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Open Book/ Closed Notes

Problem 1

An electric motor weighing 750 lb and running at 1800 rpm is supported on four steel helical springs, each of which has a stiffness of 30.5 lb/in. The rotor of the motor has a weight of 100 lb with its center of mass located at a distance of 0.01 in. from the axis of rotation.

Determine:

- 8 a. The amplitude of the steady-state displacement of the motor
- 7 b. The magnitude of the force transmitted to the foundation
- 5 c. If the motor speed is increased by 20%, will the amplitude of the steady-state displacement change? By how much?
- 5 d. If the motor speed is increased by 20%, will the magnitude of the force transmitted to the foundation change? By how much?

$$M = \frac{W}{g} = \frac{750}{386} = 1.94 \text{ lbm}$$

$$\omega = 1800 \text{ rpm} = 60\pi \text{ r/sec}$$

$$m = \frac{100}{386} = .26 \text{ lbm}$$

$$\omega_n = \sqrt{\frac{k}{M}} = \sqrt{\frac{4(30.5)}{1.94}} = 7.94$$

$$\textcircled{a} \quad r = 23.8, \beta = ? \quad \frac{Mx}{mc} = 1.00 \Rightarrow X = \frac{me}{M} = \frac{(.26)(.01)}{1.94}$$

$$\textcircled{a} \quad X = .0013$$

$$\frac{F_T}{F_0} (23.8, \beta = 0) = .0018$$

$$F_T = me\omega^2 (.0018) = (.26)(.01)(60\pi)^2 (.0018)$$

$$\textcircled{b} \quad F_T = .1663 \text{ lbs}$$

$$\textcircled{c} \quad \text{for } r = 23.8 + 20\%$$

$$\frac{Mx}{me} = 1.0$$

∴ No change in X

$$\text{d) } r = r + .2r = 28.6$$

$$\frac{F_T}{F_0} (28.6, 0) = .0012$$

$$F_T = (.26)(.01)(60\pi + 12\pi)^2 (.0012)$$

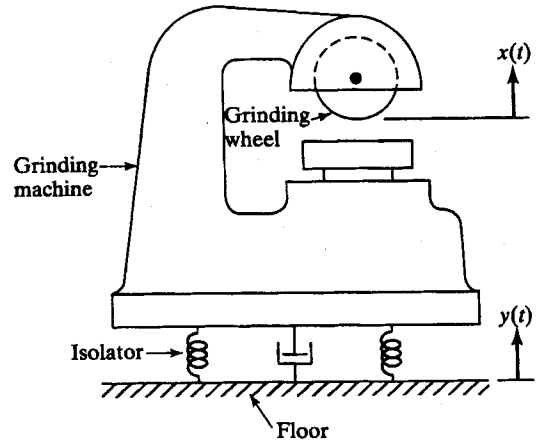
$$F_T = .159$$

$$\text{decrease by } .166 - .159 = \underline{\underline{.007 \text{ lb}}}$$

Problem # 2

A precision grinding machine is supported on two isolators that have a stiffness of 500,000 N/m, each and a damper that has a viscous damping constant of 1000 N-sec/m. The floor, on which the machine is mounted, is subjected to a harmonic disturbance due to the operation of an unbalanced pump in the vicinity of the grinding machine.

- 15
- Find the maximum acceptable displacement amplitude of the floor if the resulting amplitude of motion of the machine is limited to 10^{-6} m. The machine weighs 5000 N.
 - If the amplitude of the floor displacement is twice as large as the value you found in part (a), which of the following techniques can be used, with confidence, to reduce the motion of the machine? Circle all that apply and EXPLAIN.



$$y(t) = Y \sin(50t)$$

- Reduce the mass of the machine ✓
- Increase the mass of the machine ✓ enough
- Reduce the stiffness of the isolator ✓ enough
- Increase the stiffness of the isolator ✓
- Reduce the damping of the isolator
- Increase the damping of the isolator ✓

$$W = 5000 \text{ N} \Rightarrow m = 5000/9.81 = 510 \text{ kg}$$

$$K = 1EG \quad C = 1000$$

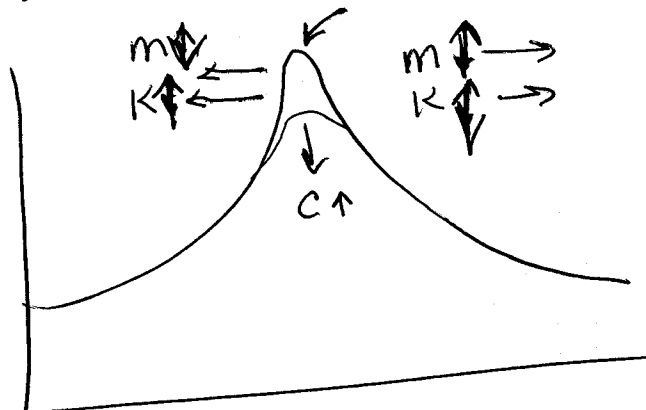
$$\omega_n = \sqrt{\frac{1EG}{510}} = 44.3 \text{ rad/sec}$$

$$r = \frac{\omega}{\omega_n} = \frac{50}{44.3} = 1.129$$

$$\zeta = \frac{1000}{2\sqrt{1EG \cdot 510}} = .0221$$

$$\frac{X}{Y} (1.13, .0221) = 3.6 \Rightarrow Y = \frac{X}{3.6} = 2.78 \times 10^{-6} \text{ m}$$

b.)



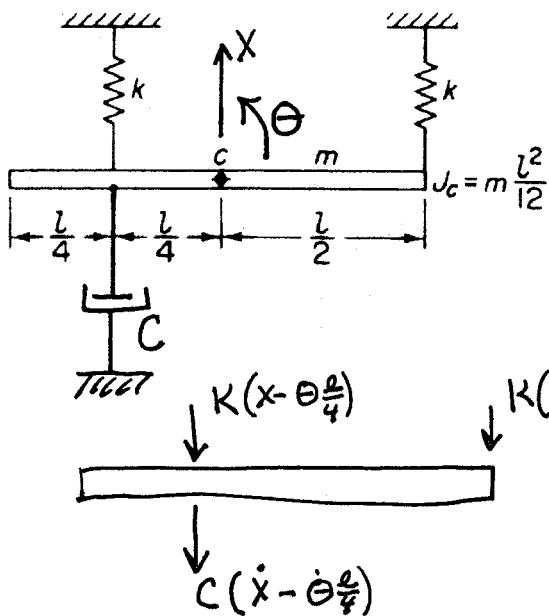
all except (c)

Problem #3

For the system shown below:

1. Determine the equations of motion and express them in matrix form
2. Determine the natural frequencies and associated mode shapes.

Use the coordinates that are shown. The mass of the bar is 2 kg. The stiffness of each spring is 1000 N/m. the length of the bar is 0.5 m. $C = 10$ n-sec/m



$$\sum F_x = m\ddot{x}$$

$$-C(\dot{x} - \dot{\theta}\frac{l}{4}) - K(x - \frac{l}{4}\theta) - K(x + \frac{l}{2}\theta) = m\ddot{x}$$

$$m\ddot{x} + (2K)x + C\dot{x} - C\dot{\theta}\frac{l}{4}$$

$$K(\frac{l}{2}\theta - \frac{l}{4}\theta) = 0$$

Fi

$$\sum M_c = J_c \ddot{\theta}$$

$$K(x - \theta\frac{l}{4})\frac{l}{4} + C(\dot{x} - \dot{\theta}\frac{l}{4})\frac{l}{4} - K(x + \theta\frac{l}{2})\frac{l}{2} = J_c \ddot{\theta}$$

$$J_c \ddot{\theta} + C\frac{l^2}{16}\dot{\theta} - C\dot{x}\frac{l}{4} + K\theta\frac{l^2}{16} - Kx\frac{l}{4}$$

$$+ Kx\frac{l}{2} + K\theta\frac{l^2}{4} = 0$$

$$\begin{bmatrix} m \\ J_c \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} C & -C\frac{l}{4} \\ -C\frac{l}{4} & C\frac{l^2}{16} \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} 2K & +K\frac{l}{4} \\ K\frac{l}{4} & K\frac{5l^2}{16} \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

with numbers

$$\begin{bmatrix} 2 & 0 \\ 0 & .042 \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\theta} \end{Bmatrix} + \begin{bmatrix} 10 & -1.25 \\ -1.25 & .156 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\theta} \end{Bmatrix} + \begin{bmatrix} 2000 & 125 \\ 125 & 78 \end{bmatrix} \begin{Bmatrix} x \\ \theta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad 15$$

eigen solution

$$\lambda_1 = \frac{28}{850} \Rightarrow \omega_1 = 21.2 \text{ rad/sec}$$

$$\lambda_2 = \frac{43}{1854} \Rightarrow \omega_2 = 47 \text{ rad/sec}$$

$$\begin{Bmatrix} 1.00 \\ -1.489 \end{Bmatrix} \quad -3.11$$

$$\begin{Bmatrix} 1.00 \\ -1.67 \end{Bmatrix} \quad 13.66$$

Problem #4

The equations of motion for a MDOF system are given as:

$$\begin{bmatrix} 20 & 0 \\ 0 & 12 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} 2600 & -1400 \\ -1400 & 4000 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 200 \sin(100t) \end{Bmatrix}$$

1. Determine the steady-state amplitude of vibration of each degree of freedom
2. If the frequency of the forcing function is varied, at what frequency will the displacement of X_2 equal zero?

~~$$\begin{bmatrix} 2600 - 20\omega^2 & -1400 \\ -1400 & 4000 - 12\omega^2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 200 \end{Bmatrix}$$~~

$$[Z(\omega)] = \begin{bmatrix} 2600 - 20\omega^2 & -1400 \\ -1400 & 4000 - 12\omega^2 \end{bmatrix}$$

$$\{X\} = [Z]^{-1} \{F\} \quad \{F\} = \begin{Bmatrix} 0 \\ 200 \end{Bmatrix}$$

$$[Z] = \begin{bmatrix} -197400 & -1400 \\ -1400 & -116000 \end{bmatrix}$$

~~$$[Z]^{-1} = \dots$$~~

$$\{X\} = \begin{Bmatrix} 1.22 \times 10^{-5} \\ -0.0017 \end{Bmatrix}$$

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$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \frac{\begin{bmatrix} 4000 - 12\omega^2 & 1400 \\ 1400 & 2600 - 20\omega^2 \end{bmatrix} \begin{Bmatrix} 0 \\ 200 \end{Bmatrix}}{\det [Z]}$$

$$x_2 = \frac{(2600 - 20\omega^2)200}{\det [Z]}$$

$$0 = 2600 - 20\omega^2$$

$$\omega = \sqrt{\frac{2600}{20}} = 11.4 \text{ rad/sec}$$