Fusion: 2 or more light nuclei fuse to form a heavier nucleus

solar: $4\,{}^{1}_{1}H \rightarrow {}^{4}_{2}He + \underbrace{2\,{}^{0}_{+1}e}_{\text{positrons}}$ } mass decrease of 0.0265 amu corresponding to 24.7 MeV

- fusion reactions called "thermonuclear" because of the very high temperatures required to trigger and sustain reactions.

Fission: heavy nucleus is split into two or more lighter nuclei

- fission can be triggered by a neutron (doesn't experience a repulsive force)

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + \underbrace{2\,{}^{1}_{0}n}_{\text{2 neutrons released}}$$

196 MeV released

- numerous fission reactions releasing distribution of energies.
  - ① on average, U-235 fission yields 193 MeV.
  - ② 1 g fissionable material generates approximately 1 MW-day of energy.

$${}^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{137}_{56}Ba + {}^{97}_{36}Kr + 2\,{}^{1}_{0}n$$

mass balance:

235.0439 amu + 1.00867 amu → 136.9061 amu + 96.9212 amu + 2(1.00867 amu)

236.0526 amu → 235.8446 amu

$\Delta m = -0.2080$ amu
$\Delta E = -193.6$ MeV } (-) indicates exothermic reaction

- "prompt" energy released at fission event
- "delayed" energy released during (1) radioactive decay of fission fragments into fission products & (2) non-fission capture of excess neutrons.

major concern for nuclear reactor control

Fission - heavy nucleus is split into two or more lighter nuclei

- fission can be triggered by a neutron, which does not experience a repulsive force as it approaches the nucleus

examples:

Excitation > Ecritical

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U^{*} \longrightarrow ^{140}_{55}Cs + ^{93}_{37}Rb + 3(^{1}_{0}n) + \gamma_d$$

"prompt energy" released at fission event

"delayed energy" released during

(1) radioactive decay of fission fragments (Cs-140 & Rb-93)

(2) non-fission capture of excess neutrons

* delayed energy is a major concern in reactor control

prompt E: mass of reactants = 235.043924 amu + 1.008665 amu = 236.052589 amu

mass of products = 92.91699 amu + 139.90910 amu + 3·1.008665 amu = 235.85208 amu

$\Delta m = +0.200509$ amu

$\Delta E = (931.5 \frac{MeV}{amu})\Delta m = +186.8 MeV$

There is also roughly 10 MeV of γ_d released.

delayed E: $^{93}_{37}Rb \xrightarrow{\beta^-} ^{93}_{38}Sr \xrightarrow{\beta^-} ^{93}_{39}Y \xrightarrow{\beta^-} ^{93}_{40}Zr \xrightarrow{\beta^-} ^{93}_{41}Nb$

$^{140}_{55}Cs \xrightarrow{\beta^-} ^{140}_{56}Ba \xrightarrow{\beta^-} ^{140}_{57}La \xrightarrow{\beta^-} ^{140}_{58}Ce$

$$\Delta E_{Rb decay} = \left(\frac{931.5 MeV}{amu}\right)\left\{(m_{Nb-93} + 4m_{\beta^-}) - (m_{Rb-93})\right\} = +7.84 MeV$$

$$\Delta E_{Cs decay} = \left(\frac{931.5 MeV}{amu}\right)\left\{(m_{Ce-140} + 3m_{\beta^-}) - (m_{Cs-140})\right\} = +1.89 MeV$$

+9.73 MeV

- other U-235 fission reactions,

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U^{*} \longrightarrow ^{140}_{54}Xe + ^{94}_{38}Sr + 2^{1}_{0}n \qquad \Delta E = 196 MeV \text{ released}$$

$$^{235}_{92}U + ^{1}_{0}n \longrightarrow ^{236}_{92}U^{*} \longrightarrow ^{137}_{56}Ba + ^{97}_{36}Kr + 2^{1}_{0}n \qquad \Delta E = 193.6 MeV \text{ released}$$

- numerous fission reactions releasing a distribution of energies.

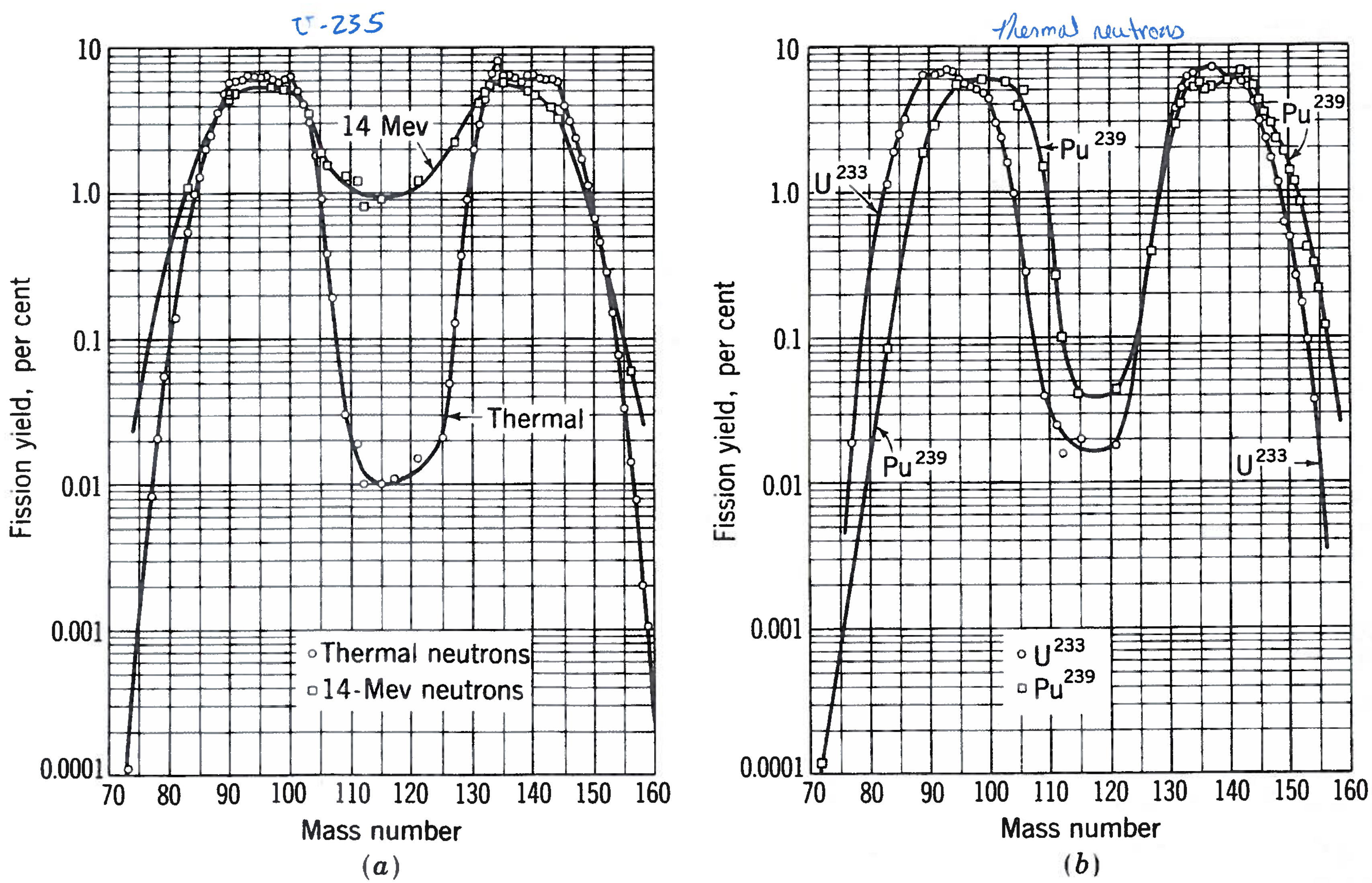

Figure 9-4 Fission product yield data for (*a*) U^{235} by thermal and 14 MeV neutrons and (*b*) U^{233} and PU^{239} by thermal neutrons [84].

Average energy distribution for energy released per fission via a <u>thermal</u> neutron absorbed by Uranium-235.

prompt energy:	167 MeV	kinetic energy of fission fragments
	5 MeV	kinetic energy of fission neutrons
	5 MeV	$\gamma_{fission}$ energy
	10 MeV	$\gamma_{capture}$ energy
	187 MeV	
delayed energy:	7 MeV	β^- energy from fission fragment decays
	6 MeV	γ_{decay} energy
	10 MeV	kinetic energy of neutrinos
	23 MeV	

- On average, 200 MeV released/usable from fission of uranium-235.
  10 MeV of neutrinos not counted because this energy can not be used.

200 MeV/fission U-235:

$$\frac{(200\ \text{MeV/nuclei U-235})(6.0225\cdot 10^{23}\ \text{nuclei/gmol U-235})}{(235.0439\ \text{g/gmol U-235})} = 5.13\cdot 10^{23}\ \text{MeV/g U-235}$$

$$= 2.276\cdot 10^{4}\ \text{kWh/g U-235}$$

$$= 8.190\cdot 10^{10}\ \text{J/g U-235}$$

$$= 0.948\ \text{MW-day/g U-235}$$

$\therefore$ 1 g fissionable U-235 ~ 1 MW day

fuel burnup:

fuel ≡ all uranium, plutonium & thorium isotopes ; some isotopes are nonfissionable

fuel material ≡ fuel + alloying & chemical compounds & mixtures

- all fissionable isotopes can not be fissioned because of accumulation of fission products, such as Xe & Kr, that absorb neutrons and eventually drop neutron flux below critical level.
- fuel burnup rates vary from 1000 to 100,000 $\frac{\text{MW-day}}{\text{ton}}$ depending on fuel

Fission Timeline

1938 : Fission discovered in Germany by Otto Hahn & Fritz Strassmann

Jan. 16, 1939 : Lise Meitener & Otto Robert Frisch published a theoretical interpretation of Hahn & Strassmon experiments in Nature (10 days after Hahn & Strassman publication)

Apr. 17, 1939 : Frederich Joliet, Hans von Halban, & Lew Kowarski publish paper dealing with possibility of nuclear chain reaction

Aug. 2, 1939 : Albert Einstein wrote a letter to Pres. F.D. Roosevelt drawing attention to the possibility of an atomic bomb

1940 : Edwin McMillon & Glenn Seaborg discover Plutonium

Dec. 1942 : First nuclear reactor went critical
- beneath the stands of the University of Chicago stadium (Chicago Pile 1)
- Fermi Chain Reaction
- core was 9 m wide, 9.5 m long, 6 m high
  (29.5 ft w) (31 ft) (19.7 ft)

  52 tons of natural uranium & 1350 tons of graphite

  Cadmium rods used for control

  0.5 W power for a few minutes

1943 : Town of Los Alamos constructed for atomic research & weapons construction

July 16, 1945 : 1st Nuclear Explosion, Plutonium bomb, Trinity site in New Mexico

1952 : Fusion Weapon, Hydrogen bomb (much less radioactive debris)

June 1, 1954 : 1st Nuclear Power Plant
- Obninsk Nuclear Power Station, near Moscow (APS-1 Obninsk)
- rated power 5 MW_e, 30 MW_{th}
- graphite moderated, water cooled
- shut down April 29, 2002

Jan. 21, 1954 : USS Nautilus (SSN-571) launched
- pressurized water reactor (Westinghouse)
- 10 MW_m
- decommissioned March 3, 1980

Neutron Energies

$E_n \equiv$ kinetic energy $\equiv \frac{1}{2g_c} m_n V_n^2$

$m_n = 1.00866$ amu

$0.075 \leq E_n \leq 17$ MeV (Fig. 9-9, El-Wakil)

- when travelling through matter, neutrons collide with nuclei and are decelerated (mainly by lighter nuclei) — scattering

- ~~3~~ 4 categories of neutron energy

  fast $> 10^5$ ~~MeV~~ eV ← fast reactors utilize

  intermediate

  slow < 1 eV ← thermal ← thermal reactors utilize

  cold $<$ 0.025 eV

- newly released fission neutrons carry about 2% of a reactor fission energy
  - prompt, released at time of fission, 10^{-14} seconds
  - delayed, released during radioactive decay of fission fragments (0.645% of fission neutrons for U-235, less for Pu-239 & U-233)

Thermal Neutrons

- Fission neutrons scattered (slowed) by materials in reactor core
  - Moderators (Scattering Media)
    - small nuclei with high neutron scattering & low neutron-absorption probability

      ^1_1H, ^2_1H in H_2O, D_2O, C (graphite), Be, BeO

  - lowest energy a neutron can reach is that which puts it in thermal equilibrium with surrounding environment
    - neutron becomes "thermalized" → thermal neutrons

$k \equiv 1.3805 \cdot 10^{-23}$ J/K $\equiv 8.617 \cdot 10^{-11} \frac{MeV}{K}$

$T \equiv [K]$

$$E_n = \frac{1}{2g_c} m_n V_n^2 = \underbrace{kT}_{\text{independent of mass}}$$

k: Boltzmann's constant $\equiv \frac{\text{universal gas constant}}{\text{Avagadro's number}}$

- most probable energy correlates to most probable velocity (Fig 9-10, El-Wakil) neutrons (Sec. 9-10).

Newly born fission neutrons carry, on an average, about 2 percent of a reactor

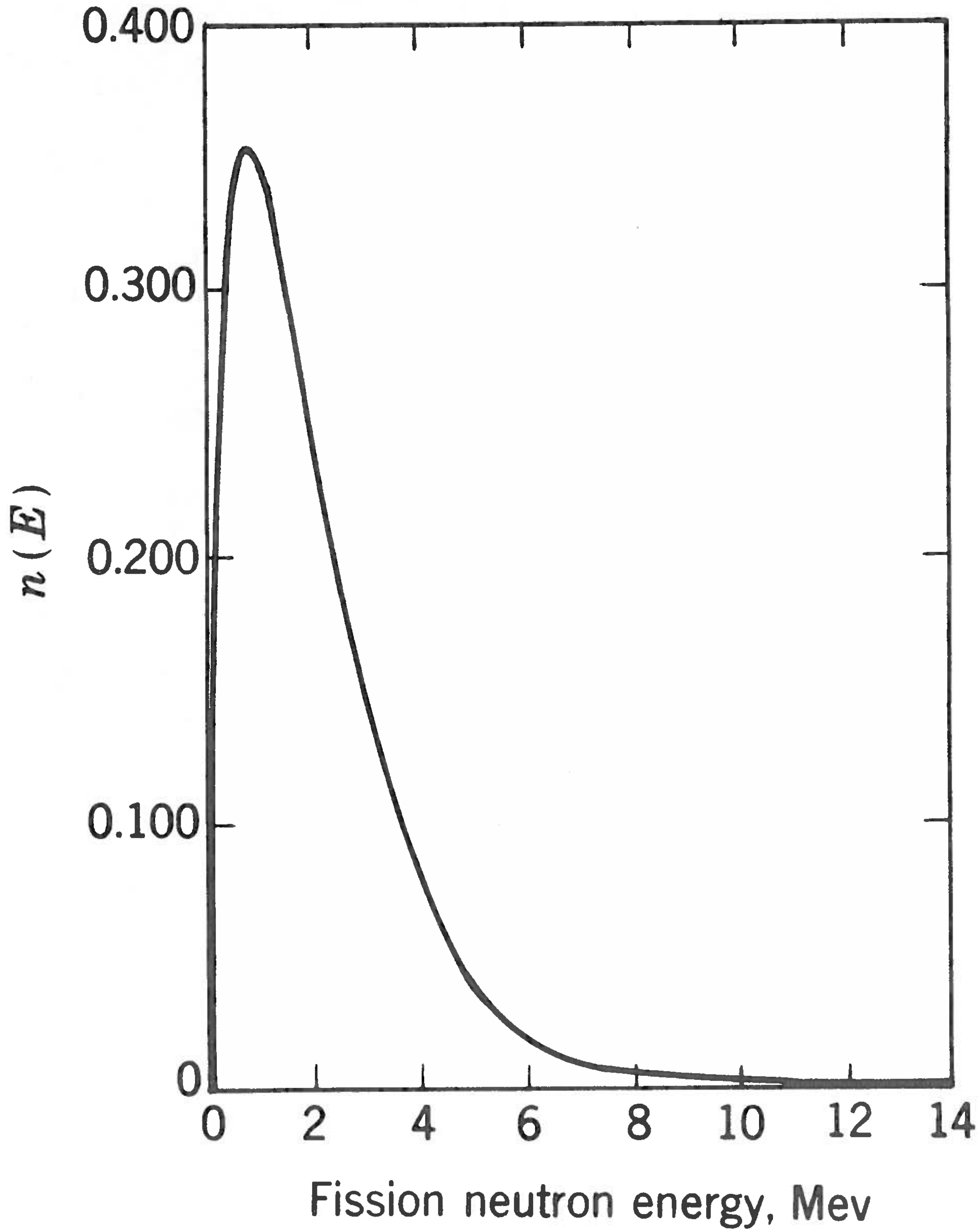

Figure 9-9 Energy spectrum of fission neutrons.

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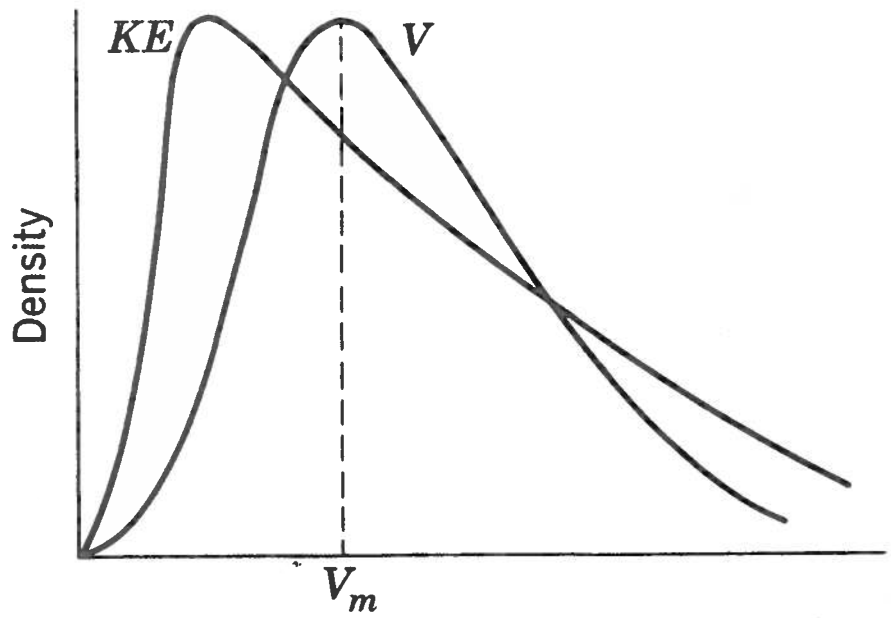
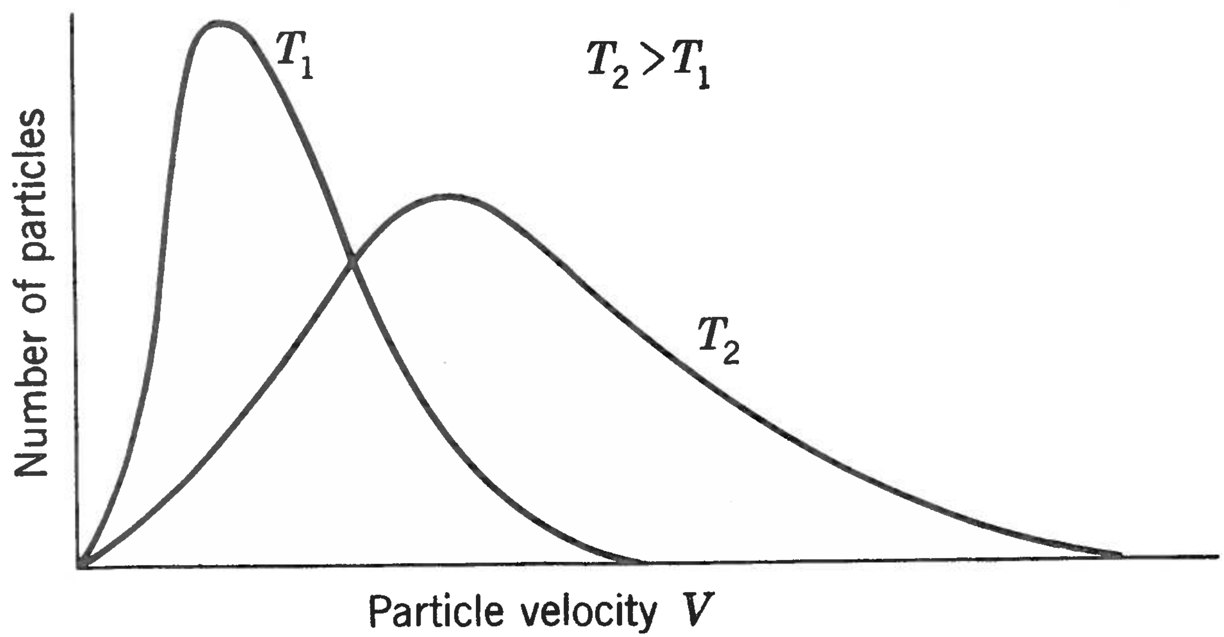


(a)



▫ Neutron Scattering Reactions

Elastic — hard sphere interaction
— absorption & reemission

Inelastic

▫ Absorption Reactions

Radiative Capture $^{238}_{92}U + ^{1}_{0}n \longrightarrow ^{239}_{92}U^{*} \longrightarrow ^{239}_{92}U + \gamma_c$

Particle Capture

⊛ Fission

▫ Fissile Materials → unstable w/ Absorption of ~~an~~ a neutron of any energy; thermalized neutrons

$^{233}U, ^{235}U, ^{239}Pu$

Fissionable Materials → fission possible if energy of absorbed neutron is sufficiently large

---

▫ Nuclear Cross Sections & Reaction Probability

Atomic Density

$$n \equiv \left[\frac{\text{particles}}{cm^3}\right] = \frac{\rho N_A}{M} \equiv \frac{\left[\frac{g}{cm^3}\right]\left[\frac{\text{particles}}{gmol}\right]}{\left[\frac{g}{gmol}\right]}$$

Aluminum, $\rho = 2.7\ g/cm^3 \longrightarrow n_{Al} = 6.024 \cdot 10^{22}$ atoms/cm^3

▫ Reaction Probability

- The probability of a neutron interacting with a nucleus is dependent on:
  - type of reaction
  - nucleus & nucleus energy
  - neutron energy

- Probability of Reaction expressed in terms of the microscopic cross section, σ, of nucleus
  - $\sigma = f$(neutron energy)
  - probability is essentially an effective area of the nucleus
  - 1 barn $= 10^{-24}\ cm^2 = 10^{-28}\ m^2$ ← $r_{nuclei} \sim \frac{1}{10{,}000} r_{atom}$

    $A_{nuclei} = \pi r_{nuclei}^2 \approx 10^{-24}\ cm^2$

- $\Sigma \equiv$ macroscopic cross section $\equiv N\sigma$

  ↑ # of nuclei

  ~~$\frac{1}{\Sigma}$~~ $\frac{1}{\Sigma} \equiv$ mean free path

Nuclear Cross Section

Atomic Density

$$n \equiv \left[\frac{\text{particles}}{cm^3}\right] = \frac{\rho N_A}{M} \doteq \frac{\left[\frac{g}{cm^3}\right]\left[\frac{\text{particles}}{g\ mol}\right]}{\left[g/gmol\right]}$$

Aluminum

$\rho = 2.7\ g/cm^3 \longrightarrow 6.024 \cdot 10^{22}\ \frac{\text{atoms}}{cm^3}$

- nuclei have radii $\sim \frac{1}{10{,}000}$ of atoms radii
- neutrons have small target
- probability of collision (moderation) $\sim \frac{\text{cross-sectional area of nuclei}}{\text{total cross-sectional area}}$
  - average cross-sectional area of nuclei is $10^{-24}\,cm^2 \approx 1\ \text{barn} = 10^{-28}\,m^2$
- effective cross-sectional area, σ, is larger than actual nuclei cross-sectional area
- different effective areas based on type of reaction
  - absorption, σ_a
  - scattering, σ_s
  - fission, σ_f
  - etc.

$$\sigma = \sigma_0\left(\frac{T_0}{T}\right)^{1/2}$$

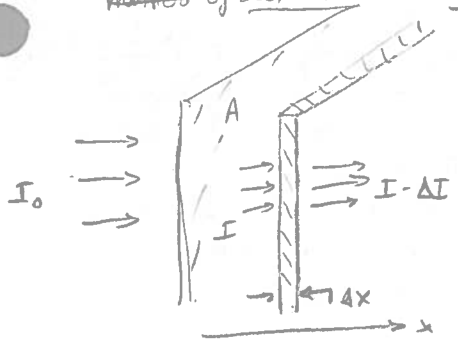
$\sigma_0 \equiv \sigma(T = 68°F)$ $(v_n = 2200\ m/s)$

$T_0 = 68°F$

U-235: $\sigma_f = 583$ barns @ 68°F

$\sigma_f = 432$ barns at 500°F

~~method of~~ determination of σ



$I_0 \equiv \left[\frac{\text{neutrons}}{s}\right]$

$A \equiv [cm^2]$

$n \equiv \left[\frac{\text{nuclei}}{cm^3}\right] \longrightarrow$ nuclear density

$N \equiv [\#\text{ of nuclei}]$

$\sigma \equiv$ microscopic cross section

$\Sigma \equiv$ macroscopic cross section $= N\sigma$

$1/\Sigma \equiv$ mean free path

- # of nuclei in volume $= n \cdot A\Delta x$
- as neutron beam passes through volume, some neutrons are absorbed or scattered.
- fraction removed equal to the ratio of effective area to the total area

$$I_2 = I_1\left\{1 - \underbrace{\frac{\sigma(nA\Delta x)}{A}}\right\}$$

fraction of neutrons removed from incident beam

$$\Delta I = -\sigma n \Delta x I_1$$

- in the limit, $\frac{dI}{I} = -\sigma n\,dx \rightarrow$ integrating

$$-\int_{I_0}^{I}\frac{dI}{I} = \sigma n\int_0^x dx \longrightarrow \boxed{I = I_0 e^{-\sigma n x}}$$

$V_n = 2200$ m/s , T = 68°F

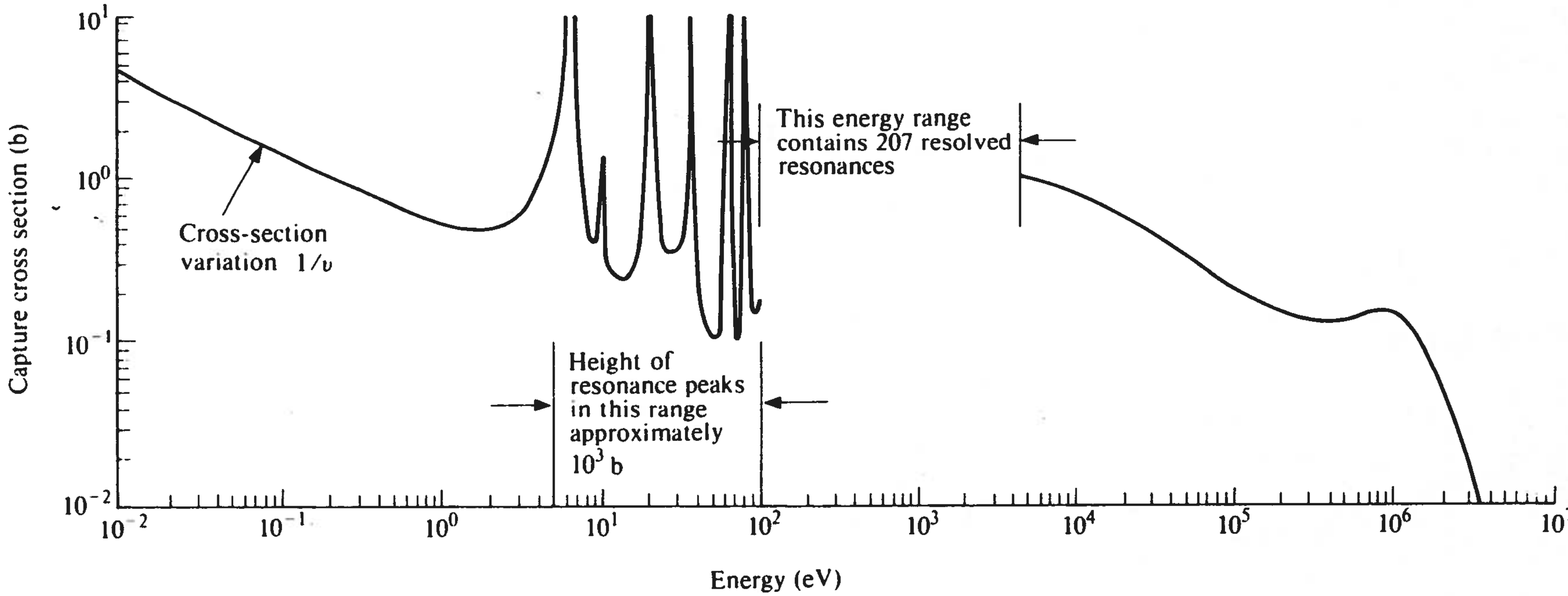


FIGURE 3.2
The microscopic neutron-absorption cross section for U-238. (*From Steam: Its Generation and Use, 1972.*)

as the moderator. The neutrons at this reduced energy are designated thermal neutrons and are the basis for BWR and PWR reactors. Figure 4.4 shows the basic reason for the choice of neutron energy. It can be seen that the probability of having the ^{235}U nucleus fission increases very rapidly as the neutron energy is reduced. Thus, it is advantageous to use very low-energy neutrons; the arrow indicates the lowest energy neutrons that can conveniently be obtained; namely, thermal neutrons at 0.025 eV. Figure 4.4 also shows that the probability of having ^{238}U fission is negligible unless neutrons above 1 MeV in energy are used. Of the two isotopes of uranium that are present to any extent in nature, ^{235}U (0.72% abundant) is of direct use in thermal reactors, whereas ^{238}U (99.27% abundant) plays an indirect role that is discussed later. A thermal reactor using water as a coolant and moderator must use enriched fuel in which the ^{235}U, instead of being 0.7% of the total uranium present, has been increased to about 3%.

The neutrons that come directly from the fissioning of ^{235}U have an average energy of 2 MeV. To reduce their energy to 0.025 eV, a moderator is positioned such that it is likely that a neutron will become thermalized before it is absorbed by a nonfissionable nucleus. In the BWR and PWR reactors, water, which is both the moderator and the coolant, surrounds the rods containing the uranium fuel. To slow neutrons down quickly, it is advantageous to have them collide with a nucleus that has about the same mass as the neutron. As any billiards player knows, when a billiard ball collides with another of the same mass head-

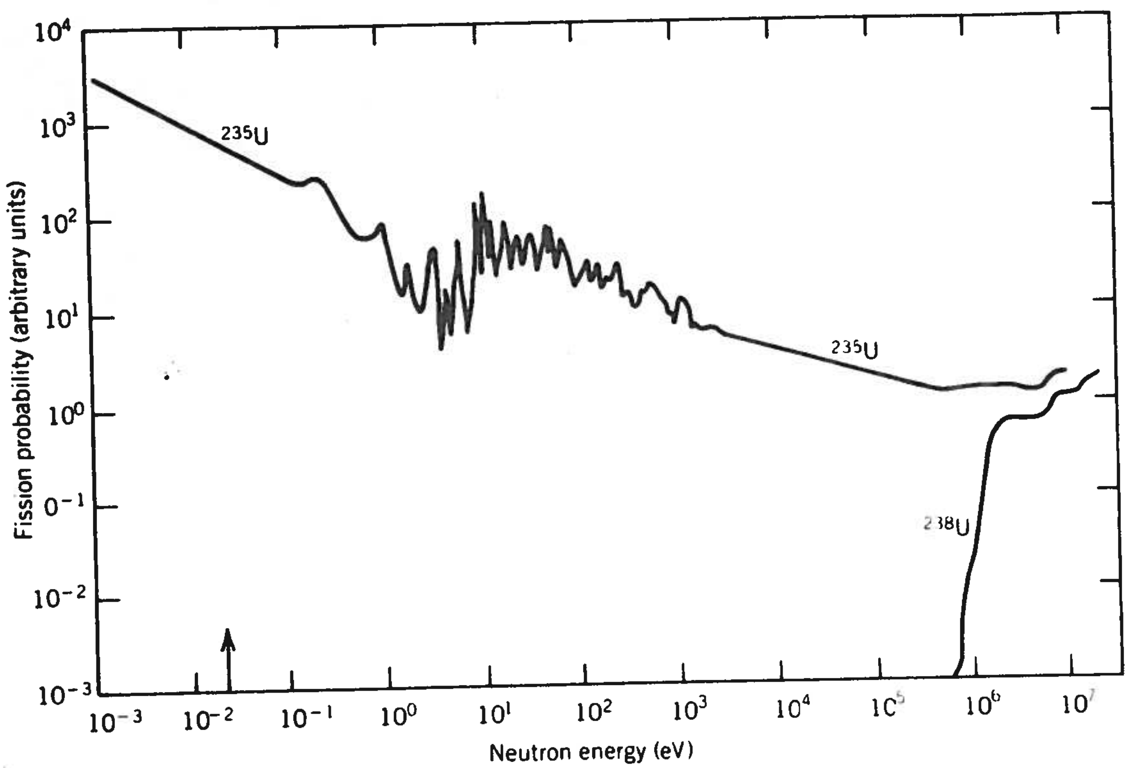


Figure 4.4 The fission probability for ^{235}U and ^{238}U as a function of neutron energy. The arrow at 0.025 eV indicates the energy of thermalized neutrons. For ^{238}U, the fission probability becomes appreciable only above 1 MeV neutron energy.

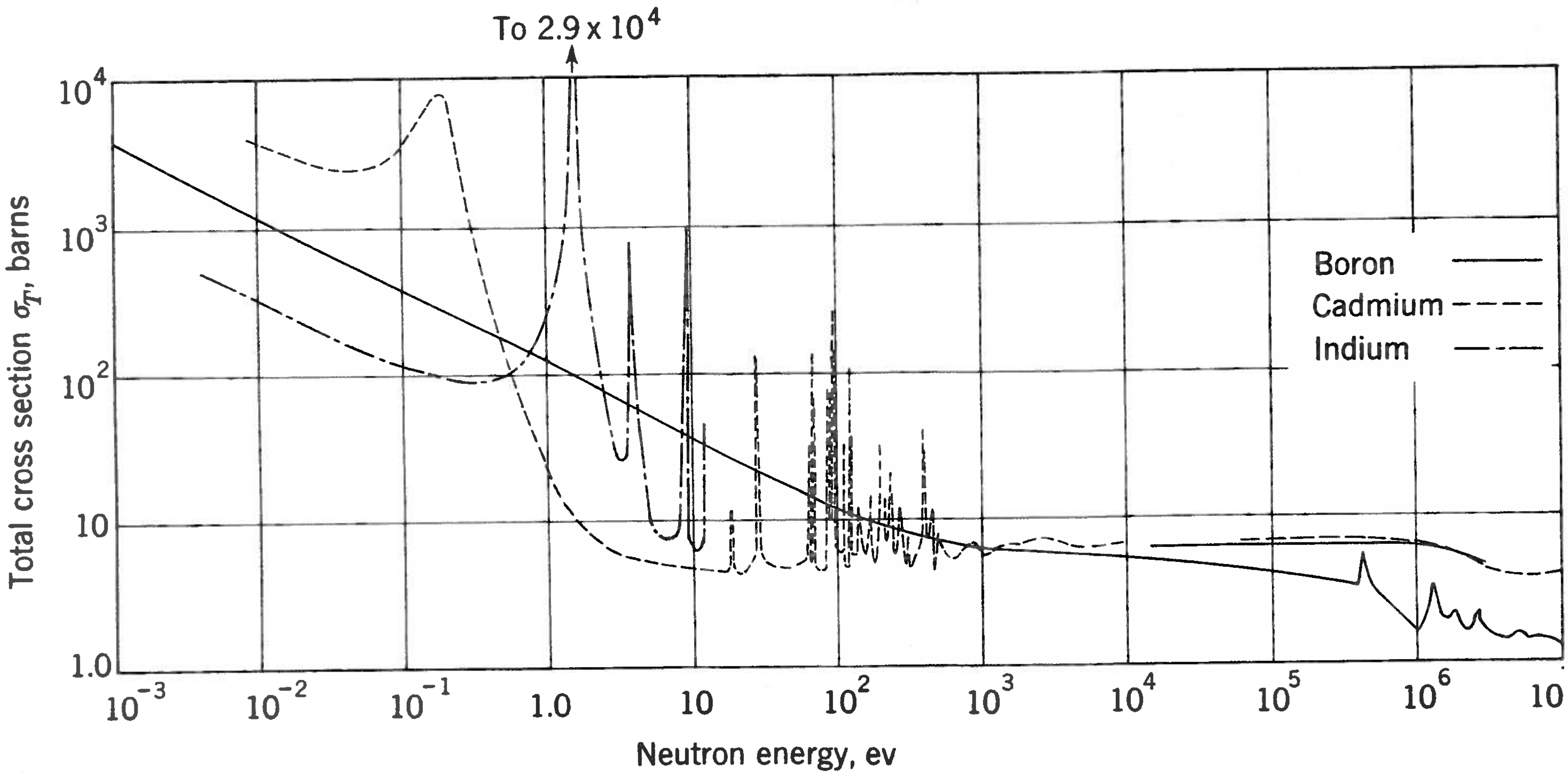


Figure 9-12 Neutron cross sections for cadmium, indium, and boron.

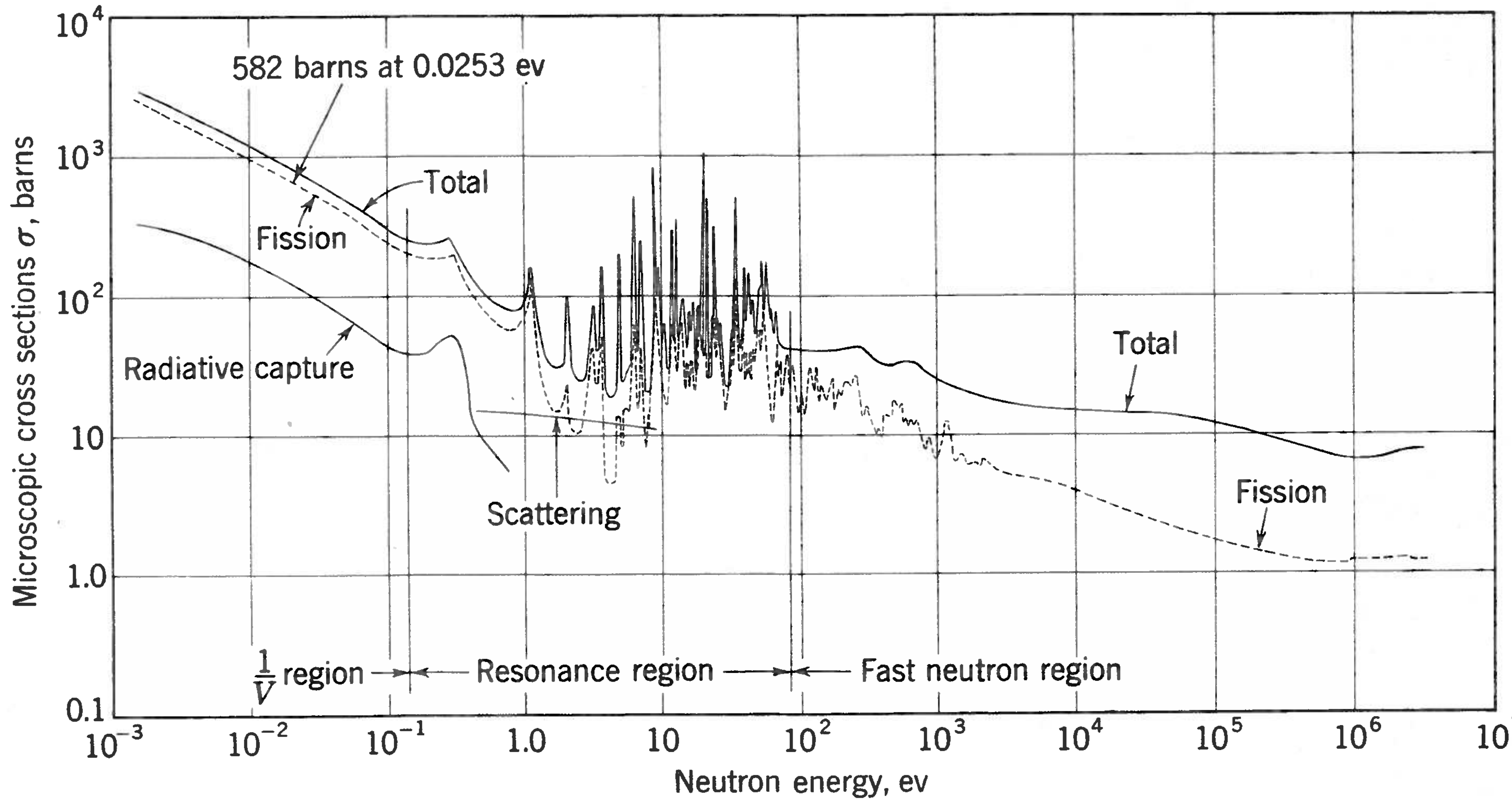


Figure 9-13 Neutron cross sections for U^{235}.

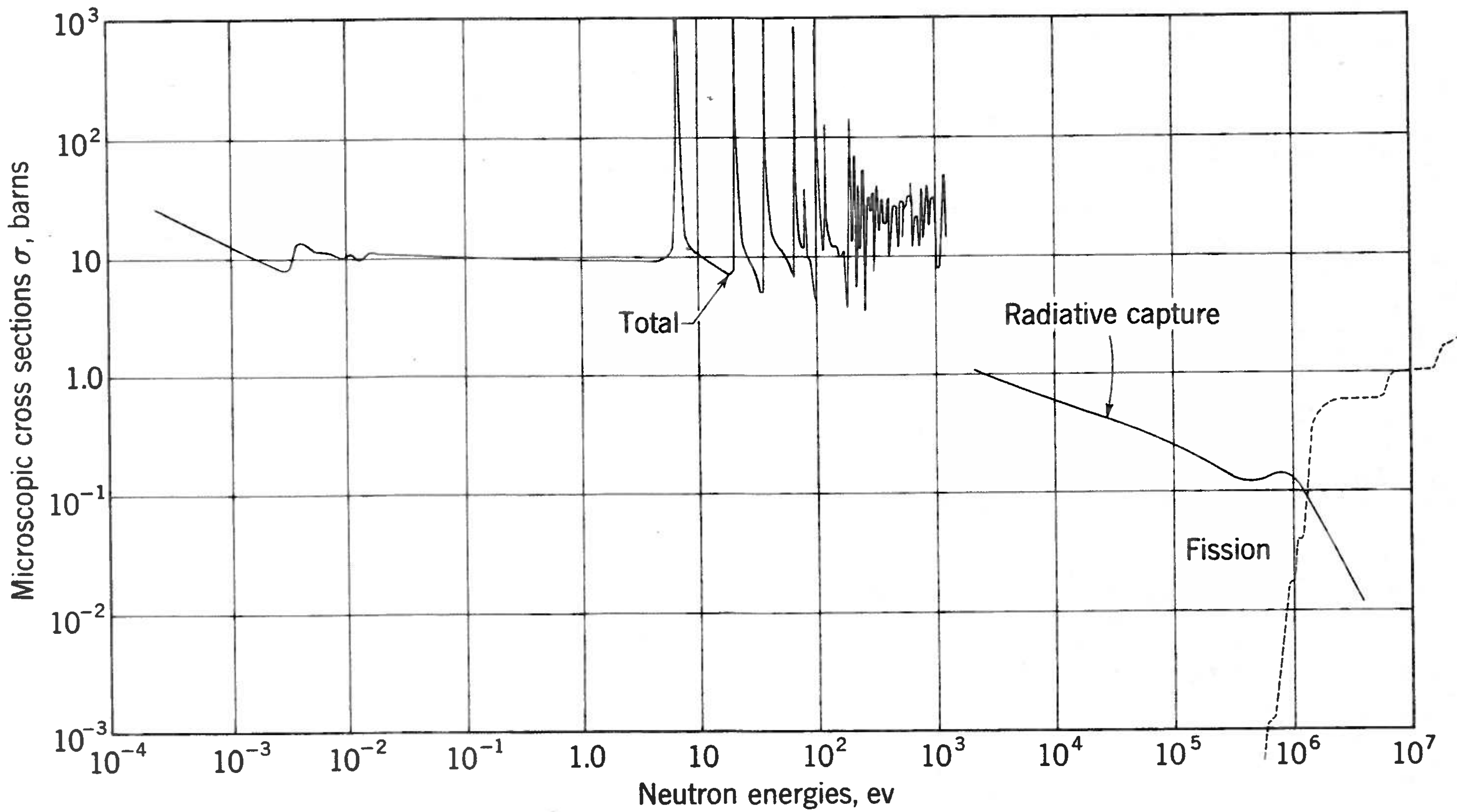


Figure 9-14 Neutron cross sections for U^{238}.

§ 3.6.2

Neutron Flux, Φ $\left[\frac{\text{neutrons}}{m^2 \cdot s}\right]$ $10^{15} - 10^{20}$

$\Phi = \underset{\sim}{n} \circ \underset{\sim}{v}$

$\uparrow$ neutron velocity $\left[\frac{m}{s}\right]$

$\uparrow$ neutron density $\left[\frac{\text{neutrons}}{m^3}\right]$

functions of spatial location in core and the kinetic energy of the neutrons

$\bar{\Phi} \equiv$ average neutron flux

$E_L \equiv$ lower energy bound

$E_H \equiv$ higher energy bound

$$\bar{\Phi} = \frac{\int_{E_L}^{E_H} \int_{\forall} \Phi(E, \underset{\sim}{v}) \, dE \, d\forall}{\int_{E_L}^{E_H} \int_{\forall} dE \, d\forall}$$

§ 3.6.3 Neutron Reaction Rates

neutron reaction rate $= \bar{\Phi} \cdot N \cdot \bar{\sigma}$

$\uparrow$ total # of ~~radioactive~~ nuclei reacting w/ neutrons

In El-Wakil, $N \equiv \frac{\text{\# of nuclei}}{\forall}$!

$\bar{\sigma}$ $\equiv$ average neutron cross-section for the given reaction $\left[\frac{\text{reactions} \cdot m^2}{\text{nucleus} \cdot \text{neutron}}\right]$

$\uparrow$ target area associated with a given neutron reaction

$\sigma_c \equiv$ radiative capture cross-section $\left[\frac{\text{captures} \cdot m^2}{\text{nucleus} \cdot \text{neutron}}\right]$

$\sigma_f \equiv$ fission cross-section $\left[\frac{\text{fissions} \cdot m^2}{\text{nucleus} \cdot \text{neutron}}\right]$

barn $\equiv 10^{-28} m^2 = \underline{10^{-24}}$ cm^2

$\sigma_s \equiv$ scattering

$\sigma_a \equiv$ absorption $= \sigma_c + \sigma_f$

$3.1 \cdot 10^{16} \frac{\text{fissions}}{\text{MWth} \cdot s}$

$\frac{\bar{\sigma} \cdot N}{\forall} \equiv$ macroscopic cross-section $\equiv \bar{\Sigma}$

Neutron reaction rate $= \bar{\Phi} N \bar{\sigma} = \bar{\Phi} \forall \bar{\Sigma}$

Culp 3.7

Determine the reactor fuel loading, in kg of U-235, in a 1200 MWe power reactor operating at a thermal efficiency of 33%, an average neutron flux of $6\times10^{17}\ \frac{\text{neutrons}}{m^2\cdot s}$ in the core, and an average fission cross section of 365 barns.

$$\text{Reactor Power} = \dot{W}_{th} = \frac{\dot{W}_e}{\eta_{th}} = \frac{1200\ MW_e}{0.33\ MW_e/MW_{th}} = 3636.36\ MW_{th}$$

$$\text{Fission Rate} = \left(3636.36\ MW_{th}\right)\left(\frac{\text{fission}}{200\ MeV}\right)\left(\frac{6.242\cdot10^{15}\ MeV/s}{kW_{th}}\right) = 1.135\cdot10^{20}\ \frac{\text{fissions}}{s}$$

$$= (\text{Neutron Flux})\underbrace{\left(\begin{array}{c}\text{atom density}\\ \text{of material}\end{array}\right)}_{\substack{\text{number density}\\ [\#/cm^3]}}(\text{fission cross-section}) = \bar{\Phi} N \bar{\sigma}_f$$

↑ avg. neutron flux

↑ fission reaction

$$= \left(6\cdot10^{17}\ \frac{\text{neutrons}}{m^2 s}\right)(365\ \text{barns})\left(10^{-28}\ m^2/\text{barn}\right) N_{fuel}$$

$$\left(1.135\cdot10^{20}\ \frac{\text{fissions}}{s}\right) = \left(6\cdot10^{17}\ \frac{\text{neutrons}}{m^2 s}\right)\underbrace{\left(365\cdot10^{-28}\ m^2\left[\frac{\text{fissions/neutron}}{\text{nuclei}\cdot U\text{-}235}\right]\right)}_{\text{probability of fission}} N_{fuel}$$

$$N_{fuel} = 5.1826\cdot10^{27}\ U\text{-}235\ \text{atoms}$$

$$m_{fuel} = \frac{N_{fuel}\cdot M_{U\text{-}235}}{N_A} = \frac{(5.1826\cdot10^{27}\ U\text{-}235\ \text{atoms})(235.0439\ kg/kgmol\ U\text{-}235)}{(6.0225\cdot10^{26}\ kg/kgmol)}$$

$$\boxed{m_{fuel} = 2022.7\ kg\ U\text{-}235}$$

If the reactor is fueled with 2.3% enriched UO_2, determine the fuel loading.

$$\text{total mass of uranium} = (2022.7\ kg\ ^{235}U)\left(\frac{1\ kg\ ^{238}U}{0.023\ kg\ ^{235}U}\right) = 87\,942\ kg\ ^{238}U$$

$$M_{^{238}UO_2} = 238 + 2(16) = 270\ \frac{kg\ ^{238}UO_2}{kgmol\ ^{238}UO_2}$$

$$\text{total mass of } UO_2 = \frac{87{,}942\ ^{238}U}{238\ kg\ ^{238}U / 270\ kg\ ^{238}UO_2}$$

$$\boxed{\text{fuel loading} = 99\,766\ kg\ UO_2}$$