• For an $\underbrace{\text{ideal gas}}$ & a $\underbrace{\text{polytropic process}}$: (both property relationships)

$P\forall = RT$ $\qquad$ $P\forall^n = \text{constant}$

$$\left.\begin{array}{l} \dfrac{P_1 \forall_1}{T_1} = \dfrac{P_2 \forall_2}{T_2} \longrightarrow \dfrac{\forall_1}{\forall_2} = \dfrac{P_2 T_1}{P_1 T_2} \\ P_1 \forall_1^n = P_2 \forall_2^n \longrightarrow \dfrac{\forall_1}{\forall_2} = \left(\dfrac{P_2}{P_1}\right)^{1/n} \end{array}\right\} \begin{array}{l} \dfrac{P_2}{P_1} \cdot \dfrac{T_1}{T_2} = \left(\dfrac{P_1}{P_2}\right)^{-1/n} \\ \dfrac{T_1}{T_2} = \left(\dfrac{P_1}{P_2}\right)\left(\dfrac{P_1}{P_2}\right)^{-\frac{1}{n}} = \left(\dfrac{P_1}{P_2}\right)^{\frac{n-1}{n}} \end{array}$$

• For an $\underbrace{\text{isentropic process}}$ & an ideal gas: (both property relationships)

$\Delta s = 0$

$$0 = s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right) \longleftarrow$$ derived from 1st law with reversible work (Pdv) & reversible heat (Tds)

• or with enthalpies:

$$0 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$\ln\left(T_2/T_1\right) = -\frac{R}{c_v} \ln\left(v_2/v_1\right) = \ln\left(v_1/v_2\right)^{R/c_v}$$

• recall $\left.\begin{array}{l} R = c_p - c_v \\ \& \quad k = c_p/c_v \end{array}\right\} \dfrac{R}{c_v} = k - 1$

<u>Ideal Gas: Isentropic Process</u>

$$\left.\begin{array}{l} \left.\dfrac{T_2}{T_1}\right|_{\Delta s=0} = \left(\dfrac{v_1}{v_2}\right)^{k-1} \\ \\ \left.\dfrac{P_2}{P_1}\right|_{\Delta s=0} = \left(\dfrac{v_1}{v_2}\right)^{k} \\ \\ \left.\dfrac{T_2}{T_1}\right|_{\Delta s=0} = \left(\dfrac{P_2}{P_1}\right)^{\frac{k-1}{k}} \end{array}\right\}$$ Table 1-3, El-Wakil

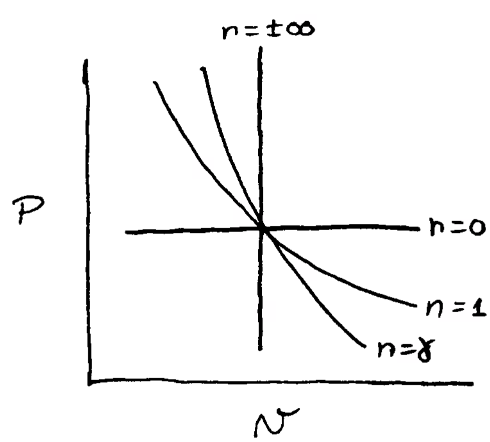
Polytropic Processes $P\forall^n = \text{constant}$

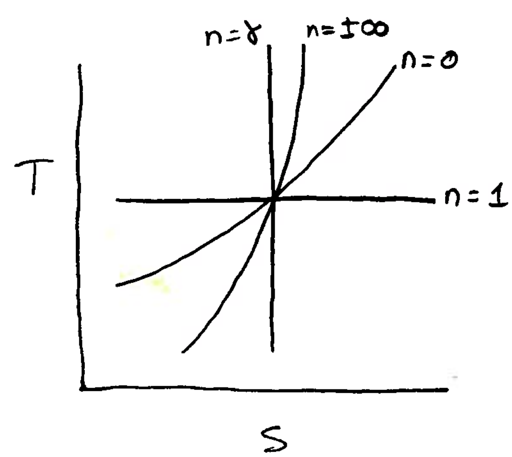
$P_1 \forall_1^n = P_2 \forall_2^n = \text{constant}$

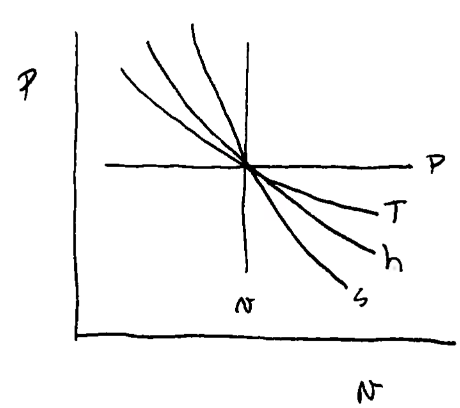
$\cancel{Pv = RT}$

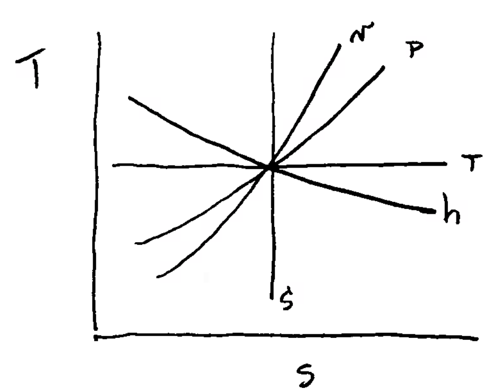
$\delta Q = c\, m\, dT$

Process	c	n
constant volume (isochoric)	c_v	∞
constant pressure (isobaric)	c_p	0
constant temperature (isothermal)	∞	1
adiabatic, reversible (isentropic)	0	$\gamma = \frac{c_p}{c_v}$
polytropic	$c_v\left(\frac{\gamma - n}{1-n}\right)$	0 to ∞



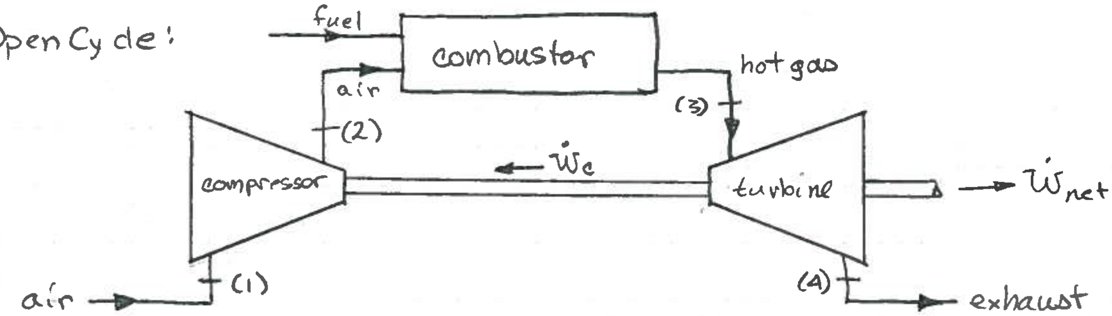






Ideal Brayton Cycle

Direct Open Cycle:

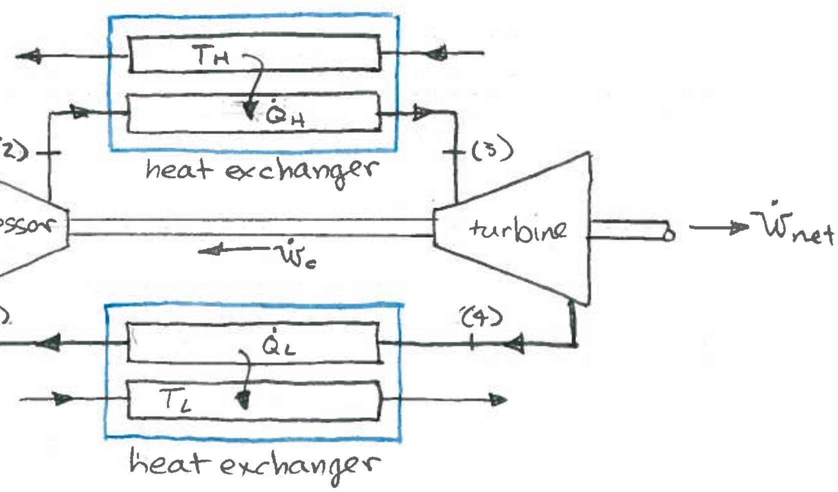


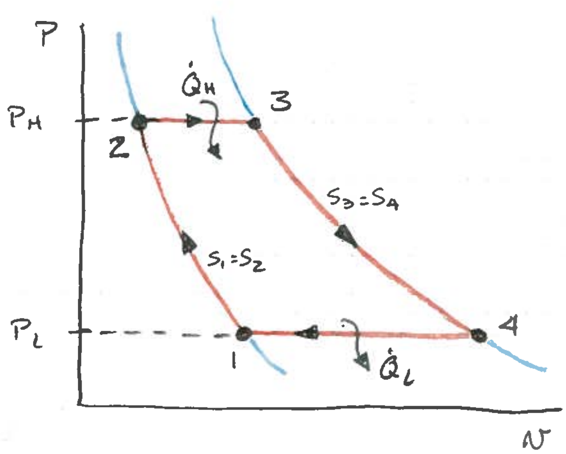
Air Standard Cycle (ASC):

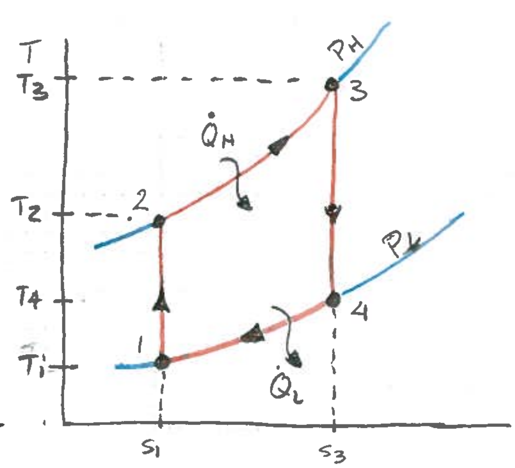
- fixed mass of air throughout the entire cycle; no inlet/exhaust process
- air is an ideal gas
- combustion process is replaced by heat transfer from an external source
- cycle is completed by heat transfer to surroundings
- all processes are internally reversible
- air has a constant specific heat

Indirect Closed Cycle:

1-2: isentropic compression
2-3: isobaric heat addition
3-4: isentropic expansion
4-1: isobaric heat rejection







First law for each process:

$$\dot{Q} - \dot{W} = H_{exit} - H_{inlet}$$
$$= \dot{m}(h_{exit} - h_{inlet})$$

uniform, steady flow neglecting changes in kinetic & potential energies

$$q - w = h_{exit} - h_{inlet}$$
$$= c_p(T_{exit} - T_{inlet})$$

ideal gas

Thermal Efficiency

$$\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_{in}} = \frac{\dot{Q}_H - \dot{Q}_L}{\dot{Q}_H} = 1 - \frac{q_L}{q_H}$$ ← $\dot{m}$ is constant for the cycle

$$q_L = -{}_4q_1 = -c_p(T_1 - T_4) = c_p(T_4 - T_1)$$

$$q_H = +{}_2q_3 = c_p(T_3 - T_2)$$

$$\eta_{th} = 1 - \left(\frac{T_4 - T_1}{T_3 - T_2}\right) = 1 - \frac{T_1}{T_2}\left[\frac{T_4/T_1 - 1}{T_3/T_2 - 1}\right]$$

- recall, process 1-2 is an isentropic compression therefore, $\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$ for an ideal gas

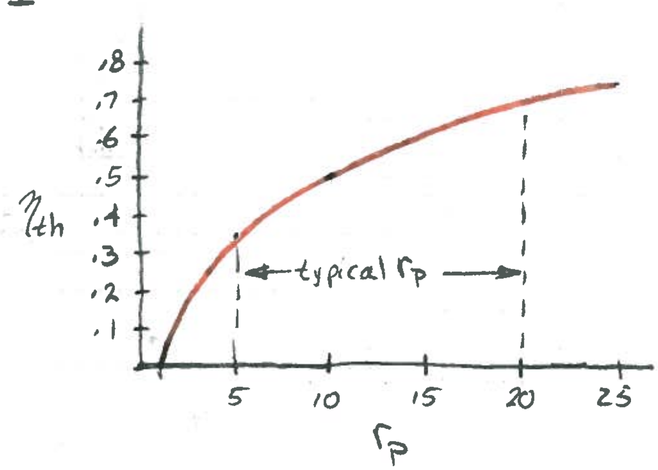
- also, $P_2 = P_3$ & $P_1 = P_4$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} = \frac{T_3}{T_4}$$

- since $\frac{T_2}{T_1} = \frac{T_3}{T_4}$, $\left\{\frac{T_4/T_1 - 1}{T_3/T_2 - 1}\right\} = 1$

$$\eta_{th} = 1 - \frac{T_1}{T_2} = 1 - \frac{1}{r_p^{\,k-1/k}}$$

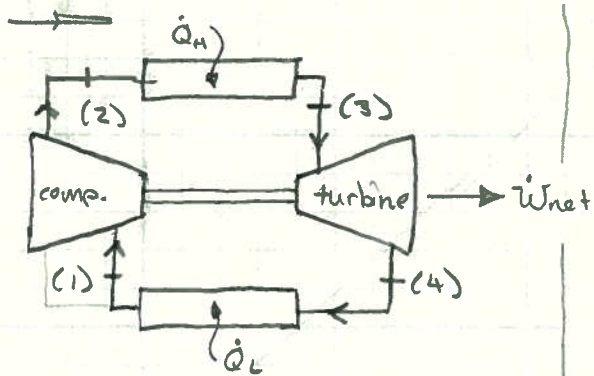
$$r_p = \frac{P_2}{P_1} \equiv \text{pressure ratio}$$

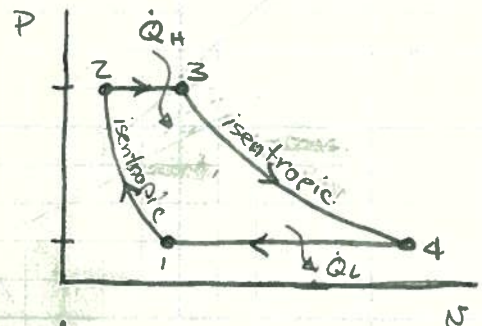


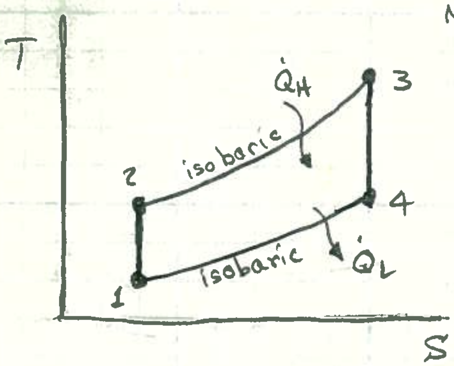
In an air-standard Brayton cycle the air enters the compressor at 0.1 MPa, 15°C. The pressure leaving the compressor is 1.0 MPa and the maximum temperature in the cycle is 1100°C. Determine:

(a) The pressure and temperature at each point in the cycle.

(b) The compressor work, turbine work, and cycle efficiency.







- assume steady, uniform flow
- negligible changes in kinetic & potential energies

Compressor, 1-2:

$$\underset{0,\text{ isentropic}}{{}_1\dot{Q}_2} - {}_1\dot{W}_2 = \dot{m}\underbrace{(h_2 - h_1)}_{\text{ideal gas}}$$

$$-{}_1w_2 = c_p(T_2 - T_1)$$

need to find T_2

- for isentropic process w/ ideal gas

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$$

$$T_2 = (15 + 273\,k)(10)^{0.286} = 556.8\,K$$

(283 °C)

$$-{}_1w_2 = (1.005\ kJ/kg\,K)(556.8 - 288\,k)$$

$${}_1w_2 = -270\ kJ/kg$$

High Temperature Heat Exchanger, 2-3:

$${}_2q_3 - \underset{0}{{}_2w_3} = h_3 - h_2 = c_p(T_3 - T_2)$$

$${}_2q_3 = (1.005\ kJ/kg\,k)(1100 - 283\,C)$$

$${}_2q_3 = 821\ kJ/kg$$

$$P_3 = P_2$$

Turbine, 3-4:

$$-{}_3w_4 = h_4 - h_3 \longrightarrow {}_3w_4 = c_p(T_3 - T_4)$$

- isentropic process with an ideal gas,

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}} \longrightarrow T_4 = 710.8\,K = 437.8\,°C$$

- turbine work

$${}_3w_4 = (1.005\ kJ/kg\,K)(1100 - 437.8\ °C) = 665\ kJ/kg$$

- net work

$$w_{net} = w_t - w_c = {}_3w_4 + {}_1w_2 = 665\ kJ/kg - 270\ kJ/kg$$

$$w_{net} = 395\ kJ/kg$$

Low Temperature Heat Exchanger, 4-1:

$${}_4q_1 = c_p(T_4 - T_1) = (1.005\ kJ/kg)(437.8\,°C - 15\,°C)$$

$${}_4q_1 = 425\ kJ/kg$$

Efficiency

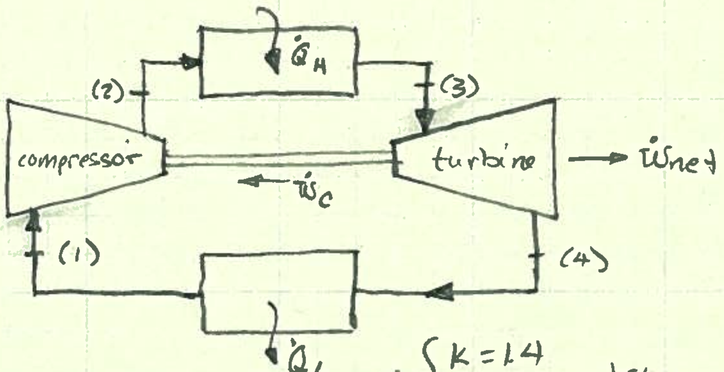
$$\eta_{th} = \frac{w_{net}}{Q_H} = \frac{395}{821} = 48\%$$

OR

$$\eta_{th} = 1 - \frac{1}{r_p^{\frac{k-1}{k}}} = 1 - \frac{1}{10^{.286}} = 48\%$$

Example of Simple Brayton Cycle

An air-standard Brayton cycle operates with a compression ratio of 5.0. The actual expansion & compression efficiencies of the gas processes are 0.88 and 0.82, respectively, and the maximum and minimum temperatures are 750°C and 16°C, respectively. Compute the compression work, the expansion work, the ratio of compression to expansion work (back-work ratio), and the actual and theoretical thermal efficiencies. If the power output of the installation is 8 MW, determine the mass flow rate, kg/min.



$r_p = 5.0$ $\quad w_c = ?$

$w_t = ?$

$T_{max} = 750°C = T_3$ $\quad BWR = ?$

$T_{min} = 16°C = T_1$ $\quad \eta_a = ?$

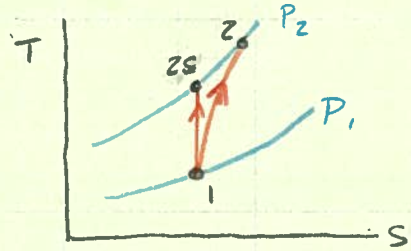
$\eta_t = 0.88$ $\quad \eta_i = ?$

$\eta_c = 0.82$ $\quad \dot{m} = ?$

$\dot{W}_{net} = 8\ MW_m$

air $\begin{cases} k = 1.4 \\ c_p = 1.005\ kJ/kg\,k \\ R = 0.287\ kJ/kg\,k \end{cases}$

Compressor Work



$${}_1\dot{Q}_2 - {}_1\dot{W}_2 = \dot{m}(e_2 - e_1) = \dot{m}(h_2 - h_1)$$

· for an ideal (isentropic) process

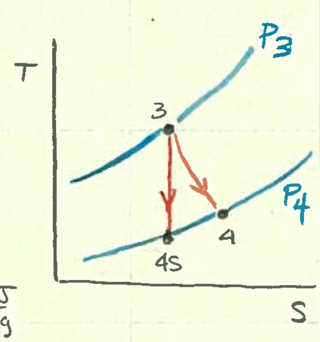
$$-w_2 = h_{2s} - h_1 = c_p(T_{2s} - T_1) \quad \text{for an ideal gas}$$

$$\frac{T_{2s}}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \longrightarrow T_{2s} = T_1\, r_p^{\frac{k-1}{k}} = (16 + 273\,k)(5)^{\frac{1.4-1}{1.4}} = 458\,k$$

$${}_1w_2\big|_{ideal} = c_p(T_1 - T_{2s}) = \left(1.005\,\frac{kJ}{kg\,k}\right)(289\,k - 458\,k) = -170\,\frac{kJ}{kg}$$

$${}_1w_2\big|_{actual} = \frac{{}_1w_2\big|_{ideal}}{\eta_c} = -207\,\frac{kJ}{kg}$$

Turbine Work



$${}_3\dot{Q}_4 - {}_3\dot{W}_4 = \dot{m}(e_4 - e_1) = \dot{m}(h_{4s} - h_3)$$

· for an ideal (isentropic) expansion process with an ideal gas

$$+{}_3w_4 = c_p(T_3 - T_{4s})$$

$$T_{4s} = T_3\left(\frac{1}{r_p}\right)^{\frac{k-1}{k}} = (750 + 273\,k)\left(\frac{1}{5.0}\right)^{\frac{1.4-1}{1.4}} = 646\,k$$

$${}_3w_4\big|_{ideal} = c_p(T_3 - T_{4s}) = (1.005\ kJ/kg\,k)(1023\,k - 646\,k) = +379\,\frac{kJ}{kg}$$

$${}_3w_4\big|_{actual} = \eta_t \cdot {}_3w_4\big|_{ideal} = (0.88)(379\ kJ/kg) = 333\,\frac{kJ}{kg}$$

$$\text{Back-Work Ratio} \equiv \left.\frac{w_c}{w_t}\right|_{actual} = \frac{207\ kJ/kg}{333\ kJ/kg} = 62\%$$

$$\text{Net Specific work} = w_t - w_c = {}_3w_4|_{actual} - |{}_1w_2|_{actual}| = 333\ \frac{kJ}{kg} - 207\ \frac{kJ}{kg} = 126\ \frac{kJ}{kg}$$

Actual Temperatures at (2) & (4) ← required to compute q_{in} & q_{out}

$$\eta_t = \frac{h_3 - h_4}{h_3 - h_{4s}} = \frac{c_p(T_3 - T_4)}{c_p(T_3 - T_{4s})}$$

$$T_4 = T_3 - \eta_t(T_3 - T_{4s}) = 1023k - (0.88)(1023k - 646k) = 691k$$

$$\eta_c = \frac{h_{2s} - h_1}{h_2 - h_1} = \frac{c_p(T_{2s} - T_1)}{c_p(T_2 - T_1)}$$

$$T_2 = T_1 + \frac{1}{\eta_c}(T_{2s} - T_1) = 289k + \frac{1}{0.82}(458k - 289k) = 495k$$

Actual Heat Transfer into Cycle ← required to compute η_{th}

$$q_{in}|_{actual} = {}_2q_3|_{actual} = c_p(T_3 - T_2) = (1.005\ \tfrac{kJ}{kg\,k})(1023k - 495k) = 530.64\ kJ/kg$$

Thermal Efficiency

$$\eta_{th}|_{actual} = \left.\frac{w_{net}}{q_{in}}\right|_{actual} = \frac{126\ kJ/kg}{530.64\ kJ/kg} = 0.237$$

$$\eta_{th}|_{ideal} = 1 - \frac{1}{r_p^{\frac{k-1}{k}}} = 1 - \left(\frac{1}{5.0}\right)^{\frac{1.4-1}{1.4}} = 0.369$$

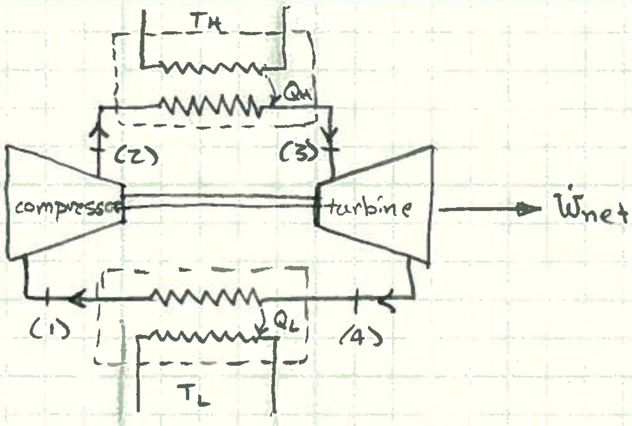
Mass Flow Rate of Air for Net Power of 8 MW

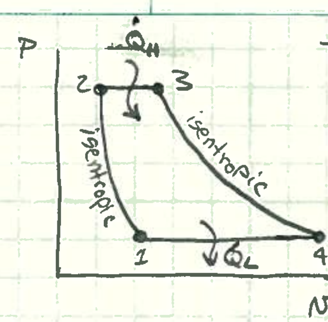
$$\dot{m} = \frac{\dot{W}_{net}}{w_{net}} = \frac{8000\ kJ/s}{126\ kJ/kg} = 63.5\ kg/s$$

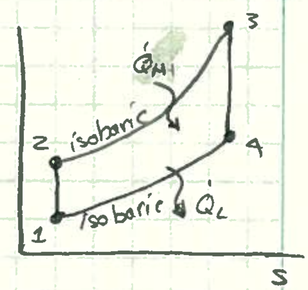
Compressor Power: $\dot{W}_c = (63.5\ kg/s)(207\ kJ/kg) = 13.14\ MW$

Turbine Power: $\dot{W}_t = (63.5\ kg/s)(333\ kJ/kg) = 21.15\ MW$

Brayton Cycle







Turbine Work: $\dot{W}_t = \dot{m} C_p (h_3 - h_4) = \dot{m} \int_{T_4}^{T_3} C_p(T)\, dT$

• for constant specific heats

$$\dot{W}_t = \dot{m} C_p (T_3 - T_4)$$

• isentropic with process using ideal gas

$$\frac{T_3}{T_4} = \left(\frac{P_3}{P_4}\right)^{\frac{k-1}{k}}$$

• pressure ratio is defined as

$$r_{p_t} = \frac{P_3}{P_4}$$

• therefore,

$$\dot{W}_t = \dot{m} C_p T_3 \left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right) \qquad (*)$$

Compressor Work: $\dot{W}_c = \dot{m} C_p (h_2 - h_1) = \dot{m} C_p T_2 \left(1 - \frac{1}{r_{p_c}^{\frac{k-1}{k}}}\right)$ ——— $(**)$

• for an ideal Brayton cycle, $r_{p_t} = r_{p_c} = r_p$

$$\dot{W}_{net} = \dot{W}_t - |\dot{W}_c| = \underbrace{\dot{m} C_p (T_3 - T_2)}_{= \dot{Q}_H} \left[1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right] \text{ ——— (!)}$$

thermal efficiency: $\eta_{th} = \frac{\dot{W}_{net}}{\dot{Q}_H} = 1 - \frac{1}{r_p^{\frac{k-1}{k}}}$ ——— $(***)$

$$\frac{\dot{W}_{net}}{\dot{m}} = w_{net} = c_p (T_3 - T_2)\left(1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right)$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} = r_p^{\frac{k-1}{k}}$$

$$W_{net} = c_p\left[T_3 - T_1 r_p^{\frac{k-1}{k}}\right]\left[1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right]$$

OR

$$w_{net} = c_p\left\{\underbrace{T_1}_{T_{min}}\left[1 - r_p^{\frac{k-1}{k}}\right] + \underbrace{T_3}_{T_{max}}\left[1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right]\right\}$$

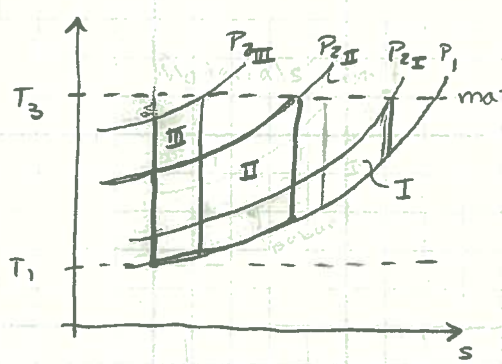
1. All other things being equal (T_1, T_3, r_p, k), the specific work of the cycle is proportional to c_p; the higher the c_p the higher the specific work. Thus, helium can produce five times as much work as can air at low temperatures.

   - specific heats for monotomic gases (He, Ar) are relatively constant and independent of temperature.
   - specific heats for diatomic gases (O_2, N_2, air) increase with temperature
   - specific heats for triatomic gases (CO_2) increase with temperature faster than diatomic gases

2. All other things being equal, gases with higher values of k, $\left(\frac{k-1}{k}\right)$, produce more specific work than gases with lower values of k.

3. For any particular gas, an increase in r_p from 1.0 (no work) decreases one portion of the net work and increases another portion.

$$w_{net} = c_p\left\{T_1 \underbrace{\left[1 - r_p^{\frac{k-1}{k}}\right]}_{\text{decreases with increase in } r_p} + T_3 \underbrace{\left[1 - \frac{1}{r_p^{\frac{k-1}{k}}}\right]}_{\text{increases with increase in } r_p}\right\}$$



Area II > Area III & Area I

$$r_{p_I} < r_{p_{II}} < r_{p_{III}}$$

The optimum pressure ratio for an <u>ideal</u> Brayton cycle can be found by differentiating the net work with respect to r_p and setting the derivative to zero.

$$r_{p_{optimum}} = \left(\frac{T_2}{T_1}\right)^{k/k-1} = \left(\frac{T_3}{T_1}\right)^{k/2(k-1)}$$

decreases w/ increasing k

Brayton Cycle

- Derivation of compression ratio as a function of the initial (T_1) and maximum (T_3) temperatures of an ideal Brayton cycle.

$$w_{net} = c_p\left\{T_1\left[1 - r_p^{\frac{k-1}{k}}\right] + T_3\left[1 - r_p^{\frac{k}{k-1}}\right]\right\}$$

· for convienence, define $\beta \equiv \frac{k-1}{k}$;

$$w_{net} = c_p\left\{T_1\left[1 - r_p^{\beta}\right] + T_3\left[1 - \frac{1}{r_p^{\beta}}\right]\right\}$$

$$\frac{dw_{net}}{dr_p} = c_p\left\{T_1\left[-\beta r_p^{\beta-1}\right] + T_3\left[+\beta r_p^{-\beta-1}\right]\right\} = 0$$

$$-T_1 r_p^{\beta-1} + T_3 \frac{1}{r_p^{\beta+1}} = 0$$

$$T_1 r_p^{\beta-1} \cdot r_p^{\beta+1} = T_3$$

$$r_p^{\beta-1} \cdot r_p^{\beta+1} = \left(\frac{T_3}{T_1}\right)$$

$$r_p^{2\beta} = \left(\frac{T_3}{T_1}\right)$$

$$2\beta = \frac{2(k-1)}{k}$$

optimum pressure ratio; for simple Brayton cycle

$$\boxed{r_p = \left(\frac{T_3}{T_1}\right)^{\frac{k}{2(k-1)}}}$$

Find the pressure ratio required by an ideal Brayton cycle to produce a net work of 600 Btu/lbm of (i) helium and (ii) air with constant specific heats. The cycle has initial and maximum temperatures of 500 R and 2500 R, respectively. Also, calculate the optimum pressure ratio for both gases.

Helium: $C_p = 1.250 \frac{Btu}{lbm\,R}$ Air: $C_p = 0.240 \frac{Btu}{lbm\,R}$

$k = 1.659$ $k = 1.4$

$\frac{k-1}{k} = 0.3972$ $\frac{k-1}{k} = 0.286$

turbine: $\dot{W}_t = \dot{m} C_p T_{max}\left(1 - 1/r_{pt}^{\frac{k-1}{k}}\right)$

compressor: $\dot{W}_c = \dot{m} C_p T_{init}\left(1 - 1/r_{pc}^{\frac{k-1}{k}}\right)$

for an ideal cycle, $r_{pt} = r_{pc} = r_p$

specific network: $w_{net} = w_t - |w_c| = c_p\left(T_{max} - T_{init}\, r_p^{\frac{k-1}{k}}\right)\left(1 - 1/r_p^{\frac{k-1}{k}}\right)$

(i) Helium: $600 \frac{Btu}{lbm} = \left(1.250 \frac{Btu}{lbm\,R}\right)\left[(2500\,R) - (500\,R)\, r_p^{0.3972}\right]\left[1 - 1/r_p^{0.3972}\right]$

• solving for r_p,

$$(r_p^{0.3972})^2 - 5.04(r_p^{0.3972}) + 5 = 0$$

$$r_p = \underline{\underline{2.16}} \text{ or } \underline{\underline{26.62}}$$

• the optimum compression ratio is

$$r_p\big|_{optimum} = \left(\frac{T_{max}}{T_{init}}\right)^{\frac{k}{2(k-1)}} = \left(\frac{2500}{500}\right)^{\frac{1.659}{2(1.659-1)}} = \underline{\underline{7.58}}$$

• this pressure ratio, $r_p\big|_{optimum}$, yields the maximum work of 954.8 Btu/lbm.

• At $r_p = 1$, the network is zero. There is also a maximum value of r_p which yields a network equal to zero. The maximum r_p occurs when $T_{max} = T_{init}$ $(w_t = w_c)$.

$$r_p\big|_{maximum} = \left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}} = \left(\frac{T_3}{T_1}\right)^{\frac{k}{k-1}}$$

$$\text{or } \left(r_p\big|_{max}\right)^{0.3972} = \left(\frac{2500}{500}\right) = 5$$

$$r_p\big|_{maximum} = \underline{\underline{57.5}}$$

iii) Air:

$$600 \frac{Btu}{lbm} = \left(0.240 \frac{Btu}{lbm\,R}\right)\left[(2500\,R) - (500\,R)\,r_p^{0.286}\right]\left[1 - 1/r_p^{0.286}\right]$$

$$\left(r_p^{.286}\right)^2 - \left(r_p^{.286}\right) + 5 = 0$$

- solving for r_p results in imaginary values which indicates that 600 Btu/lbm net work cannot be produced in this cycle using air
- the optimum pressure ratio is

$$r_p|_{opt} = \left(\frac{2500}{500}\right)^{\frac{1.4}{2(1.4-1)}} = 16.72$$

- the maximum work, achieved at $r_p|_{opt}$, is 183.3 $\frac{Btu}{lbm}$
- the maximum r_p is 279.6

summary

	He	Air
c_p [Btu/lbm R]	1.250	0.24
k	1.659	1.4
r_p for 600 $\frac{Btu}{lbm}$	2.16 or 26.62	n/a
$r_p\vert_{opt.}$	7.58	16.72
$W_{net}(r_p\vert_{opt})$ [$\frac{Btu}{lbm}$]	954.8	183.3
$r_p\vert_{max}$	57.5	279.6

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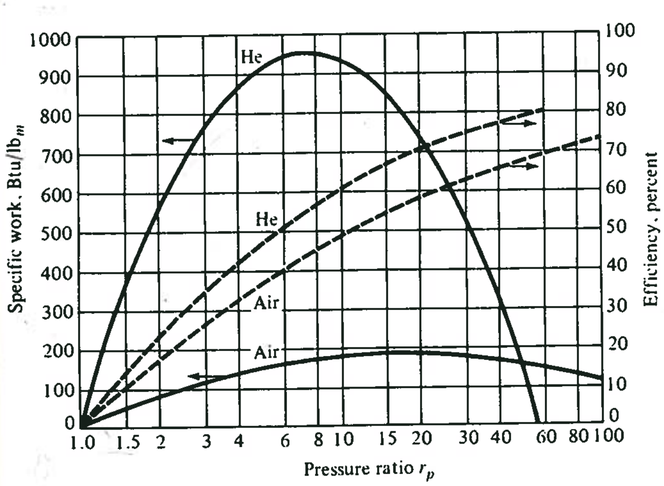
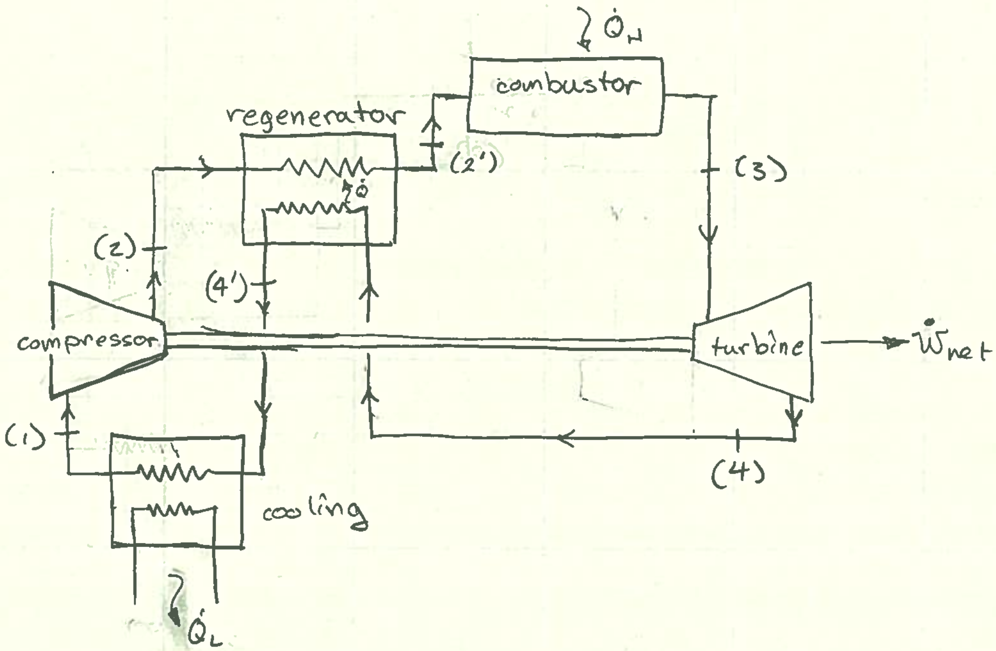
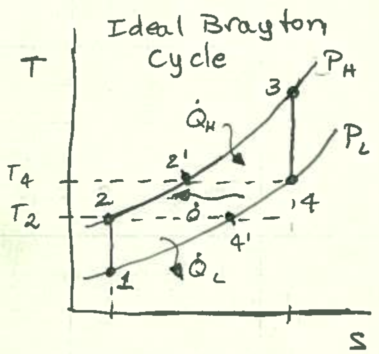


Figure 8-8 Specific power and efficiency versus pressure ratio for ideal Brayton cycles operating with helium and air.

Brayton Cycle – Regeneration

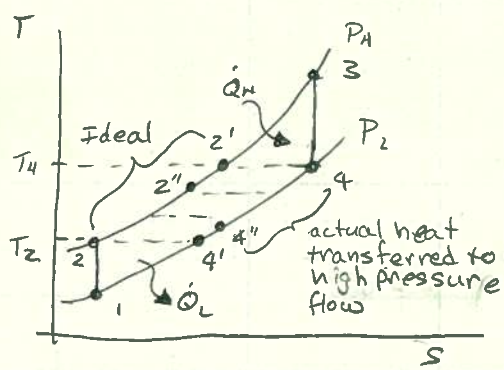




$T_4 > T_2 \Rightarrow$ thermal energy from the turbine exhaust can be transferred to the compressor exhaust thereby saving energy in the combustor

Highest temperature in the regenerator is T_4 (turbine exhaust).

In practice, the exit temperature of the high pressure flow is less than T_4.



$$q_{reg}|_{max} = h_{2'} - h_2 = h_4 - h_{4'}$$

$$q_{reg}|_{actual} = h_{2''} - h_{2'} = h_4 - h_{4''}$$

regenerator effectiveness,

$$\varepsilon_R = \frac{q_{reg}|_{actual}}{q_{reg}|_{max}}$$

$$\varepsilon_R = \frac{h_{2''} - h_2}{h_{2'} - h_2} = \frac{h_4 - h_{4''}}{h_4 - h_{4'}}$$

For an ideal gas with constant specific heats,

$$\varepsilon_R = \frac{T_2'' - T_2}{T_2' - T_2} = \frac{T_4 - T_4''}{T_4 - T_4'}$$

Note that $T_4' = T_2$ & $T_2' = T_4$

$$\boxed{\varepsilon_R = \frac{T_2'' - T_2}{T_4 - T_2} = \frac{T_4 - T_4''}{T_4 - T_2}}$$

Typical effectiveness < 0.85

- Thermal efficiency,

$$\eta_{th} = 1 - \frac{T_4'' - T_1}{T_3 - T_2''} = 1 - \frac{T_4'' - T_{min}}{T_{max} - T_2''}$$

- For an ideal cycle, $T_4'' = T_4' = T_2$ and $T_2'' = T_2' = T_4$
- The thermal efficiency with regeneration becomes:

$$\boxed{\eta_{th} = 1 - \left(\frac{T_1}{T_3}\right) r_p^{\frac{k-1}{k}}}$$

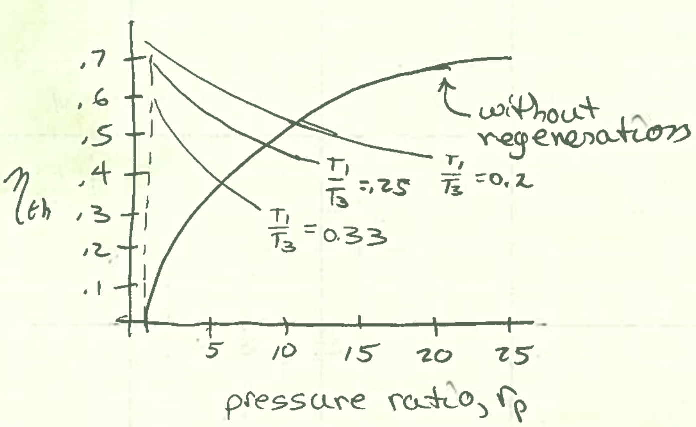
$T_3 = T_{max}$

$T_1 = T_{min}$

without regeneration

$$\boxed{\eta_{th} = 1 - \frac{1}{r_p^{\frac{k-1}{k}}}}$$

- note that the effects of k & r_p are reversed with regeneration as compared to a Brayton cycle without regeneration.



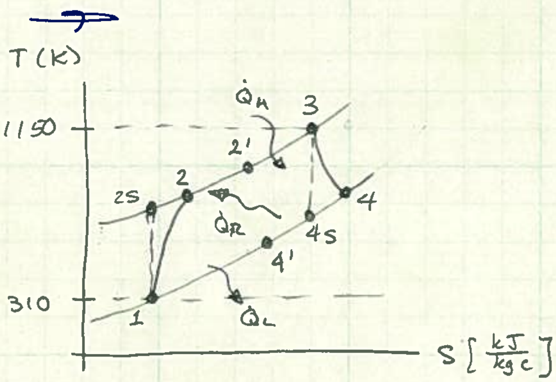
CB 9-46

A Brayton cycle with regeneration using air as the working fluid has a pressure ratio of 7. The minimum and maximum temperatures in the cycle are 310 K and 1150 K. Assuming an isentropic efficiency of 75% for the compressor and 82% for the turbine and a regenerator effectiveness of 65%, determine:

(a) the air temperature at the turbine exit,

(b) the net work output, and

(c) the thermal efficiency.



For air as an ideal gas (using tables):

$T_1 = 310\ K \rightarrow h_1 = 310.24\ kJ/kg$

• for $\Delta S = 0$, $\frac{P_{2s}}{P_1} = \frac{Pr_2}{Pr_1} \rightarrow h_{2s} = 541.26\ kJ/kg$

$\eta_c = 0.75 = \frac{h_{2s} - h_1}{h_2 - h_1} \rightarrow h_2 = 618.26\ \frac{kJ}{kg}$

$T_3 = 1150\ K \rightarrow h_3 = 1219.25\ \frac{kJ}{kg}$

• for $\Delta S = 0$, $\frac{P_{4s}}{P_3} = \frac{Pr_4}{Pr_3} \rightarrow h_{4s} = 711.80\ \frac{kJ}{kg}$

$\eta_t = 0.82 = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = 803.14\ \frac{kJ}{kg}$

$\boxed{T_4 = 782.8\ K}$ (a)

$w_{net} = w_t - |w_c| = (h_3 - h_4) - (h_2 - h_1)$

$= (1219.25 - 803.14\ \frac{kJ}{kg}) - (618.26 - 310.24\ \frac{kJ}{kg})$

$= \boxed{108.09\ kJ/kg}$ (b)

$\varepsilon_R = \frac{h_{2'} - h_2}{h_4 - h_2} = 0.65$

$h_{2'} = h_2 + \varepsilon(h_4 - h_2) = 738.43\ \frac{kJ}{kg}$

$q_H = h_3 - h_{2'} = 480.82\ kJ/kg$

$\eta_{th} = \frac{w_{net}}{q_H} = \frac{108.09\ kJ/kg}{480.82\ kJ/kg} = 22.5\%$ (c)

• Assuming Constant Specific Heats at low temperature

$C_p = 1.005\ \frac{kJ}{kg\,k}$

$k = 1.4$

$R = 0.2870\ \frac{kJ}{kg\,k}$

$T_1 = 310\ k \rightarrow h_1 = C_p T_1 = 311.55\ kJ/kg$

$S_1 = S_{2s} \rightarrow \left(\frac{T_{2s}}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}}$

$T_{2s} = 540.53\ k$, $h_{2s} = 543.2\ \frac{kJ}{kg}$

$\eta_c = 0.75 = \frac{h_{2s} - h_1}{h_2 - h_1} \rightarrow h_2 = 620.42\ kJ/kg$

$T_3 = 1150\ k \rightarrow h_3 = 1155.75\ kJ/kg$

$S_3 = S_{4s} \rightarrow \left(\frac{T_{4s}}{T_3}\right) = \left(\frac{P_4}{P_3}\right)^{\frac{k-1}{k}}$

$T_{4s} = 659.5\ k \rightarrow h_{4s} = 662.8\ \frac{kJ}{kg}$

$\eta_t = 0.82 = \frac{h_3 - h_4}{h_3 - h_{4s}} \rightarrow h_4 = 751.53\ \frac{kJ}{kg}$

(a) $T_4 = 747.8\ k$

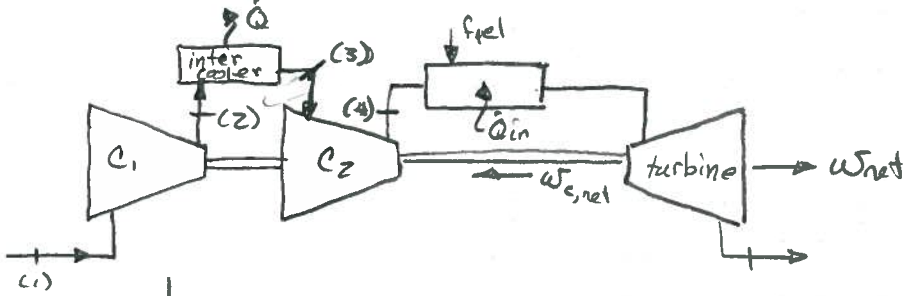
(b) $w_{net} = 95.35\ kJ/kg$

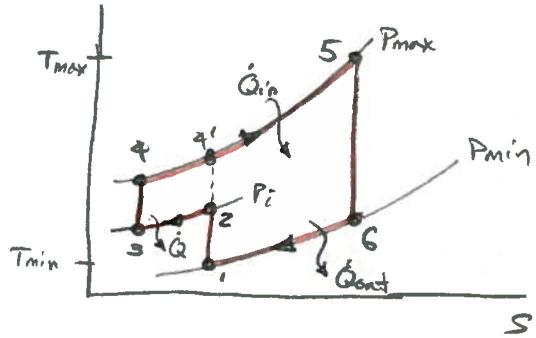
$h_{2'} = 705.64\ kJ/kg$

$q_H = 450.11\ kJ/kg$

$\eta_{th} = 21.2\%$ (c)

Brayton Cycle - Multistage Compression with Intercooling





- multistage compression with intercooling produces a lower discharge temperature than that resulting from single-stage compression; $T_4 < T_{4'}$.

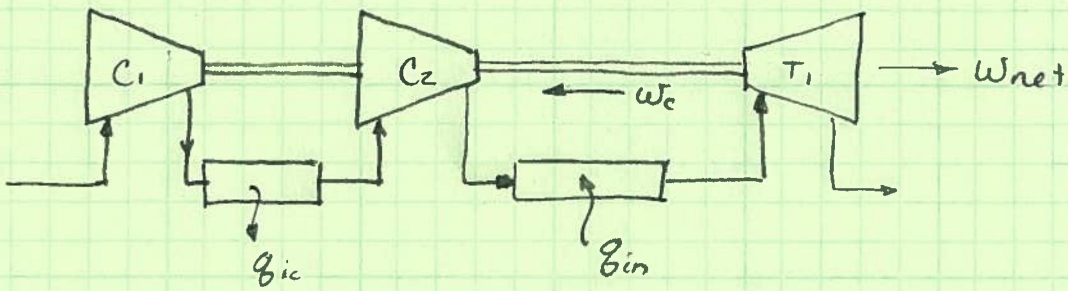
- more heat is required with multistage compression with intercooling;

  $$h_5 - h_4 > h_5 - h_{4'}$$

  unless regeneration is used the theoretical efficiency of the cycle will be lower

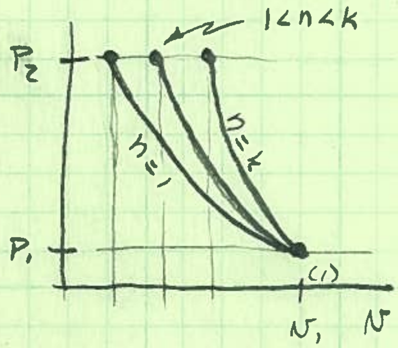
- in practice, the cycle efficiency <u>may</u> improve due to improved compressor efficiency over the smaller ΔP.

Compressor Work - Multistage with Intercooling



- which is more efficient, single-stage compression or multi-stage compression with intercooling?

- Polytropic Process, $Pv^n = \text{constant}$



- for an isentropic process, $n=k$
  - smallest area under the curve; least amount of work required to compress from P_1 to P_2
- for an isothermal process, $n=1$
  - largest area under curve
  - $Pv^1 = RT = \text{constant} \Rightarrow T$ must be constant

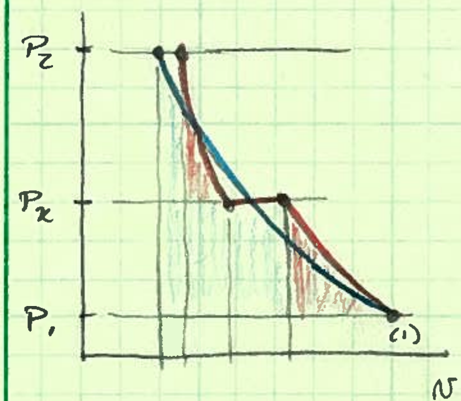
- For an ideal gas with constant specific heats,

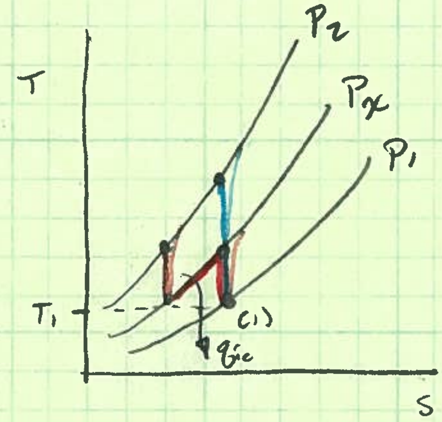
$$w_c\Big|_{reversible} = -\frac{R(T_2 - T_1)}{(1-n)/n} = -\frac{RT_1}{(n-1)/n}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right] \quad ; n \neq 1$$

$$\qquad\qquad\qquad\qquad\qquad\qquad \uparrow r_p$$

- isentropic compression, $n=k$
- for an isothermal compression ($n=1$),

$$w_c = -RT \ln(P_2/P_1)$$





$$w_c = w_{c_1} + w_{c_2} = -\frac{RT_1}{\left(\frac{n-1}{n}\right)}\left[\left(\frac{P_x}{P_1}\right)^{\frac{n-1}{n}} - 1\right] - \frac{RT_1}{\left(\frac{n-1}{n}\right)}\left[\left(\frac{P_2}{P_x}\right)^{\frac{n-1}{n}} - 1\right]$$

- what value of P_x minimizes w_c?
  - When $\left(\frac{P_x}{P_1}\right) = \left(\frac{P_2}{P_x}\right) \longrightarrow \boxed{r_{p_1} = r_{p_2}}$

- Pressure Rise per stage,

$$r_{p,stage} = \sqrt[N_c]{r_{p,Total}}$$

$N_c \equiv$ # of compressor stages

- for $r_p = 10$ with 3 stages,

$$r_{p,stage} = \sqrt[3]{10} = 2.154 \quad \left\{ \text{Not}: \frac{10}{3} = 3.33! \right\}$$

$P_1 = 1$ bar
$P_3 = 9$ bar
2 stages of compression

$$\left.\right\} \; r_{p_s} = \sqrt[2]{\frac{9}{1}} = 3$$

stage 1: $1 \text{ bar} \times r_{p_s} = 3$ bar

stage 2: $3 \text{ bar} \times r_{p_s} = 9$ bar

Ideal Gas Turbine

2 stages of compression

2 stages of expansion

$r_p|_{total} = 8$

$T_{c_{in}} = 300\,k$ (both stages)

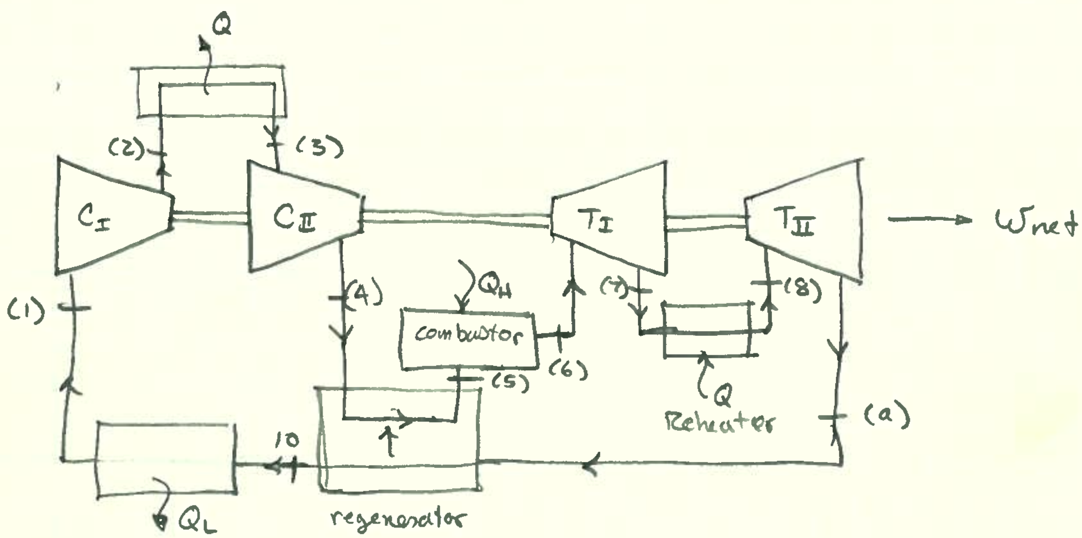
$T_{t_{in}} = 1300\,k$ (both stages)

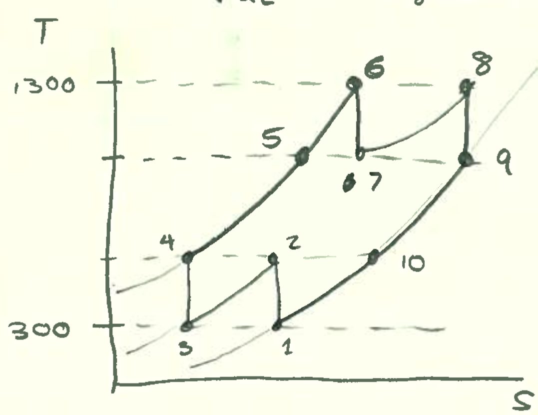
Back Work Ratio = ?

$\eta_{th} = ?$

(a) no regenerator

(b) Ideal regenerator, $\varepsilon_R = 1$





$$\frac{P_2}{P_1} = \frac{P_4}{P_3} = \sqrt[2]{8} = 2.83$$

$$\frac{P_6}{P_7} = \frac{P_8}{P_a} = \sqrt[2]{8} = 2.83$$

$T_1 = T_3,\ h_1 = h_3$
$T_2 = T_4,\ h_2 = h_4$ } compressor inlets / exits

$T_6 = T_8,\ h_6 = h_8$
$T_7 = T_9,\ h_7 = h_9$ } turbine inlets / exits

$\therefore\ w_{c_I} = w_{c_{II}}$

$w_{t_I} = w_{t_{II}}$

no regen

$T_1 = 300\,k \rightarrow h_1 = 300.19$ kJ/kg
$T_2 = 403.3\,k \rightarrow h_2 = 404.31$ kJ/kg
} from ideal gas - air tables

$T_6 = 1300\,k \rightarrow h_6 = 1395.97$ kJ/kg
$T_7 = 1006.4 \rightarrow h_7 = 1053.33$ kJ/kg

$w_c = +2\,w_{c_I} = 2(h_2 - h_1) = 208.24$ kJ/kg
$w_t = 2\,w_{t_I} = 2(h_6 - h_7) = 685.28$ kJ/kg
} $w_{net} = 477.04 \frac{kJ}{kg}$

$q_{in} = q_{primary} + q_{reheat} = (h_6 - h_4) + (h_8 - h_7) = 1334.30 \frac{kJ}{kg}$

Back Work Ratio = 0.304

$\eta_{th} = 0.358$

w/o reheat & intercooling & regen.
BWR = 0.403
$\eta_{th} = 0.426$

with regen

ideal, $\Delta P = 0$, $\varepsilon_R = 1$ → does not affect w_c or w_t

$h_5 = h_7 = h_9$

$q_{in} = q_{primary} + q_{reheat} = (h_6 - h_5) + (h_8 - h_7) = 658.28$ kJ/kg

- Back Work Ratio is same = 0.304

$\eta_{th} = 0.696$

Intercooling, reheat & regeneration can be combined in the same cycle,

$$w_{net} = w_t - |w_c| = c_p \left\{ T_3 \eta_t (N_t+1)\left[1 - \frac{1}{r_{p_t}^{\frac{k-1}{k}}}\right] - T_1\left(\frac{N_c+1}{\eta_c}\right)\left[r_{p_c}^{\frac{k-1}{k}} - 1\right]\right\}$$

$$q_{in} = c_p T_3 \left\{(N_t+1) - (N_t+\varepsilon_R)\left[1 - \eta_t\left(1 - \frac{1}{r_{p_t}^{\frac{k-1}{k}}}\right)\right]\right\} - c_p T_1 (1-\varepsilon_R)\left[1 + \frac{1}{\eta_c}\left(r_{p_c}^{\frac{k-1}{k}} - 1\right)\right]$$

$\eta_t \equiv$ turbine efficiency

$N_t \equiv$ number of reheat processes

$\eta_c \equiv$ compressor efficiency

$N_c \equiv$ number of intercoolers

changed definition of N_t & N_c

$$\left.\begin{aligned} r_{p_c} = r_{p_T} \; ; \; r_{p_{t_i}} &= \sqrt[N_t+1]{r_{p_T}} \\ r_{p_{c_i}} &= \sqrt[N_c+1]{r_{p_c}} \end{aligned}\right\}$$

A combined gas-steam-turbine power plant is designed with four 50-MW_e gas turbines and one 120-MW_e steam turbine.

Gas Turbines:

Compressor Inlet Temperature = 505°R

Turbine Inlet Temperature = 2450°R

pressure ratio (comp. & turbine) = 5

$\eta_c = \eta_t = 0.87$

$\eta_m = 0.96$

The gases leave the turbine and go to a heat-recovery boiler then to a regenerator.

$\varepsilon_R = 0.87$

200% theoretical air (use $CH_{2.145}$ as fuel)

Supplemental Firing raises gas temperature to 2000°F (full load).

Steam Cycle

Turbine Inlet Pressure = 1200 psia

Turbine Inlet Temperature = 1460°R

1 open feedwater heater, 920°R

Condenser Pressure = 1 psia

$\eta_t = 0.87$

$\eta_m = 0.96$

– ignore pump work –

Generator

$\eta_e = 0.96$

Draw the flow & T-s diagrams, find:

(a) $\dot{m}_{steam}$ [lbm steam/hr]

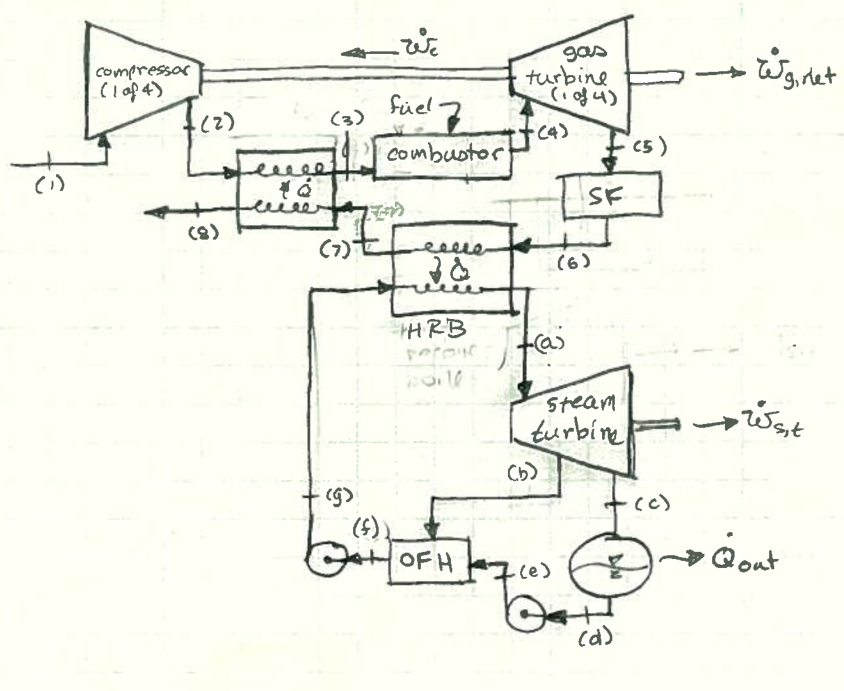
(b) $\dot{m}_{gas}$ [lbm gas/hr/turbine]

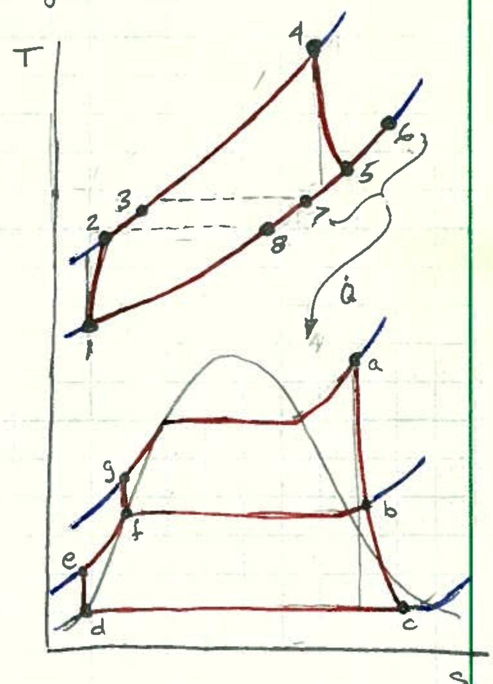
(c) $\dot{Q}_{in}$ (combuster & supplemental firing) [Btu/hr]

(d) T_{stack} of gas exhaust [°F]

(e) η_{th} at full load for combined cycle

(f) η_{th} w/ 1 gas turbine & no supplemental firing or regeneration.





Gas Cycle

(1) Dry Air (Table I-1)

$T_1 = 505\,°R$

$Pr_1 = 1.0970$

$h_1 = 120.675$ Btu/lbm air

(2) Dry Air (Table I-1)

$Pr_2 = r_p \cdot Pr_1 = 5.4848$

$h_{2s} = 191.39$ Btu/lbm air (interpolation)

$h_2 = h_1 + \frac{1}{\eta_c}(h_{2s} - h_1) = 201.96\ \frac{Btu}{lbm\ air}$; ~~$T_2 =$~~

$T_2 = 798.3\,°R$

(3) Dry Air — need to balance energy across regenerator

(4) 200% Theoretical Air (Table I-2)

$T_4 = 2450\,°R$

$Pr_4 = 511.9$

$\bar{h}_4 = 19080.7$ Btu/lbmol gas

- for $CH_{2.145}$, the 200% theoretical air is A/F = 29.82 (see problem 8-13)

$$\frac{m_{gas}}{m_{air}} = 1 + \frac{1}{A/F} = 1.0335\ \frac{lbm\ gas}{lbm\ air}$$

- M (200% Theoretical Air) = 28.880 $\frac{lbm\ gas}{lbmol\ gas}$

$$h_4 = \left(\frac{\bar{h}_4}{M}\right)\left(1 + \frac{1}{A/F}\right) = 682.82\ \frac{Btu}{lbm\ air}$$

(5) 200% Th. Air (Table I-2)

$Pr_5 = Pr_4 / r_p = 102.38$

$\bar{h}_{5s} = 12{,}539.5$ Btu/lbmol gas

$\bar{h}_5 = \bar{h}_4 - \eta_t(\bar{h}_4 - \bar{h}_{5s}) = 13{,}389.9\ \frac{Btu}{lbmol\ gas}$

$h_5 = \frac{\bar{h}_5}{M}\left(1.0335\ \frac{lbm\ gas}{lbm\ air}\right) = 479.17$ Btu/lbm air

(6) 200% Theor. Air (Table I-3)

$T_6 = 2000\,°F$

$Pr_6 = 211.6$

$\bar{h}_6 = 15{,}189.3\ \frac{Btu}{lbmol\ gas}$

$h_6 = 543.56$ Btu/lbm air

(7) 200% theoretical air — need to balance energy across heat recovery steam generator

- Gas Cycle Work:

$$\left.\begin{aligned} w_t &= h_4 - h_5 = 203.65\ \frac{Btu}{lbm\ air} \\ w_c &= h_2 - h_1 = 81.29\ \frac{Btu}{lbm\ air} \end{aligned}\right\}\ w_{net} = 122.37\ \frac{Btu}{lbm\ air}$$

$$\dot{W}_{net} = \frac{\dot{W}_{net,e}}{\eta_m \cdot \eta_e} = \frac{(50{,}000\ kW_e)(3412\ \frac{Btu/hr}{kW})}{(0.96\ kW_m/kW_t)(0.96\ kW_e/kW_m)} = 185.11 \cdot 10^6\ \frac{Btu}{hr}$$ ← single turbine

(b) $\dot{m}_{air} = \frac{\dot{W}_{net}}{w_{net}} = 1.513 \cdot 10^6\ \frac{lbm\ air}{hr}$ ← single turbine

$6.124 \cdot 10^6\ \frac{lbm\ air}{hr}$ ← 4 turbines

Steam Cycle

(a) Superheated Steam

$T_a = 1460°R$

$P_a = 1200\ psia$

$h_a = 1500\ Btu/lbm\ steam$ } from EES

$s_a = 1.6303\ Btu/lbm°R$ } (steam_IAPWS)

(b) Superheated Steam

$T_{bsat} = 920°R$

$P_b = 468.3\ psia$

$s_{bs} = s_a = 1.6303\ Btu/lbm°R$

$h_{bs} = 1370.8\ Btu/lbm$

$h_b = h_a - \eta_t \cdot (h_a - h_{bs}) = 1388\ Btu/lbm\ steam$

(c) Saturated Mixture

$P_c = 1\ psia$

$s_{cs} = s_a = 1.6303\ Btu/lbm°R$

$x_{cs} = 0.8118$

$h_{cs} = 910.5\ Btu/lbm$

$h_c = h_a - \eta_t (h_a - h_{cs}) = 987.2\ Btu/lbm\ steam$

(d) Saturated Liquid

$P_d = 1\ psia$

$h_d = h_f = 69.72\ Btu/lbm\ steam$

$v_d = v_f = 0.01614\ ft^3/lbm$

(e) Compressed Liquid

$P_e = P_b = 468.3\ psia$

$h_e = h_d + v_d (P_e - P_d) = 71.16\ \frac{Btu}{lbm\ steam}$ [watch units!]

$1\ Btu = 778.17\ ft\text{-}lbf$

$1\ ft^2 = 144\ in^2$

(f) Saturated liquid

$h_f = h_f(sat) = 441.9\ Btu/lbm\ steam$

$v_f = 0.01962\ ft^3/lbm$

$P_f = P_b = 468.3\ psia$

(g) Compressed Liquid

$P_g = P_a = 1200\ psia$

$$h_g = h_f + v_f (P_g - P_f) = 444.56\ \frac{Btu}{lbm\ steam}$$

- Open Feed Water Heater:

$$\dot{m} = \dot{m}_b + \dot{m}_c \longrightarrow 1 = M_b + M_c$$

$$\dot{m} h_f = \dot{m}_b h_b + \dot{m}_c h_c \longrightarrow M_b = \frac{h_f - h_c}{h_b - h_c} = 0.2782$$

$$M_c = 1 - M_b = 0.7218$$

- Steam Turbine:

$$w_t = (h_a - h_b) + (1 - M_b)(h_b - h_c) = 401.7 \ \frac{\text{Btu}}{\text{lbm steam}}$$

$$\dot{W}_t = \frac{\dot{W}_{s,e}}{\eta_m \cdot \eta_e} = \frac{(120{,}000 \text{ kW}_e)(3412 \ \frac{\text{Btu/hr}}{\text{kW}})}{(0.96 \ W_m/W_{sh})(0.96 \ W_e/W_m)} = 444.27 \cdot 10^6 \ \frac{\text{Btu}}{\text{hr}}$$

(a) $$\dot{m}_{steam} = \frac{\dot{W}_t}{w_t} = 1.106 \cdot 10^6 \ \frac{\text{Btu}}{\text{hr}}$$

- Heat Recovery Steam Generator:

$$\dot{Q} = \dot{m}_{steam}(h_a - h_g) = 1.167 \cdot 10^9 \ \text{Btu/hr}$$

$$\dot{Q} = 4 \cdot \dot{m}_{air}(h_6 - h_7) \longrightarrow h_7 = 350.73 \ \frac{\text{Btu}}{\text{lbm air}}$$

- we need to find T_2 to balance the regenerator

$$\bar{h}_7 = h_7 \cdot M / 1.0335 \ \frac{\text{lbm gas}}{\text{lbm air}} = 9800.76 \ \frac{\text{Btu}}{\text{lbmol gas}}$$

$$T_7 = 1343 \ ^\circ R$$

- we need to convert back to 200% Theoretical Air in order to get the correct temperature for the exhaust

- Regenerator:

$$\varepsilon_R = \frac{T_3 - T_2}{T_7 - T_2} = \frac{T_7 - T_8}{T_7 - T_2}$$

$$T_3 = 1272 \ ^\circ R \longrightarrow \bar{h}_3 = 9242.98 \ \frac{\text{Btu}}{\text{lbmol gas}}$$

(d) $$T_8 = 869 \ ^\circ R \qquad h_3 = 330.77 \ \text{Btu/lbm air}$$

- Heat Addition:

combustor: $${}_3Q_4 = 4 \dot{m}_{air}(h_4 - h_3) = 2.131 \cdot 10^9 \ \frac{\text{Btu}}{\text{hr}}$$

supplemental firing: $${}_5Q_6 = 4 \dot{m}(h_6 - h_5) = 0.394 \cdot 10^9 \ \frac{\text{Btu}}{\text{hr}}$$

$$\dot{Q}_{in} = 2.525 \cdot 10^9 \ \frac{\text{Btu}}{\text{hr}} \quad \text{(c)}$$

(e) $$\eta_{th} = \frac{\dot{W}_{t,steam} + \dot{W}_{net,gas}}{\dot{Q}_{in}} = \frac{(444.27 \cdot 10^6 \ \frac{\text{Btu}}{\text{hr}}) + 4(185.11 \cdot 10^6 \ \frac{\text{Btu}}{\text{hr}})}{(2.525 \cdot 10^9 \ \text{Btu/hr})} = 0.4691$$

cycle, not plant

$$\eta_{plant} = \frac{(120{,}000 \text{ kW} + 4 \cdot 50{,}000 \text{ kW})(3412 \ \frac{\text{Btu/hr}}{\text{kW}})}{(2.525 \cdot 10^9 \ \text{Btu/hr})} = 0.4324$$

▪ No Supplemental Firing or Regeneration, 1 gas turbine:

$$\eta_{th} = \frac{w_{net}}{h_4 - h_2} = \frac{122.37\ \text{Btu/lbm air}}{\left(682.82\ \frac{\text{Btu}}{\text{lbm air}} - 201.96\ \frac{\text{Btu}}{\text{lbm air}}\right)} = \frac{122.37}{480.86} = 0.2545$$

(f) $$\eta_{plant} = \frac{(50{,}000\ \text{kW}_e)\left(3412\ \frac{\text{Btu/hr}}{\text{kW}}\right)}{\left(1.513 \cdot 10^6\ \frac{\text{lbm air}}{\text{hr}}\right)(480.86\ \text{Btu/lbm air})} = 0.2345$$