T} \text{ microns} \qquad (7\text{-}2)$$

This equation is known as *Wien's displacement law*.

For nonblack surfaces, the emittance e, leads to actual energy emitted from a surface

$$E = \varepsilon E_b \qquad (7\text{-}3)$$

where

$$E_b = \sigma T^4$$

Table 7-1 Solar Absorptances (See book for more)

Surface	Absorptance
Brick	0.63
Paint, sandstone	0.50
Paint, white acrylic	0.26
Sheet metal, galvanized, new	0.65
Sheet metal, galvanized, weathered	0.80
Concrete	0.60-0.83
Asphalt	0.90-0.95
Grassland	0.80-0.84
Snow, fresh	0.10-0.25
Snow, old	0.30-0.55

7-2 The Earth's Motion About the Sun

The sun's position in the sky is a major factor in the effect of solar energy on a building. The mean distance from the center of earth to the center of sun is approximately $92.9x10^6$ miles ($1.5x10^8$ km). The *perihelion distance*, when the earth is closest to the sun, is 98.3 percent. The *aphelion distance* when the earth is farthest from the sun, is 101.7 percent. Because of this earth receives about 7 percent more total radiation in January than in July.

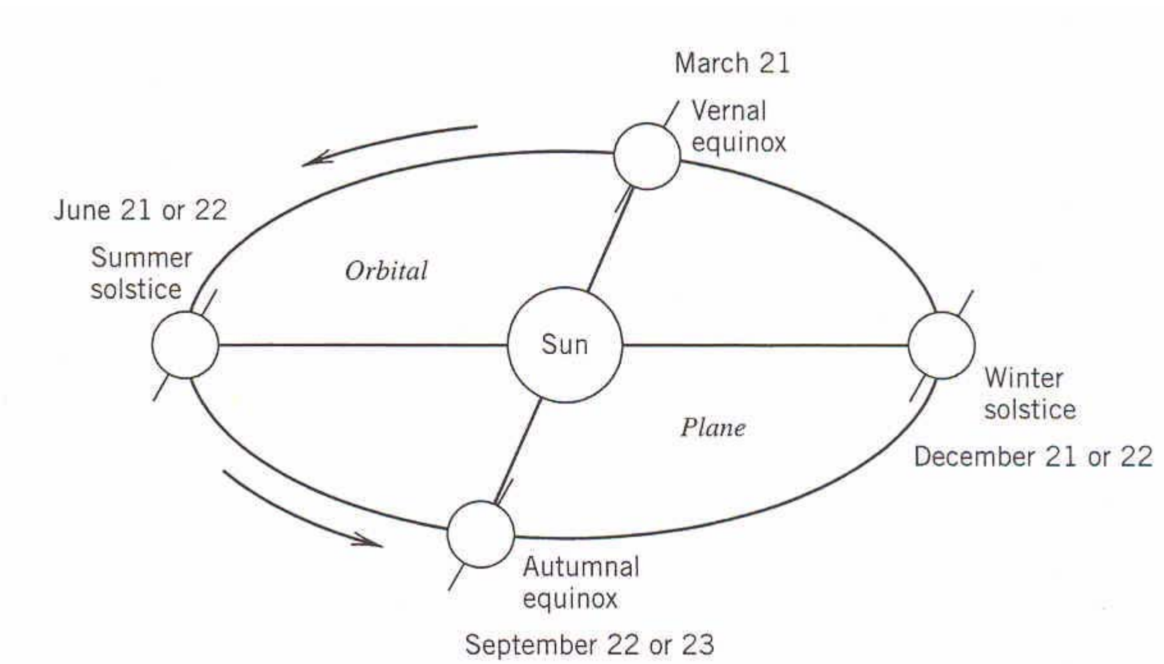

Figure 7-1.a The effect of the earth's tilt and rotation about the sun.

The earth's axis of rotation is tilted 23.5 deg. with respect to orbital plane.

At the time of the vernal equinox (March 21) and of the autumnal equinox (September 22 or 23), the sun appears to be directly overhead at the equator and the earth's poles are equidistant from the sun. Equinox means "equal nights," and during the time of the two equinoxes all points on earth (except the poles) have exactly 12 hours of darkness and 12 hours of daylight.

During the summer solstice (June 21 or 22) the north pole is inclined 23.5 deg. toward the sun. All points on the earth's surface north of (90-23.5) 66.5 deg. N latitude (the Arctic Circle) are in continuous light, whereas points south of 66.5 deg. S latitude (the Antarctic Circle) are in continuos darkness. The word solstice means *sun standing still*.

During the summer solstice the sun appears to be directly overhead at noon along the Tropic of Cancer, whereas during the winter solstice it is overhead at noon along the Tropic of Capricorn.

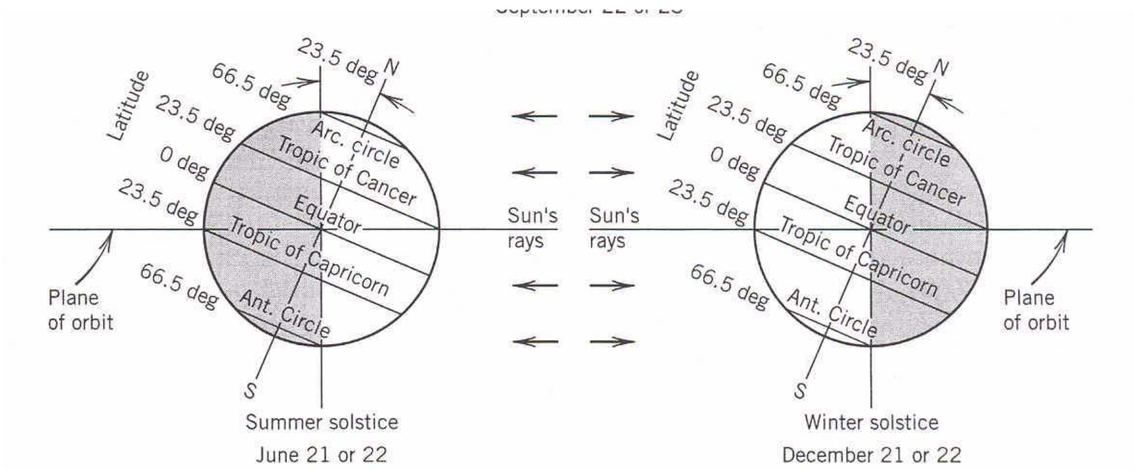

Figure 7-1.b The effect of earth’s tilt and rotation about the sun.

7-3 Time

The earth is divided into 360 deg of circular arc by longitudinal lines passing through the poles. Thus, 15 deg. of longitude corresponds to (360/15=24) 1 hour of time. A point on the earth's surface exactly 15 deg west of another point will see the sun in exactly the same position as the first point after one hour of time has passed. *Universal Time* or *Greenwich civil time* (GCT) is the time along the zero longitude line passing through Greenwich, England. Local civil time (LCT) is determined by the longitude of the observer, the difference being four minutes of time for each degree of longitude, the more advanced time being on meridians further west. Thus when is 12:00 noon GCT, it is (4x75=300, 300/60=5, 12-5=7) 7:00 A.M. LCT along the seventy-fifth deg W longitude meridian.

Clocks are usually set for the same reading throughout a zone covering approximately 15 deg of longitude. The local civil time for a selected meridian near the center of the zone is called the *standard time*. The four standard times zones in the lower 48 states and their standard meridians are

Eastern standard time, EST 75 deg
Central standard time, CST 90 deg
Mountain standard time, MST 105 deg
Pacific standard time, PST 120 deg

In much of the United States clocks are advanced one hour during the late spring, summer, and early fall season, leading to *daylight saving time*.

Time measured by the position of the sun is called *solar time*.

The *local solar time* (LST) can be calculated from the *local civil time* (LCT) with the help of a quantity called the *equation of time*:

LST = LCT+(equation of time) EOT

$$\mathrm{EOT} = 229.2(\mathrm{Term1} + \mathrm{Term2}) \tag{7-4}$$

$$\mathrm{Term1} = 0.000075 + 0.001868\cos N$$

$$\mathrm{Term2} = -0.032077\sin N - 0.014615\cos 2N - 0.04089\sin 2N$$

where

$N = \frac{(n-1)360}{365}$ degree, and *n* is the day of the year.

Values of EOT are given in Table 7-2 for the twenty-first day of each month.

If DST is in effect,

$$\mathrm{LocalStandardTime} = \mathrm{LocalDST} - 1\mathrm{hour} \tag{7-5}$$

$$\mathrm{LST} = \mathrm{LocalStandardTime} - (L_L - L_S) \times 4 + \mathrm{EOT} \tag{7-6}$$

Table 7-2 Solar Data for 21st Day of Each Month

	Equation of time	Declination δ	A	B	C
	min.	degrees	Btu/hr-ft^2	Dimensionless	
January	-11.2	-20.2	381.2	0.141	0.103
February	-13.9	-10.8	376.4	0.142	0.104
March	-7.5	0.0	369.1	0.149	0.109
April	1.1	11.6	358.3	0.164	0.120
May	3.3	20.0	350.7	0.177	0.130
June	-1.4	23.45	346.3	0.185	0.137
July	-6.2	20.6	346.6	0.186	0.138
August	-2.4	12.3	351.0	0.182	0.134
September	7.5	0.0	360.2	0.165	0.121
October	15.4	-10.5	369.7	0.152	0.111
November	13.8	-19.8	377.3	0.142	0.106
December	1.6	-23.45	381.8	0.141	0.103

Chapter 7

Example 7-1

{{McQuiston, Parker, Spitler; Chapter VII}
{Example 7-1}
{Calculate the solar time for Houghton at 12:00 P.M. on July 21}

SOLUTION

{Define dimensional unit}

```
HM = 60                                   {min/hr}
Year = 365                                {day}
Deg = 360                                 {deg}
W = 4                                     {min/deg}
NN = 1                                    {day}
H = 1                                     {hr}
DC = 23.45                                {deg}
DN = 284                                  {day}
LSolN = 12                                {hr}
HL = 15                                   {deg/hr}
{Calculate solar time for Houghton}
DS = 1                                    {Daylight saving is ON}
EST = 75                                  {eastern standard time}
Long = 88.39                              {West}
Lat = 47.90                               {North}
{Month February, Day 21}
{Jan=31, Feb=28, Mar=31, Apr=30, May=31, Jun=30, Jul=31, Aug=31, Sep=30, Oct=31, Nov=30,
      Dec=31}
day = 31+ 28 + 31 + 30 + 31 + 30 + 21 {July 21}
N = (day - NN) * Deg / Year
EOT = 229.2*(0.000075+0.001868*cos(N)-0.032077*sin(N)-0.014615*cos(2*N)-0.04089*sin(2*N))
                                          {minutes}
{GIVEN}
LDST_h = 12                               {local daylight savings time, hours}
LDST_m = 0                                {local daylight savings time, minutes}
{End of given time}
LSdT = LDST_h - DS           {eastern standard time at EST corrected by daylight saving, hours}
{Calculate local standard time}
Diff = (Long - EST) * W
LSolT_m = - Diff + EOT                    {in minutes}
Ratio = LSolT_m / HM/H
HN = H * Trunc(Ratio)
LSolT_h = LDST_h + HN
Minutes = LSolT_m - HM * HN
LSolT = LSolT_h + minutes / HM
H_deg = (LSolT - LSolN) * HL
{Describe Collector Geometry}
PSI = 0.0
alpha = 0.0
gamma = abs(phi - PSI)
delta_1 = DC * Sin(Deg * (DN + day)/Year)
Term_1 = 0.3963723 - 22.9132745*Cos(N) + 4.0254304 * Sin(N)
Term_2 = -0.3872050 * Cos (2*N) + 0.05196728 * Sin (2*N) - 0.1545267 * Cos(3*N) + 0.08479777 *
      Sin(3*N)
```

delta_2 = Term_1 + Term_2
SB = Cos(Lat) * Cos(H_deg) * Cos(delta_2) + Sin(Lat) * Sin (delta_2)
beta = arcsin(SB)
theta_Z = arccos(SB)
CP = (Sin(delta_2) * Cos(Lat) - Cos(delta_2) * Sin(Lat) * Cos(H_deg))/Cos(beta)
phi = arccos(CP)
CT = Cos(beta) * Cos(gamma) * Sin(alpha) + Sin(beta) * Cos(alpha)
theta = arccos(CT)
sum = beta + theta_Z

SOLUTION Example 7-1

Unit Settings: [F]/[psia]/[Ibm]/fdegrees]

α = 90.00 [deg]	β = 53.61 [deg]	CP = -0.615
CT = 0.365 [-]	day = 202 [day]	DC = 23.450 [deg]
Deg = 360.000 [deg]	δ_1 = 20.442 [deg]	δ_2 = 20.637 [deg]
Diff = 53.56 [min.]	DN = 284 [day]	DS = 1[hr]
EOT = -6.354 [min.]	EST = 75 [deg]	γ = 0 [deg]
H = 1 (hrl	HL = 15 [deg/hr]	HM = 60 [min./hr]
HN = 0 [hr]	H_{deg} = -29.979 [deg]	Lat = 47.90 [deg]
$LDST_h$ = 12.000 [hr]	$LDST_m$ = 0.000 [min.]	Long = 88.39 [deg]
LSdT = 11.000 [hr]	$LSol_N$ = 12.000 [hr]	$LSol_T$ = 10.001 [hr]
$LSolT_h$ = 11.000 [hrl	Min = -59.91 [min.]	Minutes = -59.91 [min.]
N = 198.25 [deg)	NN = I [day]	ϕ = 0. [deg]
ψ = 0 [deg]	Ratio = 0.999 [-]	RHr = -0.999 [hr
R_{vh} = 0.751 SB = 0.805 [-]	Sig_R = 1 [-]	sum = 90.00 [deg]
$Term_1$ = 20.897 [deg]	$Term_2$ = -0.260 [deg]	θ = 36.392 [deg)
θ_z = 36.392 [deg]	W = 4 [min./deg]	Year= 365 (day)

No unit problems were detected.

7-4 Solar Angles

Nomenclature:

l	latitude
h	hour angle
n	day number
δ	declination
N	parameter
β	solar altitude angle
θ_Z	sun's zenith angle
ϕ	solar azimuth angle
γ	surface solar azimuth angle
θ	angle of incidence
α	tilt angle

The direction of sun rays can be described if the following three quantities are known

1. Location on the earth's surface (latitude, *l*)
2. Time of day
3. Day of the year (*n*)

It is convenient to describe these three quantities by giving the *latitude l*, *the hour angle h*, and the *sun's declination* δ, respectively, Figure 7-2.. The latitude is the angle between the line OP and the projection of OP on the equatorial plane. This is the same latitude that is commonly used on globes and maps. The hour angle is the angle between the projection of OP on the equatorial plane and the projection on that plane of a line from the center of the sun to the center of the earth. Fifteen degrees of hour angle corresponds to one hour of time. The Solar noon occurs when the sun is at the highest point in the sky. Hour angles are symmetrical with respect to solar noon, negative in the morning positive in the afternoon.

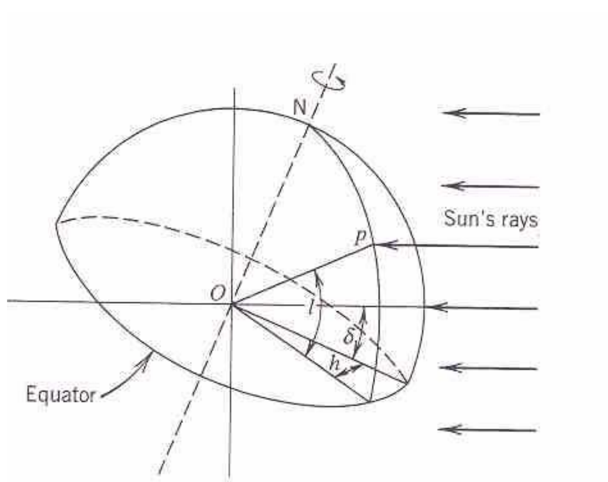

Figure 7-2. Latitude, hour angle, and sun's declination

The sun's declination is the angle between a line connecting the center of the sun and earth and the projection of this line on equatorial plane. It varies between +23.45 and -23.45 degrees.

$$\delta = 23.45 \sin\left(360 \frac{284 + n}{365}\right) \tag{7-7}$$

NOTE: Compare (7-7) with that in the text, page 188.

$$\delta = 0.3963723 - 22.9132745 \text{ CosN} + 4.0254304 \text{ SinN} - 0.3872050 \text{ Cos2N}$$
$$+ 0.05196728 \text{ Sin2N} - 0.1545267 \text{ Cos3N} + 0.08479777 \text{ Sin3N}$$

Table 7-2 shows typical values of the sun's declination for the twenty-first day of each month.

It is convenient in HVAC computations to define the sun's position in the sky relative to a surface in mind in terms of the solar altitude β and the solar azimuth ϕ, which depend on the fundamental quantities *l,h*, and δ.

The *solar altitude* β (sun's altitude angle) is the angle between the run's ray and the projection of that ray on a horizontal surface Figure 7-4. It is the angle of the sun above horizon.

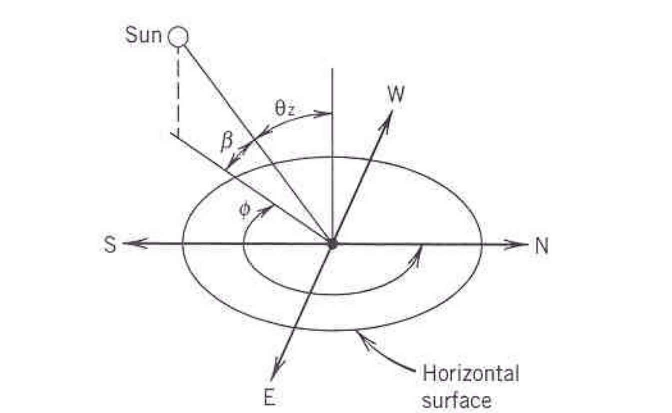

Figure 7-4. The solar altitude β and azimuth angle ϕ.

$$\sin\beta = \cos l \cosh \cos\delta + \sin l \sin\delta \tag{7-8}$$

The sun's zenith angle θ_Z is the angle between the sun's rays and the perpendicular to the horizontal plane at point P (local zenith line).

$$\beta + \theta_Z = 90 \text{ degrees} \tag{7-9}$$

The daily maximum altitude (solar noon) of the sun at a given location can be shown to be

$$\beta_{noon} = 90 - |l - \delta| \text{ degrees} \tag{7-10}$$

The *solar azimuth angle* ϕ is the angle in the horizontal plane measured, in the clockwise direction, between north and the projection of the sun's rays on that plane.

$$\cos\phi = \frac{\sin\delta\cos l - \cos\delta\sin l\cos h}{\cos\beta} \tag{7-11}$$

Note that, when calculating ϕ by taking the inverse of Cosϕ, it is necessary to check which quadrant ϕ is in.

For a vertical or tilted surface the angle measured in the horizontal plane between the projection of the sun's rays on that plane and the normal to the vertical surface is called *wall solar azimuth* γ, Figure 7-5. If ψ is the wall azimuth (facing direction) measured clockwise from north, then

$$\gamma = |\phi - \psi| \tag{7-12}$$

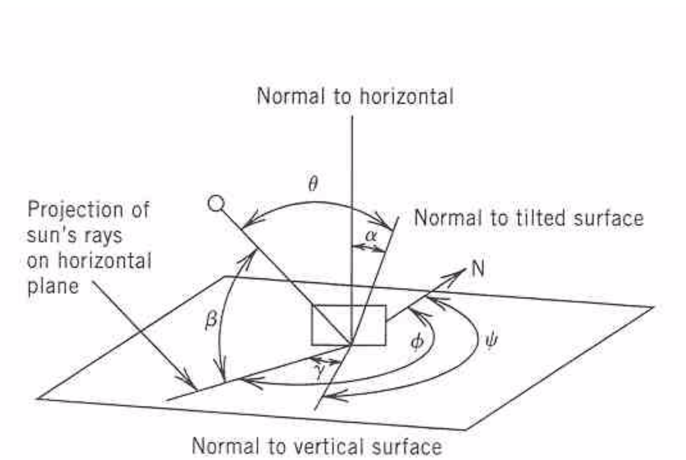

Figure 7-5. Wall solar azimuth γ, wall azimuth ψ, and angle of tilt α for an arbitrary tilted surface.

The *angle of incidence* θ is the angle between the sun's rays and the normal to the surface. The *tilt angle* α is the angle between the normal to the surface and the normal to the horizontal surface.

$$\cos\theta = \cos\beta\cos\gamma\sin\alpha + \sin\beta\cos\alpha \tag{7-13a}$$

For a vertical surface

$$\cos\theta = \cos\beta\cos\gamma \tag{7-13b}$$

For a horizontal surface

$$\cos\theta = \sin\beta \tag{7-13c}$$

Example 7-2

Find the solar altitude β and azimuth θ_Z, the angle of incidence θ at 12:00 A.M. for Houghton on July 21.

SOLUTION

```
{McQuiston, Parker, Spitler; Chapter VII}
{Example 7-2}
{Calculate solar altitude beta and solar azimuth angle phi and angle of incidence theta for Houghton at
        12:00 P.M. on
July 21}
{Define dimensional unit}
HM = 60                                   {min/hr}
Year = 365                                {day}
Deg = 360                                 {deg}
W = 4                                     {min/deg}
NN = 1                                    {day}
H = 1                                     {hr}
DC = 23.45                                {deg}
DN = 284                                  {day}
LSolN = 12                                {hr}
HL = 15                                   {deg/hr}
{Calculate solar time for Houghton}
DS = 1                                    {Daylight saving is ON}
EST = 75                                  {eastern standard time}
Long = 88.39                              {West}
Lat = 47.90                               {North}
{Month February, Day 21}
{Jan=31, Feb=28, Mar=31, Apr=30, May=31, Jun=30, Jul=31, Aug=31, Sep=30, Oct=31, Nov=30,
        Dec=31}
day = 31+ 28 + 31 + 30 + 31 + 30 + 21 {July 21}
N = (day - NN) * Deg / Year
EOT = 229.2*(0.000075+0.001868*cos(N)-0.032077*sin(N)-0.014615*cos(2*N)-0.04089*sin(2*N)
                                          {minutes}
{GIVEN}
LDST_h = 12                               {local daylight savings time, hours}
LDST_m = 0                                {local daylight savings time, minutes}
{End of given time}
LSdT = LDST_h - DS          {eastern standard time at EST corrected by daylight saving, hours}
{Calculate local standard time}
Diff = (Long - EST) * W
Min = - Diff + EOT                        {in minutes}
Ratio = abs(Min / HM / H)
Sig_R = Sign(Ratio)
HN = H * Trunc(Ratio) * Sig_R
Minutes = Min - HN * HM
LSolT_h = LSdT + HN
RHr = Minutes / HM
LSolT = LSolT_h + RHr
H_deg = (LSolT - LSolN) * HL
{Describe Collector Geometry}
gamma = 0
alpha = 0
```

delta_1 = DC * Sin(Deg * (DN + day)/Year)
Term_1 = 0.3963723 - 22.9132745*Cos(N) + 4.0254304 * Sin(N)
Term_2 = -0.3872050 * Cos (2*N) + 0.05196728 * Sin (2*N) - 0.1545267 * Cos(3*N) + 0.08479777 * Sin(3*N)
delta_2 = Term_1 + Term_2
SB = Cos(Lat) * Cos(H_deg) * Cos(delta_2) + Sin(Lat) * Sin (delta_2)
beta = arcsin(SB)
theta_Z = arccos(SB)
CP = (Sin(delta_2) * Cos(Lat) - Cos(delta_2) * Sin(Lat) * Cos(H_deg))/Cos(beta)
phi = arccos(CP)
CT = Cos(beta) * Cos(gamma) * Sin(alpha) + Sin(beta) * Cos(alpha)
theta = arccos(CT)
sum = beta + theta_Z

SOLUTION Example 7-2

Unit Seffings: [Fl/fpsia]/[Ibm]/[degrees]

α = 0.00 [deg]	β = 53.61 [deg]	CP = -0.615 [-]
CT = 0.805 [-]	day = 202 [day]	DC = 23.45 [deg]
Deg = 360 [deg]	δ_1 = 20.442 [deg]	δ_2 = 20.637 [deg]
Diff = 53.56 [min.]	DN = 284 [day]	DS = 1 [hr]
EOT = -6.354 [min.]	EST = 75.0 [deg]	γ = 0.00 [deg]
H = I [hr]	HL = 15 [deg/hr]	HM = 60 [min./hr]
HN = 0.0 [hr]	H_{deg} = -29.98 [deg]	Lat = 47.90 [deg]
$LDST_h$ = 12.0000 [hr]	$LDST_m$ = 0.0000 [min.]	Long = 88.39 [deg]
LSdT = 11.0000 [hri	L_{SolN} = 12.0000 [hr]	L_{SolT} = 10.0014 [hrl
$LSolT_h$ =11.0000 [hr]	Min = -59.91 [min.]	Minutes = -59.91 [min.]
N = 198.25 [deg]	NN = 1.0 [day]	ϕ = 127.99 [deg]
Ratio = 0.999	RHr = -0.9986 [hrl	SB = 0.80 [-]
SigR = I	sum = 90.0 [deg]	$Term_1$ = 20.897 [deg]
$Term_2$ = -0.260 [deg]	θ = 36.39 (deg)	θ_z = 36.39 [deg]
W = 4 [min./deg]	Year= 365 [day]	

No unit problems were detected.

7-5 Solar Irradiation

The *mean solar constant* G_{sc} is the rate of irradiation on the surface normal to the sun's rays beyond the earth's atmosphere and the mean earth-sun distance.

$$G_{sc} = 433.4 \text{ Btu/(hr-ft}^2) = 1367 \text{ W/m}^2.$$

The irradiation from the sun varies by ± 3.5 percent.

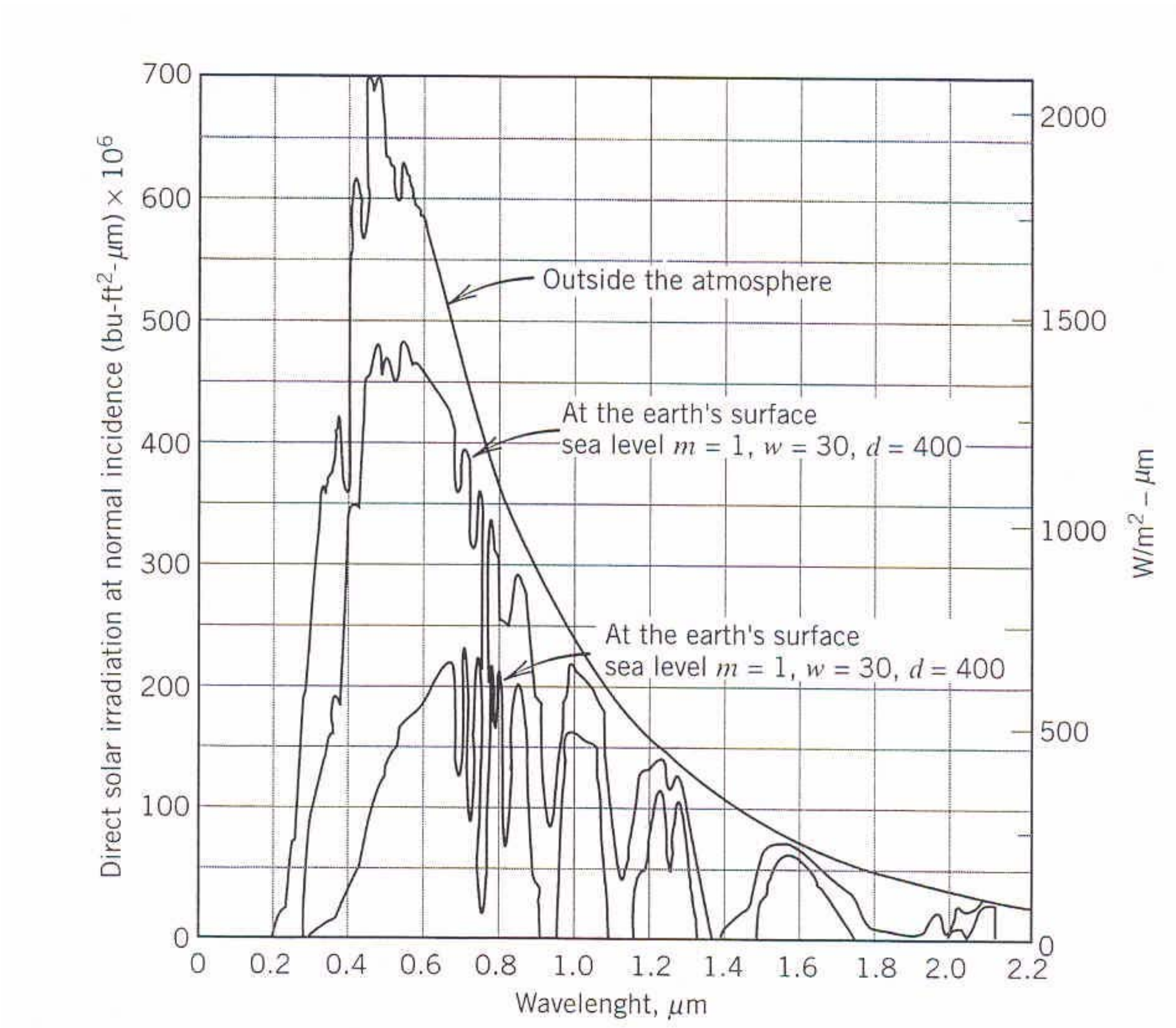

Figure 7-6. Spectral distribution of direct solar irradiation at normal incidence during clear days.

The total irradiation G_t on a surface normal to the sun's rays is made up of normal direct irradiation G_{ND}, diffuse irradiation G_d, and reflected irradiation G_R:

$$G_t = G_{ND} + G_d + G_R \qquad (7\text{-}14)$$

ASHRAE Clear Sky Model

The value of the solar constant does not take into account the absorption and scattering of the earth's atmosphere.

$$G_{ND} = \frac{A}{\exp\left(\frac{B}{\mathrm{Sin}\beta}\right)} C_N \tag{7-15}$$

G_{ND} = normal direct irradiation, Btu/(hr-ft^2) or (W/m^2)

A = apparent solar irradiation at air mass equal to zero, Btu/(hr-ft2) or (W/m2)

B = atmospheric extinction coefficient

β = solar altitude

C_N = clearness number

Values of *A* and *B* are given in Table 7-2. C_N is clearness number.

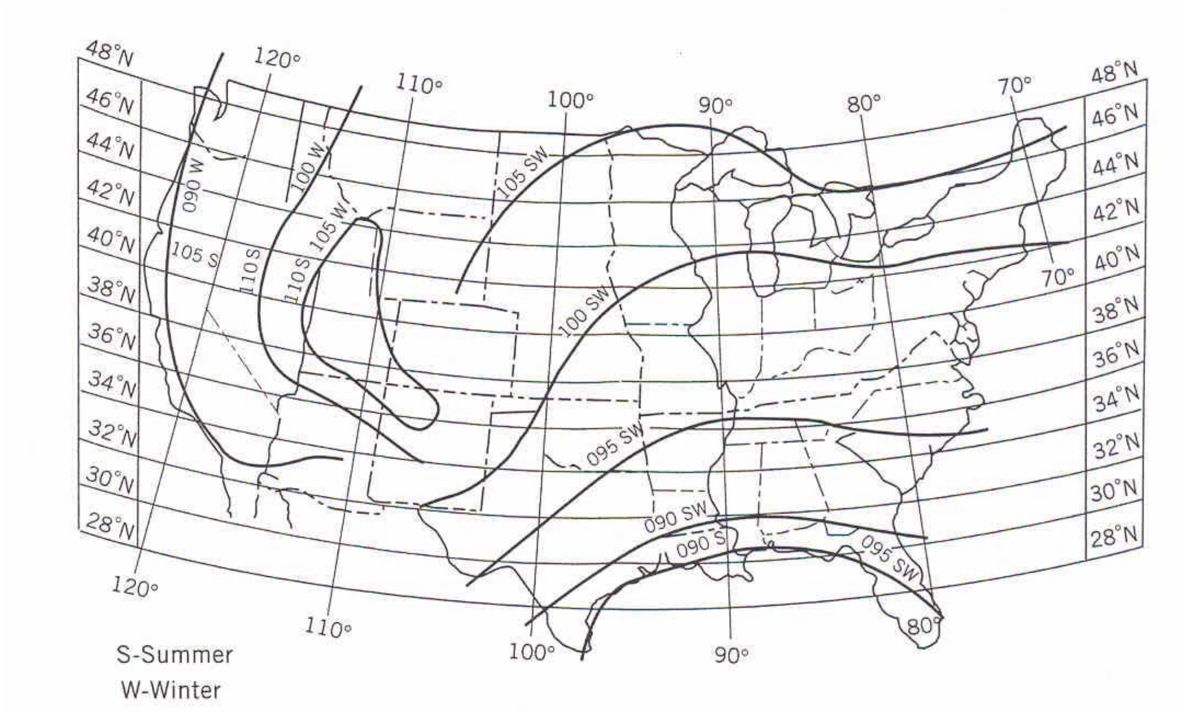

Figure 7-7. Estimated atmospheric clearness numbers C_N in the United States for nonindustrial localities, percent.

On a surface of arbitrary orientation, the direct radiation, corrected for clearness, is:

$$G_D = G_{ND} \bullet \max(\cos\theta, 0) \tag{7-16}$$

where θ is the angle of incidence. The diffuse irradiation on a horizontal surface is given by the use of factor C from Table 7-2. Read text for details.

$$G_d = CG_{ND} \tag{7-17}$$

Here C is the ratio of diffuse irradiation on a horizontal surface to direct normal irradiation. Refer to Figure 7-0. (not available in the text). C coefficient is taken from Table 7-2.

To evaluate the rate at which diffuse radiation $G_{d\theta}$ strikes a non vertical surface on a clear day, the following approximation can be made, (isotropic sky),

$$G_{d\theta} = CG_{ND}F_{ws} = CG_{ND}\left(\frac{1+\cos\alpha}{2}\right) \tag{7-18}$$

For a vertical surface,

$$\frac{G_{dV}}{G_{dH}} = 0.55 + 0.437\cos\theta + 0.313(\cos\theta)^2 \tag{7-21}$$

when $\cos\theta > -0.2$; otherwise $G_{dV}/G_{dH} = 0.45$. See Figure 7-8. Then for a vertical surface,

$$G_{d\theta} = CG_{ND}\frac{G_{dV}}{G_{dH}} \tag{7-22}$$

Energy reflected from ground and surroundings is approximated (diffuse reflection)

$$G_R = G_{tH}\rho_g\frac{(1-\text{Cos}\alpha)}{2} \tag{7-23}$$

- G_R = rate at which energy is reflected on to wall, Btu/(hr-ft^2) or (W/m^2)
- G_{tH} = rate at which the total radiation (direct plus diffuse) strikes the horizontal surface or ground in front of the wall, Btu/(hr-ft^2) or (W/m^2)
- ρ_g = reflectance of ground or horizontal surface

Total solar radiation incident on a non vertical surface,

$$G_{t,\beta} = G_D + G_d + G_R$$

$$G_{t,\beta} = \left[\max(\cos\theta,0) + C\frac{1+\text{Cos}\alpha}{2} + \rho_g\frac{1-\text{Cos}\alpha}{2}(\sin\beta + C)\right]G_{ND} \tag{7-25}$$

For a horizontal surface,

$$G_{t,H} = G_D + G_d = \left[\max(\cos\theta,0) + C\frac{1+\text{Cos}\alpha}{2}\right]G_{ND}$$

For a vertical surface,

$$G_{t,V} = \left[\max(\cos\theta, 0) + C\frac{G_{dV}}{G_{dH}} + \rho_g \frac{1-\text{Cos}\alpha}{2}(\sin\beta + C) \right] G_{ND} \tag{7-26}$$

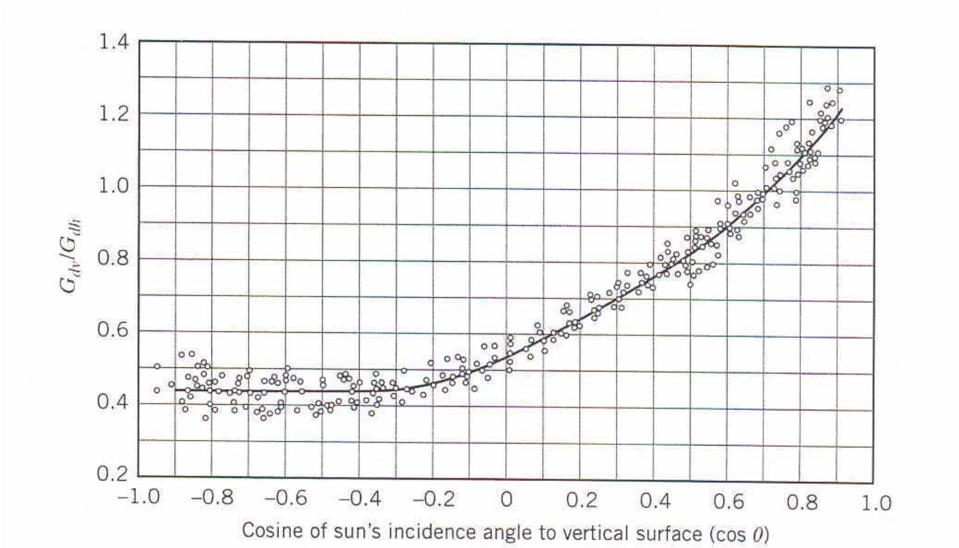

Figure 7-8. Ratio of diffuse sky radiation incident on a vertical surface to that incident on a horizontal surface during clear days.

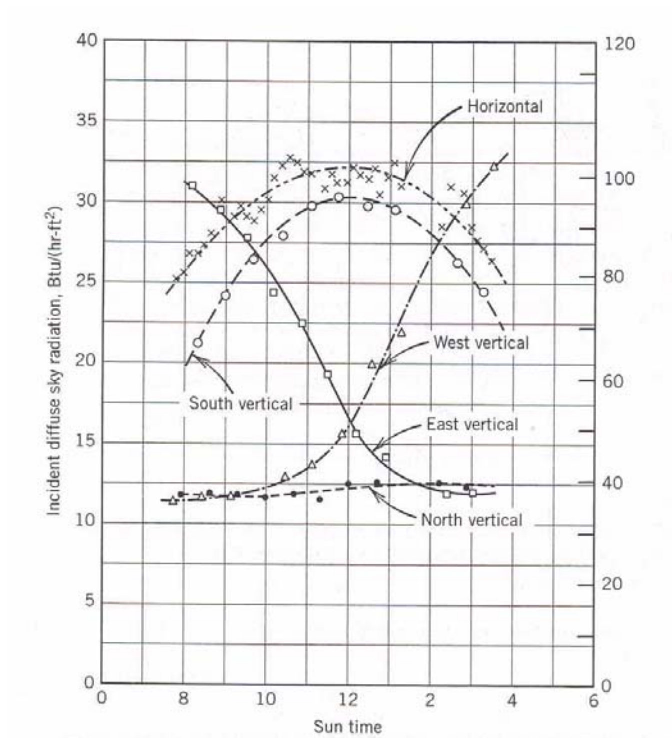

Figure 7-0. Variations of diffuse solar radiation from clear sky.

Example 7-3

Calculate the clear day direct, diffuse, and total solar radiation rate on a horizontal surface for Houghton, July 21, at 12:00 P.M. Local time.

SOLUTION

```
{McQuiston, Parker, Spitler; Chapter VII}
{Example 7-3}
{Calculate the clear day direct, diffuse, and total solar radiation for Houghton at 12:00 P.M. on
July 21}
{Define dimensional unit}
HM = 60                                   {min/hr}
Year = 365                                {day}
Deg = 360                                 {deg}
W = 4                                     {min/deg}
NN = 1                                    {day}
H = 1                                     {hr}
DC = 23.45                                {deg}
DN = 284                                  {day}
LSolN = 12                                {hr}
HL = 15                                   {deg/hr}
{Calculate solar time for Houghton}
DS = 1                                    {Daylight saving is ON}
EST = 75                                  {eastern standard time}
Long = 88.39                              {West}
Lat = 47.90                               {North}
{Month February, Day 21}
{Jan=31, Feb=28, Mar=31, Apr=30, May=31, Jun=30, Jul=31, Aug=31, Sep=30, Oct=31, Nov=30,
     Dec=31}
day = 31+ 28 + 31 + 30 + 31 + 30 + 21 {July 21}
N = (day - NN) * Deg / Year
EOT = 229.2*(0.000075+0.001868*cos(N)-0.032077*sin(N)-0.014615*cos(2*N)-0.04089*sin(2*N))
                                          {minutes}
{GIVEN}
LDST_h = 12                               {local daylight savings time, hours}
LDST_m = 0                                {local daylight savings time, minutes}
{End of given time}
LSdT = LDST_h - DS          {eastern standard time at EST corrected by daylight saving, hours}
{Calculate local standard time}
Diff = (Long - EST) * W
Min = - Diff + EOT                        {in minutes}
Ratio = abs(Min / HM / H)
Sig_R = Sign(Ratio)
HN = H * Trunc(Ratio) * Sig_R
Minutes = Min - HN * HM
LSolT_h = LSdT + HN
RHr = Minutes / HM
LSolT = LSolT_h + RHr
H_deg = (LSolT - LSolN) * HL
{Describe Collector Geometry}
gamma = 0
alpha = 0
delta_1 = DC * Sin(Deg * (DN + day)/Year)
```

```
Term_1 = 0.3963723 - 22.9132745*Cos(N) + 4.0254304 * Sin(N)
Term_2 = -0.3872050 * Cos (2*N) + 0.05196728 * Sin (2*N) - 0.1545267 * Cos(3*N) + 0.08479777 *
         Sin(3*N)
delta_2 = Term_1 + Term_2
SB = Cos(Lat) * Cos(H_deg) * Cos(delta_2) + Sin(Lat) * Sin (delta_2)
beta = arcsin(SB)
theta_Z = arccos(SB)
CP = (Sin(delta_2) * Cos(Lat) - Cos(delta_2) * Sin(Lat) * Cos(H_deg))/Cos(beta)
phi = arccos(CP)
CT = Cos(beta) * Cos(gamma) * Sin(alpha) + Sin(beta) * Cos(alpha)
theta = arccos(CT)
sum = beta + theta_Z
{Solar irradiation data - ASHRAE Clear sky model}
A = 346.4
B = 0.186
C = 0.138
C_N = 1.0                                  {Figure 7-7, page 193}
{Start calculations}
G_ND = A * C_N / exp(B/Sin(beta))          {normal direct}
G_dirE = G_ND * max(Cos(theta), 0)
G_difE = C * G_ND
G_horizE = G_dirE + G_difE
G_dirS = G_dirE * Convert(Btu/hr-ft^2, W/m^2)
G_difS = G_difE * Convert(Btu/hr-ft^2, W/m^2)
G_horizS = G_horizE * Convert(Btu/hr-ft^2, W/m^2)
```

SOLUTION   Example 7-3

Unit Settings: [F]/[psiaj/[Ibml/[degrees]

A = 346.40 [Btu/hr-ft^2]	α = 0.00 [deg)	B = 0.186 [-]
β = 53.61 [deg]	C = 0.138 [-]	CP = -0.615 [-]
CT = 0.805 [-]	C_N = 1.00 [-]	day = 202 [day]
DC 23.45 [deg]	Deg = 360 [deg]	δ_1 = 20.442 [deg]
δ_2 = 20.637 [deg]	Diff = 53.56 [min.]	DN = 284 [day]
DS = 1 [hr]	EOT = -6.354 [min.]	EST = 75 [deg)
y = 0.00 [deg]	G_{difE} = 37.94 [Btu/hr-ft^2]	G_{difS} = 119.69 [VV/m^2]
G_{dirE} = 221.31 [Btu/hr-ft^2]	G_{dirS} = 698.15 [W/m^2]	G_{horizE} = 259.25 [Btu/hr-ft^2]
G_{horizS} = 817.84 [W/m^2]	G_{ND} = 274.93 [Btu/hr-ft^2]	H = 1[hr)
HL = 15 [deg/hr]	HM = 60 [min./hr]	HN = 0.0 [hr]
H_{deg} = -29.98 [deg]	Lat = 47.90 [deg]	$LDST_h$ = 12.000 [hr]
$LDST_m$ = 0.000 [min.]	Long = 88.39 [deg]	LSdT = 11.000 [hr]
LSolN = 12.000 [hr]	LSolT=10.001 [hr]	$LSolT_h$ = 11.000 [hr]
Min = -59.91 [min.]	ϕ = 127.99 [deg	Ratio = 0.999 [-]
RHr = -0.9986 [hr]	SB = 0.805 [-]	Sig_R = 1 [-]
sum = 90.0 [deg]	$Term_1$= 20.90 [deg]	$Term_2$ = -0.26 [deg]
θ = 36.39 [deg]	θ_z = 36.39 [deg]	W = 4 [min./deg]
Year = 365 (day)		

No unit problems were detected.

Example 7-4

For Houghton, MI calculate the total solar energy available on a south facing vertical window (surface) with no setback, at 12:00 P.M. local time, on July 21. Use the clear day mode.

SOLUTION

```
{McQuiston, Parker, Spitler; Chapter VII}
{Example 7-4}
{Calculate the clear day solar radiation on a south facing vertical surface for Houghton at 12:00 P.M. on
July 21}
{Define dimensional unit}
HM = 60                         {min/hr}
Year = 365                      {day}
Deg = 360                       {deg}
W = 4                           {min/deg}
NN = 1                          {day}
H = 1                           {hr}
DC = 23.45                      {deg}
DN = 284                        {day}
LSolN = 12                      {hr}
HL = 15                         {deg/hr}
{Calculate solar time for Houghton}
DS = 1                          {Daylight saving is ON}
EST = 75                        {eastern standard time}
Long = 88.39                    {West}
Lat = 47.90                     {North}
{Month February, Day 21}
{Jan=31, Feb=28, Mar=31, Apr=30, May=31, Jun=30, Jul=31, Aug=31, Sep=30, Oct=31, Nov=30,
     Dec=31}
day = 31+ 28 + 31 + 30 + 31 + 30 + 21 {July 21}
N = (day - NN) * Deg / Year
EOT = 229.2*(0.000075+0.001868*cos(N)-0.032077*sin(N)-0.014615*cos(2*N)-0.04089*sin(2*N))
                                {minutes}
{GIVEN}
LDST_h = 12                     {local daylight savings time, hours}
LDST_m = 0                      {local daylight savings time, minutes}
{End of given time}
LSdT = LDST_h - DS        {eastern standard time at EST corrected by daylight saving, hours}
{Calculate local standard time}
Diff = (Long - EST) * W
LSolT_m = - Diff + EOT          {in minutes}
Ratio = LSolT_m / HM/H
HN = H * Trunc(Ratio)
LSolT_h = LDST_h + HN
Minutes = LSolT_m - HM * HN
LSolT = LSolT_h + minutes / HM
H_deg = (LSolT - LSolN) * HL
{Describe Collector Geometry}
PSI = 180
alpha = 90
gamma = abs(phi - PSI)
```

```
delta_1 = DC * Sin(Deg * (DN + day)/Year)
Term_1 = 0.3963723 - 22.9132745*Cos(N) + 4.0254304 * Sin(N)
Term_2 = -0.3872050 * Cos (2*N) + 0.05196728 * Sin (2*N) - 0.1545267 * Cos(3*N) + 0.08479777 *
         Sin(3*N)
delta_2 = Term_1 + Term_2
SB = Cos(Lat) * Cos(H_deg) * Cos(delta_2) + Sin(Lat) * Sin (delta_2)
beta = arcsin(SB)
theta_Z = arccos(SB)
CP = (Sin(delta_2) * Cos(Lat) - Cos(delta_2) * Sin(Lat) * Cos(H_deg))/Cos(beta)
phi = arccos(CP)
CT = Cos(beta) * Cos(gamma) * Sin(alpha) + Sin(beta) * Cos(alpha)
theta = arccos(CT)
sum = beta + theta_Z
{Solar irradiation data - ASHRAE Clear sky model}
A = 346.4
B = 0.186
C = 0.138
C_N = 1.0                                          {Figure 7-7, page 193}
R_vh = 0.55 + 0.437 * Cos(theta) + 0.313 * (Cos(theta))^2
rho_g = 0.5                                        {assumed}
{Start calculations; Horizontal plane}
G_ND = A * C_N / exp(B/Sin(beta))                  {normal direct}
G_dirE = G_ND * max(Cos(theta_Z), 0)
G_difE = C * G_ND
G_horizE = G_dirE + G_difE
G_dirS= G_dirE * Convert(Btu/hr-ft^2, W/m^2)
G_difS = G_difE * Convert(Btu/hr-ft^2, W/m^2)
G_horizS = G_horizE * Convert(Btu/hr-ft^2, W/m^2)
{Start calculations; Inclined plane}
{Equation 7-26 page 196}
G_dirEt = G_ND * max(Cos(theta), 0)
G_dirSt = G_dirEt * Convert(Btu/hr-ft^2, W/m^2)
G_difvE = C * G_ND * R_vh
G_refE = rho_g * (1 - Cos(alpha))/2 * G_horizE
G_totalE = G_dirEt + G_difvE + G_refE
G_difvS = G_difvE * Convert(Btu/hr-ft^2, W/m^2)
G_refS = G_refE * Convert(Btu/hr-ft^2, W/m^2)
G_totalS = G_totalE * Convert(Btu/hr-ft^2, W/m^2)
```

SOLUTION Example 7-4

Unit Settings: [F]/[psia]/[Ibm]/fdegrees]

A = 346.40 [Btulhr-ft^2]	α = 90.00 [deg]	B = 0.19 [-]
β = 53.61 [deg]	C = 0.138 [-]	CP = -0.615
CT = 0.365 [-]	C_N = I.000 [-]	day = 202 [day]
DC = 23.450 [deg]	Deg = 360.000 [deg]	δ_1 = 20.442 [deg]
δ_2 = 20.637 [deg]	Diff = 53.56 [min.]	DN = 284 [day]
DS = I [hr]	EOT = -6.354 [min.]	EST = 75 [deg]
γ = 52.012 [deg]	G_{difE} = 37.941 [Btu/hr-ft^2]	G_{difS} = 119.688 [W/m^2]
G_{difvE} = 28.506 [Btu/hr-ft^2]	G_{difvS} = 89.925 [W/m^2]	G_{dirE} = 100.401 [Btu/hr-ft^{2}I
G_{dirEt} = 100.4 [Btu/hr-ft^2]	G_{dirS} = 316.724 [W/m^2]	GdirSt = 316.7 [W/m^2]
G_{horizE} = 259.3[Btu/hr-ft^2]	G_{horizS} = 817.8[W/m ^{2}i]	G_{ND} = 274.934 [Btu/hr-ft^2]
G_{refE} = 64.814 [Btu/hr-ft^2]	G_{refS} = 204.461 [W/m^2]	G_{totalE} = 193.721 [Btu/hr-ft^2]
G_{totalS} = 611.109 [W/m^2]	H = 1 (hrl	HL = 15 [deg/hr]
HM = 60 [min./hr]	HN = 0 [hr]	H_{deg} = -29.979 [deg]
Lat = 47.90 [deg]	$LDST_h$ = 12.000 [hr]	$LDST_m$ = 0.000 [min.]
Long = 88.39 [deg]	LSdT = 11.000 [hr]	$LSol_N$ = 12.000 [hr]
$LSol_T$ = 10.001 [hr]	$LSolT_h$ = 11.000 [hrl	Min = -59.91 [min.]
Minutes = -59.91 [min.]	N = 198.25 [deg)	NN = I [day]
ϕ = 127.988 [deg]	ψ = 180.000 [deg]	Ratio = 0.999 [-]
ρ_g = 0.50 [-]	RHr = -0.999 [hr]	R_{vh} = 0.751
SB = 0.805 [-]	Sig_R = 1 [-]	sum = 90.00 [deg]
$Term_1$ = 20.897 [deg]	$Term_2$ = -0.260 [deg]	θ = 68.581 (deg)
θ_z = 36.392 [deg]	W = 4 [min./deg]	Year= 365 (day)

No unit problems were detected.

City: HOUGHTON, MI

WBAN No: 94814; Lat(N): 47.17; Long(W): 88.50; Elev(m): 329; Pres(mb): 974
SOLAR RADIATION FOR FLAT-PLATE COLLECTORS FACING SOUTH
(kWh/m^2/day), Percentage Uncertainty = 9

Tilt(deg)		J	F	M	A	M	J	J	A	S	O	N	D	Yr
0	Average	1.3	2.2	3.5	4.6	5.5	6.0	6.0	5.0	3.6	2.3	1.3	1.1	3.6
	Minimum	1.0	1.7	2.8	4.0	4.6	5.2	5.5	4.3	3.0	2.0	1.1	0.7	3.3
	Maximum	1.6	2.6	4.1	5.3	6.6	7.0	6.7	5.7	4.3	2.7	1.6	1.3	3.8
Lat - 15	Average	2.1	3.2	4.5	5.2	5.6	5.9	6.0	5.5	4.4	3.2	1.9	1.6	4.1
	Minimum	1.3	2.4	3.5	4.3	4.7	5.0	5.4	4.6	3.3	2.6	1.3	1.0	3.7
	Maximum	2.8	4.0	5.5	6.3	6.9	6.9	6.8	6.3	5.6	3.9	2.6	2.1	4.4
Lat	Average	2.3	3.5	4.7	5.2	5.3	5.4	5.6	5.2	4.4	3.4	2.1	1.8	4.1
	Minimum	1.4	2.5	3.5	4.2	4.3	4.6	5.1	4.4	3.3	2.7	1.3	1.0	3.7
	Maximum	3.2	4.4	5.8	6.3	6.5	6.3	6.3	6.1	5.7	4.2	2.9	2.3	4.4
Lat +15	Average	2.5	3.6	4.7	4.8	4.7	4.7	4.8	4.7	4.2	3.4	2.1	1.9	3.8
	Minimum	1.4	2.5	3.4	3.8	3.8	4.0	4.4	4.0	3.1	2.7	1.2	1.0	3.4
	Maximum	3.4	4.6	5.9	6.1	5.7	5.4	5.5	5.5	5.4	4.3	3.1	2.5	4.1
90	Average	2.4	3.3	4.1	3.7	3.0	2.9	3.0	3.2	3.2	2.9	2.0	1.8	3.0
	Minimum	1.2	2.2	2.7	2.7	2.5	2.5	2.8	2.8	2.3	2.3	1.0	0.9	2.5
	Maximum	3.3	4.4	5.3	5.0	3.6	3.2	3.3	3.8	4.3	3.7	2.8	2.5	3.2

AVERAGE CLIMATIC CONDITIONS

Element		J	F	M	A	M	J	J	A	S	O	N	D	Yr
Temp	deg C	-8.4	-7.7	-2.1	5.7	12.3	17.1	19.7	18.4	14.1	8.2	1.8	-5.2	6.2
Daily Min	deg C	-13.1	-13.2	-7.7	-0.2	5.6	10.3	13.0	12.1	8.3	3.1	-2.0	-9.1	0.6
Daily Max	deg C	-3.7	-2.2	3.4	11.5	18.9	23.8	26.4	24.7	19.9	13.2	5.6	-1.4	11.7
Record Lo	deg C	-32.2	-36.7	-30.6	-16.1	-6.1	-1.7	0.6	-1.7	-6.1	-8.9	-18.9	-29.4	-36.7
Record Hi	deg C	11.7	15.0	24.4	30.0	32.2	35.0	36.7	34.4	33.3	29.4	21.1	17.2	36.7
HDD-Base	18.3 C	828	728	634	380	199	67	19	39	130	315	497	730	4566
CDD-Base	18.3 C	0	0	0	0	11	28	62	41	0	0	0	0	142
Rel Hum	%	79	76	74	68	65	69	71	75	78	77	81	82	75
Wind Spd	(m/s)	4.5	4.1	4.4	4.5	4.1	3.8	3.4	3.3	3.5	3.9	4.3	4.3	4.0

NREL Data,

94814 HOUGHTON MI -5 N(47 10) W(088 30) 329m											
Year	Month	Day	Hour	Extraterrestrial		Global Horizontal direct+diffuse		Direct Normal		Diffuse Horizontal	
				Horizontal	Direct Normal						
Solar radiation in Wh/m^2											
A	B	C	D	E	F	G	H	I	K	L	M
90	2	21	1	0	0	0	?0	0	?0	0	?0
90	2	21	2	0	0	0	?0	0	?0	0	?0
90	2	21	3	0	0	0	?0	0	?0	0	?0
90	2	21	4	0	0	0	?0	0	?0	0	?0
90	2	21	5	0	0	0	?0	0	?0	0	?0
90	2	21	6	0	0	0	?0	0	?0	0	?0
90	2	21	7	0	0	0	?0	0	?0	0	?0
90	2	21	8	20	93	6	G5	7	G5	5	G5
90	2	21	9	138	1398	54	G5	68	G5	48	G5
90	2	21	10	351	1398	191	G5	219	G5	136	G5
90	2	21	11	529	1398	339	G5	461	G5	164	G5
90	2	21	12	657	1398	405	G5	354	G5	239	G5
90	2	21	13	728	1398	549	G5	746	G5	160	G5
90	2	21	14	737	1398	568	G5	593	G5	256	G5
90	2	21	15	682	1398	545	G4	700	G4	204	G5
90	2	21	16	568	1398	385	G4	523	G4	173	G5
90	2	21	17	402	1398	274	G4	520	G4	124	G5
90	2	21	18	197	1398	108	G5	312	G4	64	G5
90	2	21	19	48	466	16	G5	26	G4	14	G5
90	2	21	20	0	0	0	?0	0	?0	0	?0
90	2	21	21	0	0	0	?0	0	?0	0	?0
90	2	21	22	0	0	0	?0	0	?0	0	?0
90	2	21	23	0	0	0	?0	0	?0	0	?0
90	2	21	24	0	0	0	?0	0	?0	0	?0

94814 HOUGHTON MI -5 N(47 10) W(088 30) 329m											
Year	Month	Day	Hour	Extraterrestrial		Global Horizontal (dirct+diffuse)		Direct Normal		Diffuse Horizontal	
				Horizontal	Direct Normal						
Solar radiation in Wh/m^2											
A	B	C	D	E	F	G	H	I	K	L	M
90	7	21	1	0	0	0	?0	0	?0	0	?0
90	7	21	2	0	0	0	?0	0	?0	0	?0
90	7	21	3	0	0	0	?0	0	?0	0	?0
90	7	21	4	0	0	0	?0	0	?0	0	?0
90	7	21	5	0	0	0	?0	0	?0	0	?0
90	7	21	6	66	794	17	G5	17	G5	15	G5
90	7	21	7	233	1323	75	G5	140	G5	50	G5
90	7	21	8	452	1323	201	G5	273	G5	108	G5
90	7	21	9	663	1323	433	G4	639	G4	112	G5
90	7	21	10	854	1323	591	G5	698	G5	140	G5
90	7	21	11	1009	1323	731	G5	750	G5	159	G5
90	7	21	12	1119	1323	748	G5	716	G5	143	G5
90	7	21	13	1176	1323	827	G5	705	G5	200	G5
90	7	21	14	1177	1323	787	G5	694	G5	169	G5
90	7	21	15	1120	1323	633	G5	439	G5	261	G5
90	7	21	16	1011	1323	421	G4	111	G4	337	G5
90	7	21	17	856	1323	320	G4	193	G4	194	G5
90	7	21	18	666	1323	162	G4	32	G4	146	G5
90	7	21	19	454	1323	86	G4	6	G4	84	G5
90	7	21	20	235	1323	53	G5	4	G4	52	G5
90	7	21	21	68	794	14	G5	0	G4	14	G5
90	7	21	22	0	0	0	?0	0	?0	0	?0
90	7	21	23	0	0	0	?0	0	?0	0	?0
90	7	21	24	0	0	0	?0	0	?0	0	?0

Example 7-

For Houghton, MI calculate the total solar energy available on a south facing vertical window (surface) with no setback, at 12:00 P.M. local time, on July 21. Use NREL data.

SOLUTION

94814 HOUGHTON MI -5 N(47 10) W(088 30) 329m											
Year	Month	Day	Hour	Extraterrestrial		Global Horizontal (dirct+diffuse)		Direct Normal		Diffuse Horizontal	
				Horizontal	Direct Normal						
Solar radiation in Wh/m^2											
A	B	C	D	E	F	G	H	I	K	L	M
90	7	21	12	1119	1323	748	G5	716	G5	143	G5
90	7	21	13	1176	1323	827	G5	705	G5	200	G5
				1148	**1323**	**788**		**711**		**172**	

From the table above,

G_{ND} = 711 Wh/m^2

G_d = 172 Wh/m^2

ρ_g = 0.5 assumed

G_R = 788 * 0.5 * 1/2 = 197 Wh/m^2

From Example 7-4, $\theta = 68.58^o$, $Cos(\theta)$ = CT = 0.365, and

$\theta_Z = 36.39^o$, $Cos(\theta_Z) = 0.8050$

G_{tH} = 788 Wh/m^2, but using clear day approach we found G_{horizS} = 818 Wh/m^2

From Equ. (7-25),

$$G_{t,\beta} = G_{ND} \cdot \max(\cos\theta, 0) + G_d \frac{1 + Cos\alpha}{2} + \rho_g G_{tH} \frac{1 - Cos\alpha}{2}$$

$G_{t,V}$ = 711 * 0.365 + 172 * 1/2 + 197 = 542.5 Wh/m^2 (as opposed to 611 Wh/m^2)

7-6 Heat Gain Through Fenestrations

The term *fenestration* refers to any glazed aperture in a building envelope.

- Glazing material, either glass or plastic
- Framing, mullions, muntin, and dividers
- External shading devices
- Internal shading devices
- Integral (between-glass) shading system

Fenestrations are important for energy use in a building, since they affect rates of beat transfer into and out of the building, are a source of air leakage, and provide day-lighting, which may reduce the need for artificial lighting. The solar radiation passing inward through the fenestration glazing permits heat gains into a building that are quite different from the heat gains of the non transmitting parts of the building envelope. This behavior is best seen by referring to Figure 7-9.. When solar radiation strikes an unshaded window about 8 percent of the radiant energy is typically reflected back outdoors, from 5 to 50 percent is absorbed within the glass, depending on the composition and thickness of the glass, and the remainder is transmitted directly indoors, to become part of the cooling load. The solar gain is the sum of the transmitted radiation and the portion of the absorbed radiation that flows inward. Because heat is also conducted through the glass whenever there is an outdoor-indoor temperature difference, the total rate of heat admission is

Total heat admission through glass = (Solar) Radiation transmitted through glass +Inward flow of absorbed solar radiation + Conduction heat gain.

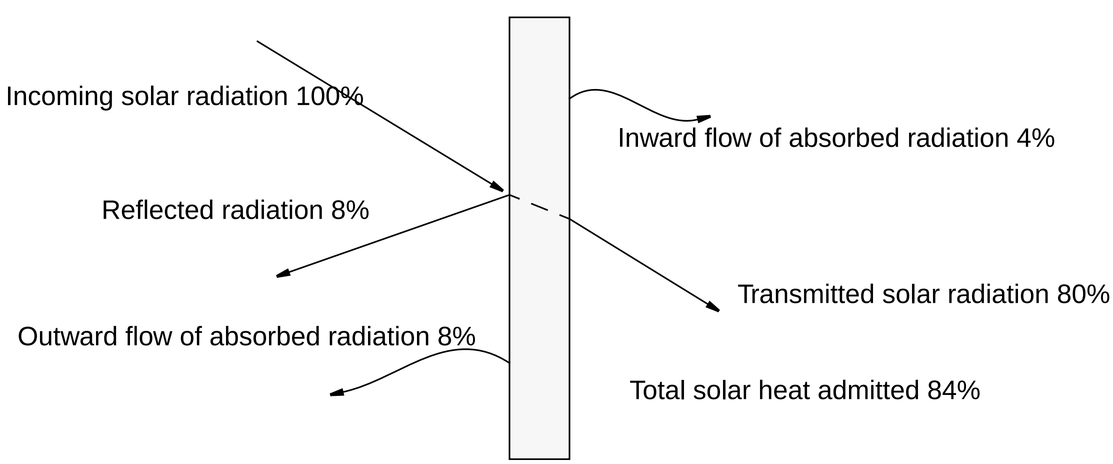

Figure 7-9. Distribution of solar radiation falling on clear plate glass.

The first two quantities on the right are related to the amount of solar radiation falling on the glass, and the third quantity occurs whether or not the sun is shining. In winter the conduction heat flow may well be outward rather than inward. The total heat gain becomes

Total heat gain = Solar heat gain + Conduction heat gain

The inward flow of absorbed solar radiation and the conduction heat gain are not independent, but they are often approximated as if they are. In this case, the conduction heat gain per unit area is simply the product of the overall coefficient of heat transfer U for the existing fenestration and the outdoor-indoor temperature difference (T_o - T_i). Values of U for a number of widely used glazing systems are given in Table 5-5. Additional values may be found in the *ASHRAE Handbook, Fundamentals Volume (5)* and in manufacturers' literature. More detailed approach, which accounts for the conduction heat gain simultaneously with the inward flowing absorbed solar radiation will be seen later in Section 8-9, Interior Surface Heat Balance-Opaque Surfaces.

Solar Heat Gain Coefficients

The heat gain through even the simplest window is complicated by the fact that the window is finite in size, it is framed, and the sunlight striking it does so at varying angles throughout the day. To fully take all of the complexities into account requires the use of not only spectral methods (using monochromatic radiation properties) but also the angular radiation characteristics involved. The equations required become quite complex, the required properties are sometimes difficult to determine. and lengthy computer calculations are involved. Early steps in this process are described by Harrison and van Wonderen (Ref. 10) and by Arasteh (Ref. 11). For a more complete description of the method refer to the fenestration chapter in the most recent edition of the *ASHRAE Handbook, Fundamentals Volume (5)*.

A simplified method utilizes a spectrally-averaged *solar heat gain coefficient* (SHGC), the fraction of the incident irradiance (incident solar energy) that enters the glazing and becomes heat gain:

$$q = (G_i)SHGC \tag{7-27}$$

The SHGC includes the directly transmitted portion, the inwardly flowing fraction of the absorbed portion, and, in some forms, the inwardly flowing fraction of that absorbed by the window frame. It does not include the portion of the fenestration heat gain due to a difference in temperature between the inside and outside air. In multiple pane glazings. the determination of the SHGC requires several assumptions to estimate the inward flowing fraction of absorbed radiation for each of the layers. Values of SHGC at a range of incidence angles for several types of glazings are found in Table 7-3. A broader selection may be found in the *ASHRAE Handbook, Fundamentals Volume (5)*, or they may be calculated with the WINDOW 5.2 software (Ref. 12).

It should be noted that. with respect to the procedures described here, it is usually the case that window data provided by the manufacturer do not include incident angle-dependent SHGC, transmittances, etc. Rather. it is more common to give SHGC for normal irradiation; both SHGC and the U-factor are often given for the entire window, including the frame. They may also be given for the center-of-glazing. If this is all that is available. it is suggested that the engineer compare these numbers to those for similar-type windows (e.g., number of panes. configuration, type of frame, coatings, etc.) in Table 7-3 or the *ASHRAE Handbook. Fundamentals Volume (5)* and choose angle-dependent properties for a similar window.

Unfortunately the SHGC approach does not directly allow for separate treatment of transmitted and absorbed components of the solar heat gain. However, for detailed cooling load calculations, it is desirable to be able to separate the two components. Fortunately, new data (transmittance and layer-by-layer absorptance) available in Table 7-3 and the *ASHRAE Handbook. Fundamentals Volume (5)* and calculable for any window with the WINDOW 5.2 software (Ref. 12) do allow a separate estimation of the transmitted and absorbed components. Two procedures are described below: a “simplified” procedure that utilizes SHGC and, hence. blends together the transmitted and absorbed components. and a “detailed” procedure that estimates them separately.

The procedure may be described from “outside to inside.”

- First, the direct and diffuse solar radiation incident on an unshaded surface with the same orientation as the window is calculated with the procedures described in Section 7-3 through 7-5.
- Second. the effects of external shading on the solar radiation incident on the window are determined.
- Third. the solar radiation transmitted and absorbed is analyzed for the window. assuming no internal shading.
- Fourth. If there is internal shading, its effects on the total amount of solar radiation transmitted and absorbed are calculated.

For the third and fourth parts, both simplified and detailed procedures are described.

Table 7-3 Solar Heat Gain Coefficients (SHGC), Solar Transmittance (T), Front Reflectance (R_f), Back Reflectance (R_b), and Layer Absorptances ($\mathcal{A}_{fn}$) for Glazing Window Systems

				Center-of-Glazing Properties							Total Window SHGC at Normal Incidence			
				Incidence Angles							Aluminum		Other Frames	
ID	Glass Thick. in. (mm)			Normal 0.00	40.0	50.0	60.0	70.0	80.0	Diffuse	Operable	Fixed	Operable	Fixed
la	1/8 (3.2)	Uncoated Single	SHGC	0.86	0.84	0.82	0.78	0.67	0.42	0.78	0.75	0.78	0.64	0.75
		Glazing, CLR	T	0.83	0.82	0.80	0.75	0,64	0.39	0.75				
			R_f	0.08	0.08	0.10	0.14	0.25	0.51	0.14				
			R_b	0.08	0.08	0.10	0.14	0.25	0.51	0.14				
			A	0-09	0.10	0.10	0.11	0.11	0.11	0.10				
5a	1/8 (3.2)	Uncoated Double	SHGC	0.76	0.74	0.71	0.64	0.50	0.26	0.66	0.67	0.69	0.56	0.66
		Glazing, CLR CLR	T	0.70	0.68	0.65	0.58	0.44	0.21	0.60				
			R_f	0.13	0.14	0.16	0.23	0.36	0.61	0.21				
			R_b	0.13	0.14	0.16	0.23	0.36	0.61	0.21				
			A_f,	0.10	0.11	0.11	0.12	0.13	0.13	0.11				
			A_4	0.07	0.08	0.08	0.08	0.07	0.05	0.07				
5b	1/4 (6.4)	Uncoated Double	SfiGC	0.70	0.67	0.64	0.58	0.45	0.23	0.60	0.61	0.63	0.52	0.61
		Glazing, CLR CLR	T	0.61	0.58	0.55	0.48	0.36	0.17	0.51				
			R_f	0.11	0.12	0.15	0.20	0.33	0.57	0.18				
			R_b	0.11	0.12	0.15	0.20	0.33	0.57	0.18				
			A_t,	0.17	0.18	0.19	0.20	0.21	0.20	0.19				
			A_4	0.11	0.12	0.12	0.12	0.10	0.07	0.11				
21a	1/8 (3.2)	Low-e Double	SfiOC	0.65	0.64	0.62	0.56	0.43	0.23	0.57	0.48	0.50	0.41	0.47
		Giazing, e = 0. I on	T	0.59	0.56	0.54	0.48	0.36	0.18	0.50				
		surface 2, LE CLR	R_f	0.15	0.16	0.18	0.24	0.37	0.61	0.22				
			R_b	0.17	0.18	0.20	0.26	0.38	0.61	0.24				
			bb	0.20	0.21	0.21	0.21	0.20	0.16	0.20				
			cc	0.07	0.07	0.08	0.08	0.07	0.05	0.07				
21c	118 (3.2)	Low-e Double	SHGC	0.60	0.58	0.56	0.51	0.40	0.22	0.52	0.53	0.55	0.45	0.53
		Glazing, e = 0. I on	T	0.48	0.45	0.43	0.37	0.27	0.13	0.40				
		surface 3, CLR LE	R_f	0.26	0.27	0.28	0.32	0.42	0.62	0.31				
			R_b	0.24	0.24	0.26	0.29	0.38	0.58	0.28				
			A_f	0.12	0.13	0.14	0.14	0.15	0.15	0.13				
			A_4	0.14	0.15	0.15	0.16	0.16	0.10	0.15				

Triple glazing data have been excluded. See text for details.

Source: ASHRAE Handbook, Fundamentals Volume, American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc., 2001.

External Shading

A fenestration may be shaded by roof overhangs. awnings. side fins or other parts of the building, trees. shrubbery, or another buildIng. External shading of fenestrations is effective in reducing solar heat gain to a space and may produce reductions of up to 80 percent. In order to determine the solar radiation Incident on the fenestration, it is necessary to determine the area of the fenestration that is shaded and the area that is sunlit. The areas on which external shade falls can be calculated from the geometry of the external objects creating the shade and from knowledge of the sun angles for that particular time and location. It is generally assumed that shaded areas have no incident direct radiation, but that the diffuse irradiation incident on the shaded area is the same as that on the sunlit area. This is a conservative approximation, if more accuracy is desired, it would be possible to refine the configuration factor to the sky $[(1 + \mathrm{Cos}\alpha)/2]$.

In general, shading devices may have almost any geometry. A general algorithm for determining shading caused by any shape with any orientation is given by Walton (Ref. 13). Procedures for other specific shapes are given in references reviewed by Spitler (Ref. 14). Here, we will describe a procedure suitable for horizontal or vertical shading devices that are long enough to cast a shadow along the entire fenestration.

Figure 7-10. illustrates a window that is set back into the structure, where shading may occur on the sides and top, depending on the time of day and the direction the window faces.

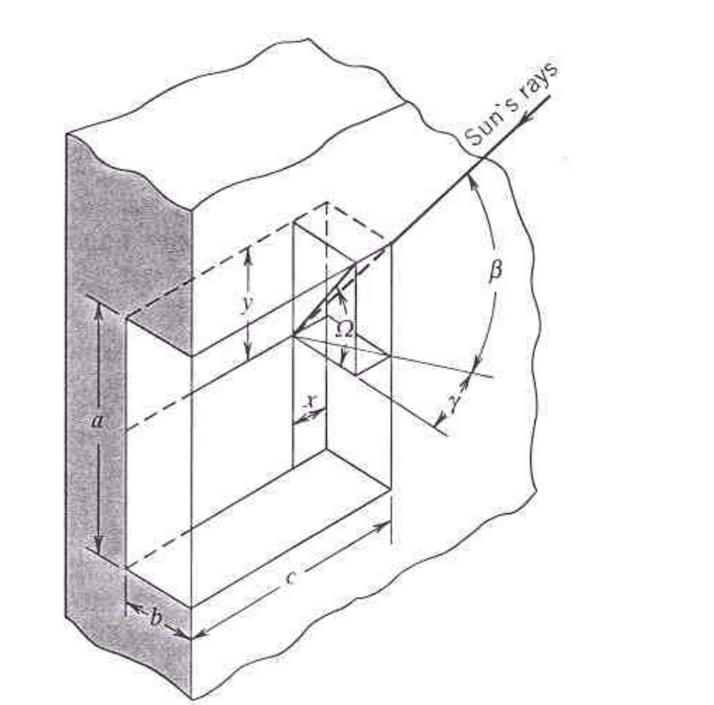

Figure 7-10. Shading of window set back from the plane of a building surface.

It can be shown that the dimensions x and y are given by

$$x = b\mathrm{Tan}\gamma \quad (7\text{-}28)$$

$$y = b\mathrm{Tan}\Omega \quad (7\text{-}29)$$

where:

$$\mathrm{Tan}\Omega = \frac{\mathrm{Tan}\beta}{\mathrm{Cos}\gamma} \quad (7\text{-}30)$$

Here:

β = sun's altitude angle

γ = wall solar azimuth angle = $|\phi - \psi|$

ϕ = solar azimuth measured clockwise from north

ψ = wall azimuth, measured clockwise from north

If γ is greater than 90 deg, the surface is in the shade. Equations (7-29) and (7-30) can be used for an overhang at the top and perpendicular to the window provided that the overhang- is wide enough for the shadow to extend completely across the window.

Example 7-5

A 4 ft high x 5 ft wide double-glazed window faces southwest ($\psi = 225$). The window has a frame with width of 1.5 in. around the outside edge. (The actual glazed area has dimensions of 3.75 ft high x 4.75 ft wide.) The top of the window has a 2 ft overhang that extends a great distance on each side of the window. Compute the shaded area of the frame and glazing on July 21 (n = 202) at 3:00 PM. solar time (h = 45), at l = 40 deg N latitude.

SOLUTION

Note

4x5 = 20 ft^2, 3.75x4.75 = 17.81 ft^2, the difference is 2. 1875 ft^2 yet the text indicates the frame area to be 2.63 ft^2. We will stay with that value to be consistent with the text. Remember,

$$\sin\beta = \cos l \cos h \cos\delta + \sin l \sin\delta, \ \beta = 47.0 \text{ deg}$$

$$\cos\phi = \frac{\sin\delta\cos l - \cos\delta\sin l\cos h}{\cos\beta}, \ \phi = 256.6 \text{ deg}$$

$$\gamma = \text{abs}(\phi - \psi) = 31.6 \text{ deg}$$

With (1/8 in) set back from the edge of the frame actual frame area needs to be increased by the following amount to

$$2.63 + ((1/8)/12)x2x(3.75 + 4.75) = 2.8071 \sim 2.81 \ ft^2.$$

This info will be used later.

$$x = b\text{Tan}\gamma = 2\text{Tan}(31.6) = 1.23 \ ft$$

$$y = b\frac{\text{Tan}\beta}{\text{Cos}\gamma} = 2\frac{\text{Tan}(47)}{\text{Cos}(31.6)} = 2.52 \ ft$$

The shading on the window is illustrated in Figure 7-11. For the shaded area of the frame,

$$A_{sh,\,f} = 2.52x0.125x2 + 0.125x4.75 = 1.22 \ ft^2$$

The sunlit portion of the frame has an area of

$$A_{sl,\,f} = 2.63 - 1.22 = 1.41 \ ft^2$$

For the shaded and sunlit areas the glazing,

$$A_{sh,g} = (2.52 - 0.125)4.75 = 11.38 \ ft^2$$

$$A_{sh,\,g} = 17.8125 - 11.3763 = 6.4363 \ ft^2$$

The sunlit portion of the areas receive direct (beam) and diffuse radiations, whereas shaded portions receive only indirect (diffuse) radiation.

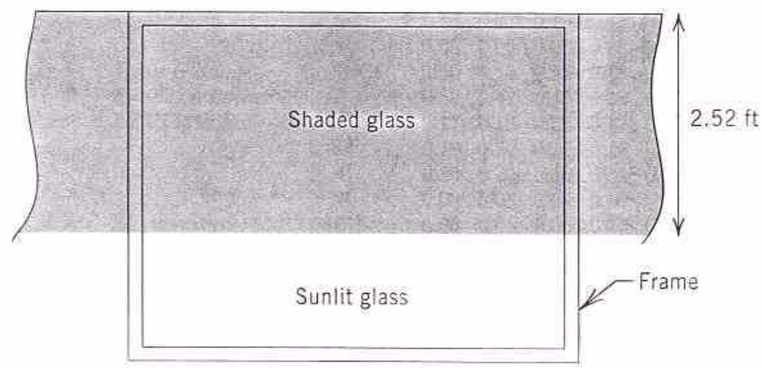

Figure 7-11. Shading of window for Example 7-5

Chapter 7

Transmission and Absorption of Fenestration Without Internal Shading, Simplified

In order to determine solar heat gain with the simplified procedure, it is assumed that, based on the procedures described above,

- the direct irradiance on the surface (G_D),
- the diffuse irradiance on the surface (G_d),
- the sunlit area of the glazing ($A_{sl,g}$),
- the sunlit area of the frame ($A_{sl,f}$)
- the areas of the glazing (A_g) and frame (A_f)
- and the basic window properties must be known.

The solar heat gain coefficient of the frame ($SHGC_f$) may be estimated as

$$SHGC_f = \alpha_f^s\left(\frac{U_f A_{frame}}{h_f A_{surf}}\right) \quad (7\text{-}31)$$

where

- A_{frame} is the projected area of the frame element, and
- A_{surf} is the actual surface area.
- α_f^s is the solar absorptivity of the exterior frame surface (see Table 7-1).
- U_f is the U-factor of the frame element (see Table 5-6);
- h_f is the overall exterior surface conductance (see Table 5-2a).
- If other frame elements like dividers exist, they may be analyzed in the same way.

The solar heat gain coefficient of the glazing may be taken from Table 7-3 for a selection of sample windows. For additional windows, the reader should consult the *ASHRAE Handbook, Fundamentals Volume (5)* as well as the WINDOW software (Ref. 12).

There are actually two solar heat gain coefficients of interest, one for direct radiation at the actual incidence angle ($SHGC_{gD}$) and a second for diffuse radiation ($SHGC_{gd}$). $SHGC_{gD}$ may be determined from Table 7-3 by linear interpolation. Values of $SHGC_{gd}$ may be found in the column labeled “Diffuse.”

Once the values of $SHGC_f$, $SHGC_{gD}$ and $SHGC_{gd}$ have been determined, the total solar heat gain of the window may be determined by applying direct radiation to the sunlit portion of the fenestration and direct and diffuse radiation to the entire fenestration:

$$\dot{q}_{SHG} = [SHGC_{gD}A_{sl,g} + SHGC_f A_{sl,f}]G_{D\theta} + [SHGC_{gd}A_g + SHGC_f A_f]G_{d\theta} \quad (7\text{-}32)$$

To compute the total heat gain through the window, the conduction heat gain must be added, which is estimated as

$$\dot{q}_{CHG} = U(T_o - T_i) \quad (7\text{-}33)$$

where U for the fenestration may be taken from Table 5-5, the ASHRAE Handbook, Fundamentals Volume (5), or the WINDOW 5.2 software (Ref. 12); and (T_o - T_i) is the outdoor-indoor temperature difference.

Transmission and Absorption of Fenestration Without Internal Shading, Detailed

Incorporates transmitted solar irradiation and that portion of absorbed radiation that is transmitted inward. EXCLUDED.

Example 7-6

Consider the 4 ft high x 5 ft wide, fixed (inoperable) double-glazed window, facing southwest from Example 7-5. The glass thickness is 1/8 In., the two panes are separated by a 1/4 in. air space, and surface 2 (the inside of the outer pane) has a low-e coating with an emissivity of 0.1. The frame, painted with white acrylic paint, is aluminum with thermal break; the spacer is insulated. The outer layer of glazing, is set back from the edge of the frame 1/8 in. On July 21 at 3:00 P.M. solar time at 40 deg N latitude, the incident angle is 54.5 deg, the incident direct irradiation is 155.4 Btu/hr-ft^2, and the incident diffuse irradiation is 60.6 Btu/hr-ft^2. Find the solar heat gain of the window.

SOLUTION

The window corresponds to ID 21a in Table 7-3.

- $SHGC_{gD}$ is found to be 0.59; $SHGC_{gd}$ is 0.57.
- The frame U-factor may be determined from Table 5-6 to be 1.04 Btu/hr-ft^2-F.
- The solar absorptance of white acrylic paint, from Table 7- 1, is 0.26.
- The outside surface conductance, from Table 5-2, is 4.0 Btu/hr-ft^2-F.
- The projected area of the frame is 2.63 ft^2; the actual surface area, 2.81 ft^2, is slightly larger, because the glass is set back 1/8 in. from the outer edge of the frame.
- $SHGC_f$ may be estimated with Eq. (7-31)

$$SHGC = 0.26\left(\frac{1.04 \times 2.63}{4.0 \times 2.81}\right) = 0.063$$

Then, from Eq. (7-32), the solar heat gain may be estimated:

$$\dot{q} = [0.59 \times 6.43 + 0.063 \times 1.41]155.4 + [0.57x17.81 + 0.063x2.63]60.6 = 1228.6$$

Btu/hr.

Transmission and Absorption of Fenestration with Internal Shading, Simplified

Internal shading, such as Venetian blinds, roller shades, and draperies, further complicate the analysis of solar heat gain. Shading devices are successful in reducing solar beat gain to the degree that solar radiation is reflected back out through the window. Solar radiation absorbed by the shading device will be quickly released to the room. Limited availability of data precludes a very detailed analysis, and angle of incidence dependence is usually neglected. To calculate the effect of internal shading, it is convenient to recast Eq. (7-2) to separate the beat gain due to the glazing and frame. Then, the solar radiation transmitted and absorbed by the glazing is multiplied by an interior solar attenuation coefficient (IAC)

$$\dot{q}_{SHG} = [SHGC_f A_{sl,f} G_{D\theta} + SHGC_f A_f G_{d\theta}] + \tag{7-41}$$

$$= [SHGC_{gD} A_{sl,g} G_{D\theta} + SHGC_{gd} A_g G_{d\theta}] IAC$$

Interior solar attenuation coefficients for Venetian blinds and roller shades may be found in Table 7-4. Since the effect of the shading device depends partly on the window, the values of IAC given in this table depend on both the shading device and the type of glazing, characterized by configuration and SHGC at normal incidence.

For draperies, the IAC depends on the color and weave of the fabric. Although other variables also have an effect, reasonable correlation has been obtained using only color and openness of the weave. Figure 7-12. may be used to help characterize openness.

- Openness is classified as open, I; semiopen, II; and closed, III;
- Color is classified as dark, D; medium, M; and light, L; A light-colored, closed-weave material would then be classified III_L.

Once the category has been established, an index letter (A to J) may be read and used to determine the IAC from Table 7-5. For any category, several index letters may be chosen, and judgment based on the color and weave is required in making a final selection.

Transmission and Absorption of Fenestration with Internal Shading, Detailed

EXCLUDED

Example 7-8

If an opaque white roller shade were added to the window in Example 7-6, what would be the effect on the solar heat gain?

SOLUTION

From Table 7-4 the interior solar attenuation coefficient for an opaque white roller shade installed on a residential double-pane Window is 0.41. From Eq. (7-41). the resulting solar heat gain may be calculated:

$$\dot{q}_{SHG} = [0.063x1.41x155.4 + 0.063x2.63x60.6] +$$

$$[0.59x6.43x155.4 + 0.57x17.81x60.6]0.41 = 493.9 \text{ Btu/hr}$$

This is 42 percent of the solar beat gain without the shade; the heat transfer through the frame is not affected by the shade, so the reduction in the total heat gain is slightly less than might be inferred from the IAC.

Table 7-4 Interior Solar Attenuation Coefficients (IAC) for Single or Double Glazings Shaded by Interior Venetion Blinds or Roller Shades

Glazing System[a]	Nominal Thickness[b] Each Pane, in.	Glazing Solar Transmittance		Glazing SHGC[b]	IAC				
					Venetian Blinds		Roller Shades		
		Outer Pane	Single or Inner Pane		Medium	Light	Opaque Dark	Opaque White	Translucent Light
Single Glazing System									
Clear, residential	1/8[c]		0.87 to 0.80	0.86	0.75[d]	0.68[d]	0.82	0.40	0.45
Clear, commercial	1/4 to 1/2		0.80 to 0.71	0.82					
Clear, pattern	1/8 to 1/2		0.87 to 0.79						
Tinted	3/16, 7/32		0.74, 0.71						
Above glazings, automated blinds[e]				0.86	0.64	0.59			
Above glazings, tightly closed vertical blinds				0.85	0.30	0.26			
Heat absorbing[f]	1/4		0.46	0.59	0.84	0.78	0.66	0.44	0.47
Reflective coated glass				0.26 to 0.52	0.83	0.75			
Double Glazing Systems[g]									
Clear double, residential	1/8	0.87	0.87	0.76	0.71[d]	0.66[d]	0.81	0.40	0.46
Clear double, commercial	1/4	0.80	0.80	0.70					
Heat absorbing double[f]	1/4	0.46	0.80	0.47	0.72	0.66	0.74	0.41	0.55
Reflective double				0.17 to 0.35	0.90	0.86			
Other Glazings (Approximate)					0.83	0.77	0.74	0.45	0.52
Range of variation[h]					0.15	0.17	0.16	0.21	0.21

[a] Systems listed in the same table block have the same IAC

[b] Values or ranges given for dentification or appropriate IAC value; where paired, solar transmittances and thicknesses correspond. SHGC is for unshaded glazing at normal incidence

[c] Typical thicknesses for residential glass.

[d] From measurements by Van Dyke and Konen (1980) for 45 deg open Venetian blinds, 35 deg solar incidence, and 35 deg profile angle

[e] Use these values only when operation is automated for exclusion of beam solar (as opposed to daylight maximization). Also applies to tightly closed horizontal blinds.

[f] Refers to gray-, bronze-, and green-tinted heat-absorbing glass (on exterior pane in double glazing).

[g] Applies either to factory-fabricated insulating glazing units or to prime windows plus storm windows.

[h] The listed approximate IAC value may be higher or lower by this amount, due to glazing/shading interactions and variations in the shading properties (e.g. manufacturing tolerances).

Source: *ASHRAE Handbook, Fundamentals Volume 2001.*

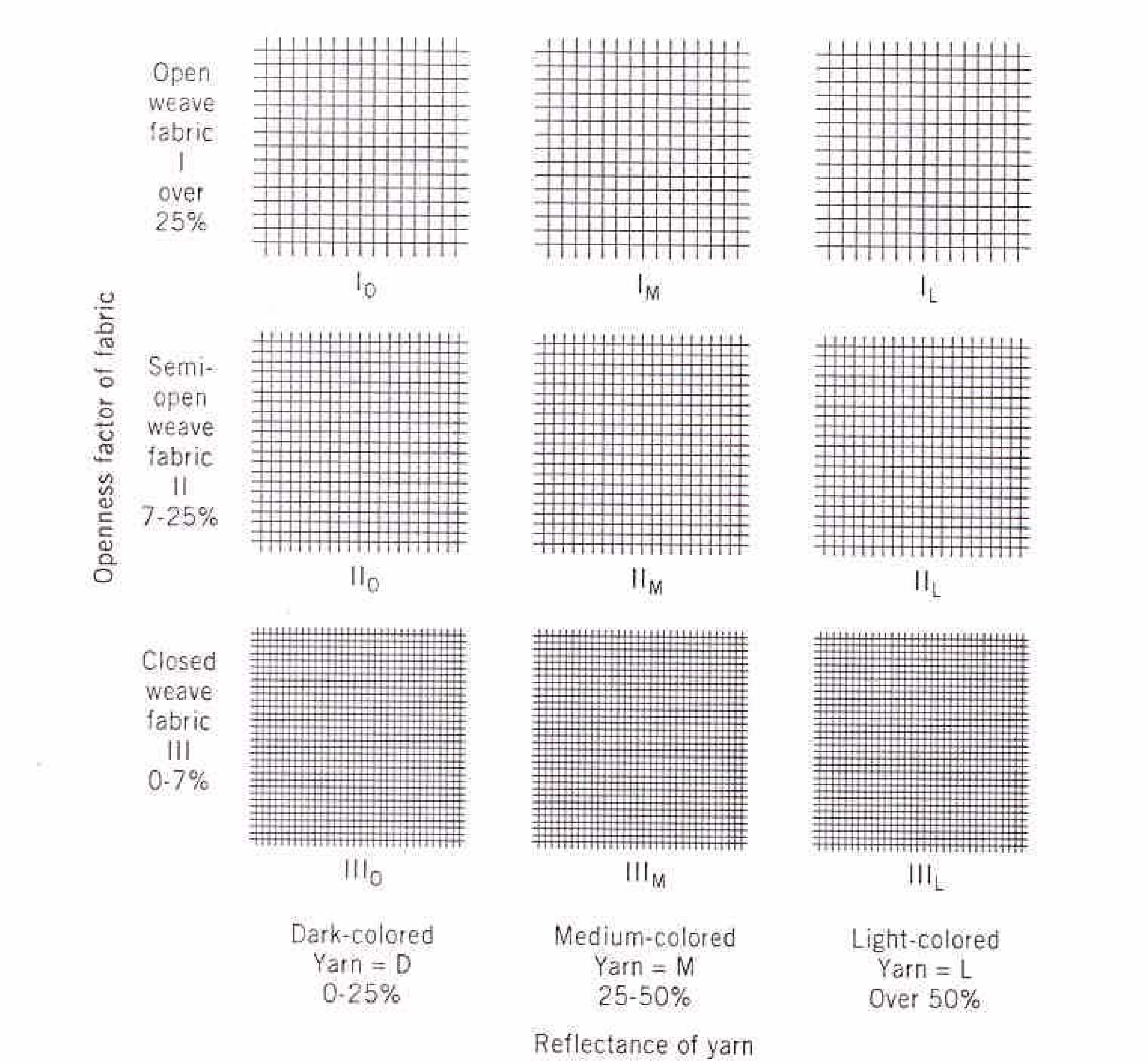

Figure 7-12. Characterization of Drapery fabrics.

Note: Classes may be approximated by eye. With closed fabrics, no objects are visible through the material but large light or dark areas may show. Semi-open fabrics do not permit details to be seen, and large objects are clearly defined. Open fabrics allow details to be seen, and the general view is relatively clear with no confusion of vision. The yarn color or shade of light or dark may be observed to determine whether the fabric is light, medium, or dark.

Table 7-5 Shading Coefficients for Single and Insulating Glass with Draperies

Glazing	Glass Trans- mission	Glazing SHGC (no drapes)	A	B	C	D	E	F	G	H	I	J
Single Glass												
1/8 in clear	0.86	0.87	0.87	0.82	0.74	0.69	0.64	0.59	0.53	0.48	0.42	0.37
1/4 in clear	0.80	0.83	0.84	0.79	0.74	0.68	0.63	0.58	0.53	0.47	0.42	0.37
Reflective coated		0.52	0.95	0.90	0.85	0.82	0.77	0.72	0.68	0.63	0.60	0.55
		0.35	0.90	0.88	0.85	0.83	0.80	0.75	0.73	0.70	0.68	0.65
Insulating Glass, 1/4 in. airspace (1/8 in. out and 1/8 in. in)	0.76	0.77	0.84	0.80	0.73	0.71	0.64	0.60	0.54	0.51	0.43	0.40
Insulating Glass, 1/2 in. air space												
Clear out and clear in	0.64	0.72	0.80	0.75	0.70	0.67	0.63	0.58	0.54	0.51	0.45	0.42
Heat-abs. out and clear in	0.37	0.48	0.89	0.85	0.82	0.78	0.75	0.71	0.67	0.64	0.60	0.58
Reflective coated												
		0.35	0.95	0.93	0.93	0.90	0.85	0.80	0.78	0.73	0.70	0.70
		0.26	0.97	0.93	0.90	0.90	0.87	0.87	0.83	0.83	0.80	0.80
		0.17	0.95	0.95	0.90	0.90	0.85	0.85	0.80	0.80	0.75	0.75

Table 7-6 Properties of Representative Indoor Shading Devices Shown in Table 7-4

Indoor Shade	Solar-Optical Properties (Normal Incidence)		
	Transmittance	Reflectance	Absorptance
Venetian blinds[a] (ratio of slat width to slat spacing 1.2, slat angle 45 deg)			
Light colored slat	0.05	0.55	0.40
Medium colored slat	0.05	0.35	0.60
Vertical blinds			
White louvers	0.00	0.77	0.23
Roller shades			
Light shade (translucent)	0.25	0.60	0.15
White shade (opaque)	0.00	0.65	0.35
Dark colored shade (opaque)	0.00	0.20	0.80

[a] Values in this table and in Table 7-4 are based on horizontal Venetian blinds. However, tests show that these values can be used for vertical blinds with good accuracy.

Source: *ASHRAE Handbook, Fundamentals Volume*. 2001 American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc. 2001.

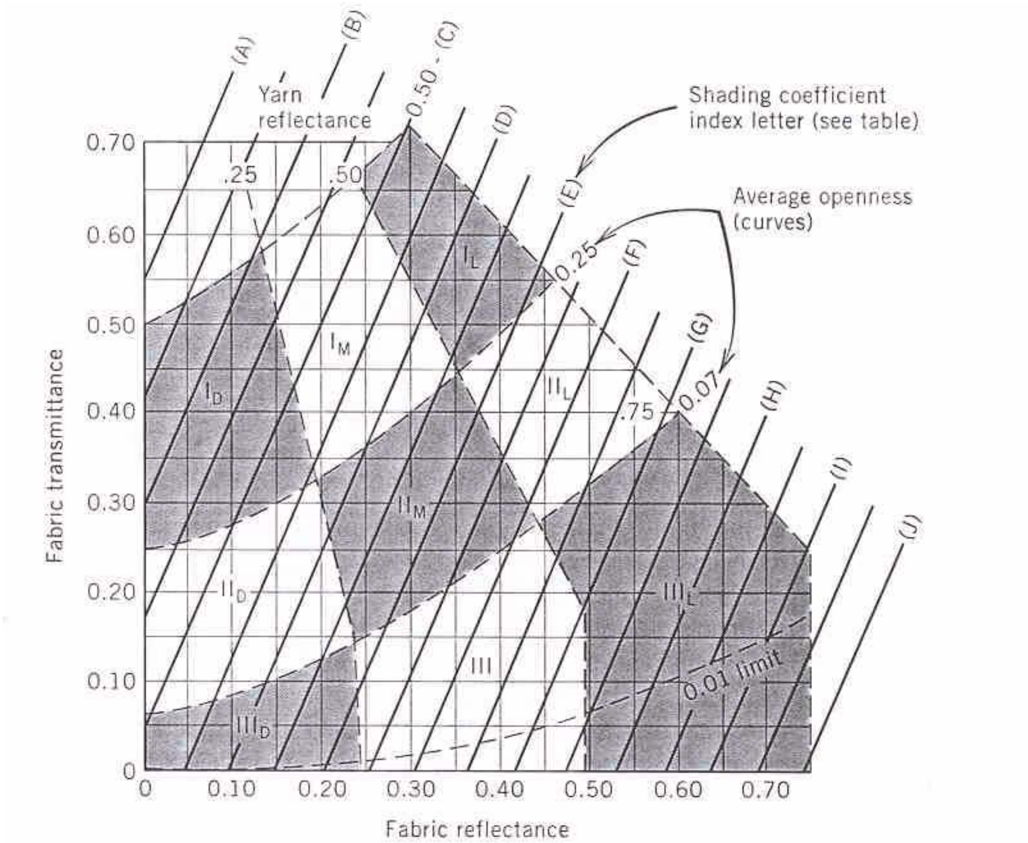

Figure 7-13. Indoor shading properties of drapery fabrics.