# MEEM 3700
# Mechanical Vibrations


Mohan D. Rao

Chuck Van Karsen

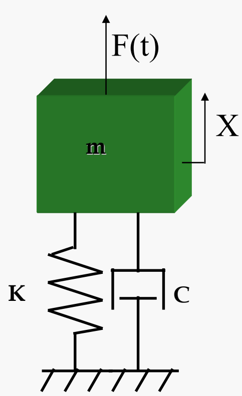
Mechanical Engineering-Engineering Mechanics

Michigan Technological University

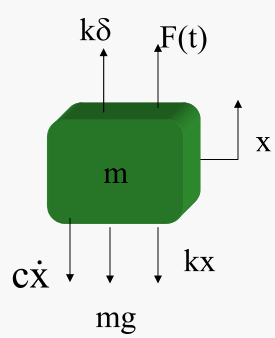





## Single Degree of Freedom Forced Vibration



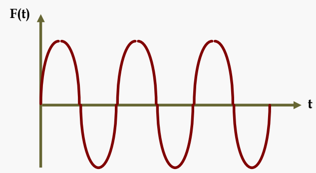

Free Body Diagram



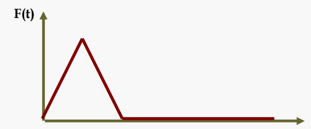

$$m\ddot{x} + c\dot{x} + kx = F(t)$$

## There are 4 categories of F(t) :

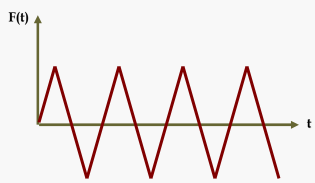
**1) Harmonic (sin, cos)**



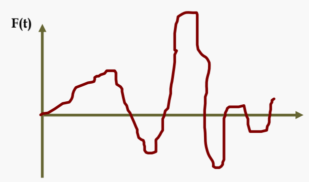

**3) Transient**




**2) Periodic**




**4) Random**



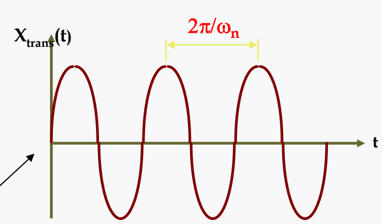



***If F(t) = F$_o$ cos (ωt)*** ⟹ $m\ddot{x} + c\dot{x} + kx = F_o\cos(\omega t)$

***If c = 0*** ⟹ $m\ddot{x} + kx = F_o\cos(\omega t)$

$$x(t) = x_h(t) + x_p(t)$$

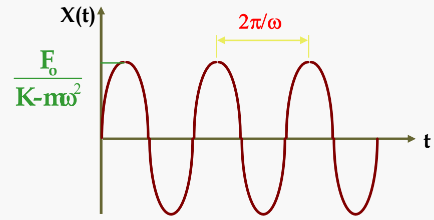
$$\text{or } x(t) = x_{trans}(t) + x_{ss}(t)$$




$$x_{trans}(t) = A\cos(\omega_n t) + B\sin(\omega_n t)$$

To find $x_{ss}$ Let

$$x_{ss}(t) = X\cos(\omega t)$$
$$\dot{x}_{ss}(t) = -\omega X\sin(\omega t)$$
$$\ddot{x}_{ss}(t) = -\omega^2 X\cos(\omega t)$$

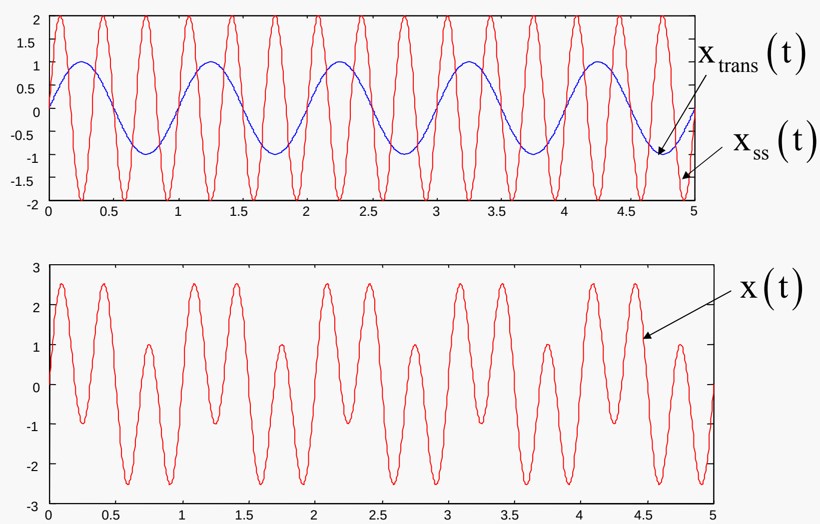



EOM becomes

$$-m\omega^2 X\cos(\omega t) + kX\cos(\omega t) = F_o\cos(\omega t)$$

Solving $X = \dfrac{F_o}{k - m\omega^2}$ $\qquad x_{ss}(t) = \dfrac{F_o}{k - m\omega^2}\cos(\omega t)$





MichiganTech

$$x(t) = A\cos(\omega_n t) + B\sin(\omega_n t) + \frac{F_0}{k-m\omega^2}\cos(\omega t)$$

$$x(0) = A + \frac{F_o}{K-m\omega^2} \quad\Rightarrow\quad A = x(0) - \frac{F_o}{K-m\omega^2}$$

$$\dot{x}(t) = -A\omega_n\sin(\omega_n t) + B\omega_n\cos(\omega_n t) - X\omega\sin(\omega t)$$

$$\dot{x}(0) = \omega_n B \quad\Rightarrow\quad B = \frac{\dot{x}(0)}{\omega_n}$$

$$x(t) = \left(x(0) - \frac{F_o}{k - m\omega^2}\right)\cos(\omega_n t) + \frac{\dot{x}(0)}{\omega_n}\sin(\omega_n t) + \frac{F_o}{k - m\omega^2}\cos(\omega t)$$



MichiganTech

**Case 1:** ***if*** $\omega << \omega_n$

$$\frac{F_o}{k - m\omega^2} = \frac{F_o}{k - \frac{k\omega^2}{\omega_n^2}} \approx \frac{F_o}{k} = \delta_{st}$$

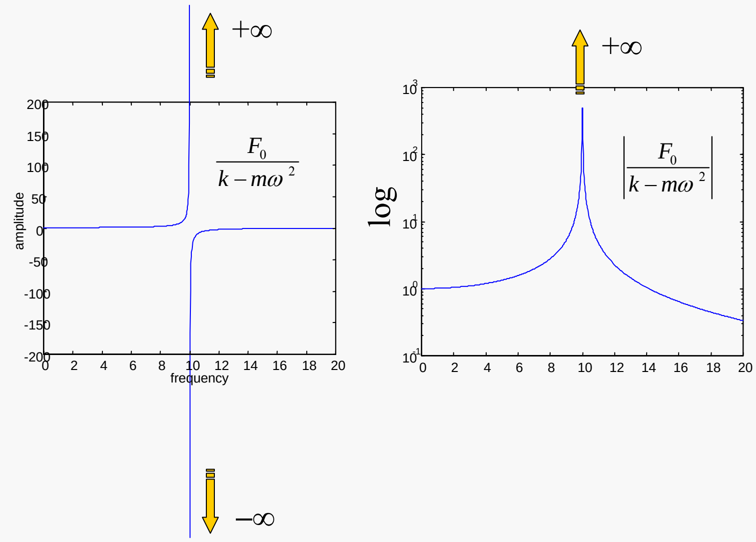
**Case 2:** ***if*** $\omega >> \omega_n$

$$\frac{F_o}{k - m\omega^2} = \frac{F_o}{k - \frac{k\omega^2}{\omega_n^2}} \approx -\frac{F_o}{\frac{k\omega^2}{\omega_n^2}}$$

**Case 3:** ***if*** $\omega = \omega_n$ ***(Resonance!!)***

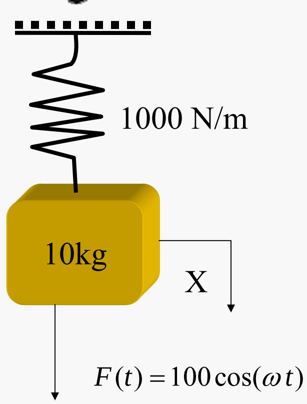
$$\frac{F_o}{k - m\omega^2} = \frac{F_o}{0} \approx \infty \quad\Rightarrow\quad x(t) = \frac{F_o\omega_n t}{2k}\sin(\omega_n t) \Rightarrow$$

using L' Hospital's rule








Example

To Find the Steady State Response

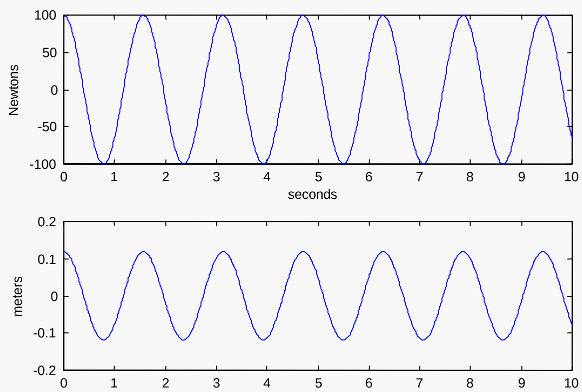
$$x_{ss}(t) = \frac{F_o}{k - m\omega^2}\cos(\omega t)$$

$$x_{ss}(t) = \frac{100}{1000 - 10\omega^2}\cos(\omega t)$$

$$\omega_n = \sqrt{\frac{1000}{10}} = 10 \text{ rad/sec}$$

MichiganTech

## Case 1: *if $\omega \ll \omega_n$*



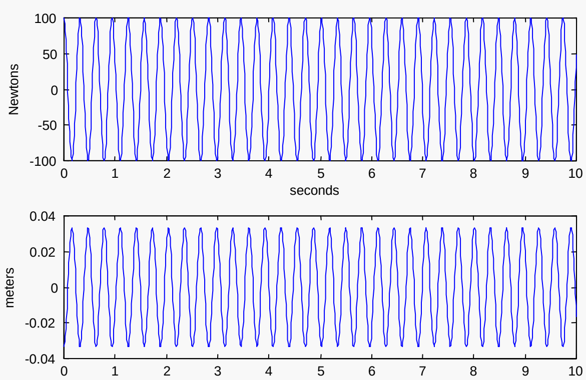

$$F(t) = 100\cos(4t)$$

$$x_{ss}(t) = \frac{100}{840}\cos(4t)$$



MichiganTech

## Case 2: *if $\omega \gg \omega_n$*

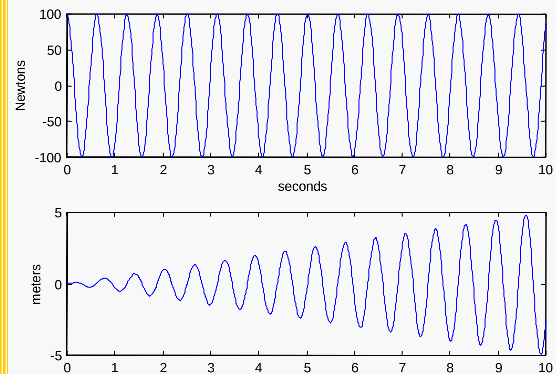



$$F(t) = 100\cos(20t)$$

$$x_{ss}(t) = -\frac{100}{3000}\cos(20t)$$

MichiganTech

## Case 3: *if $\omega = \omega_n$ (Resonance!!)*




$$F(t) = 100\cos(10t)$$

$$x_{ss}(t) = \frac{1000t}{2000}\sin(10t)$$