Heat Transfer Asymptote In Laminar Flow Of Non-Linear Viscoelastic Fluids In Straight Non-Circular Tubes And Interplay Of Elasticity And Inertia In Heat Transfer Enhancement

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ABSTRACT

A survey of secondary flows of viscoelastic liquids in straight tubes is given including recent work pointing at striking analogies with transversal deformations associated with the simple shearing of solid materials. The importance and implications of secondary flows of viscoelastic fluids in heat transfer enhancement are explored. Recent new work by the author and colleagues of both the steady and unsteady pressure gradient driven secondary flows of constitutively non-linear simple fluids in straight pipes of non-circular shape is discussed. Specifically the steady secondary flow of non-affine viscoelastic fluids in pipes of unconventional contours together with work which explores for the first time the structure of the secondary flow field in the pulsating flow in straight tubes of arbitrary cross-sections is summarized.

The fully developed thermal field in constant pressure gradient driven laminar flow of a class of non-linear viscoelastic fluids with instantaneous elasticity in straight pipes of arbitrary contour $\partial D$ with constant wall flux is investigated. The non-linear fluids considered are constitutively represented by a class of single mode, non-affine constitutive equations. The driving forces can be large. Asymptotic series in terms of the Weissenberg number $Wi$ are employed to expand the field variables. A continuous one-to-one mapping is used to obtain arbitrary tube contours from a base tube contour $\partial D_0$. The analytical method presented is capable of predicting the velocity and temperature fields in tubes with arbitrary cross-section. Heat transfer enhancement due to shear-thinning is identified together with the enhancement due to the inherent elasticity of the fluid. The latter is to a very large extent the result of secondary flows in the cross-section but there is a component due to first normal stress differences as well. Increasingly large enhancements are computed with increasing elasticity of the fluid as compared to its Newtonian counterpart. Order of magnitude larger enhancements are possible even with slightly viscoelastic fluids.

The coupling between inertial and viscoelastic nonlinearities is crucial to enhancement. The asymptotic independence of $Nu = f(Pe, Wi) \rightarrow Nu = f(Pe)$ from elasticity with increasing $Wi$ is shown analytically for the first time. Isotherms for the temperature field are discussed for non-circular contours such as the ellipse and the equilateral triangle together with the behavior of the average Nusselt number $Nu$; a function of the Reynolds $Re$, the Prandtl $Pr$ and the Weissenberg $Wi$ numbers. The change of type of the vorticity equation governs the trends in the behavior of $Nu$ with increasing $Wi$ and $Re$. The implications on the heat transfer enhancement is discussed in particular for slight deviations from Newtonian behavior where a rapid rise in enhancement seems to occur as opposed to the behavior for larger values of the Weissenberg number where the rate of increase is much slower. The asymptotic independence of $Nu$ from elasticity with increasing $Wi$ is related to the extent of the supercritical region controlled by the interaction of the viscoelastic Mach number $M$ and the Elasticity number $E$, which mitigates and ultimately cancels the effect of the increasingly strong secondary flows with increasing $Wi$ to level off the enhancement. The physics of the interaction of the effects of the Elasticity $E$, Viscoelastic Mach $M$, Reynolds $Re$ and Weissenberg $Wi$ numbers on generating the heat transfer enhancement is discussed. The existence of a heat transfer asymptote in laminar flow of non-linear viscoelastic fluids is shown for the first time. Similar to the heat transfer asymptote in turbulent pipe flows the counterpart in laminar flows in non-circular tubes delineates the region where enhancement is a function only of inertia. A different asymptote corresponds to different cross-sectional shapes in straight tubes.