Energy & Mass

$$\Delta E = \frac{\Delta m \cdot c^2}{g_c}$$

c = speed of light in a vacuum, $2.998 \cdot 10^8$ m/s

$g_c = 1$ kg m/N s^2

$= 1$ g cm^2/erg s^2

$= 32.2$ lbm·ft/lbf·s^2

$= 4.17 \cdot 10^8$ lbm·ft/lbf·h^2

$= 0.965 \cdot 10^{18}$ amu·cm^2/MeV·s^2

$\Delta E\,[J] = 9 \cdot 10^{16}\, \Delta m\,[kg]$

$\Delta E\,[J] = 1.49 \cdot 10^{-10}\, \Delta m\,[amu]$

1 amu = $1.66 \cdot 10^{-27}$ kg

$\Delta E\,[MeV] = 931.5\,[amu]$

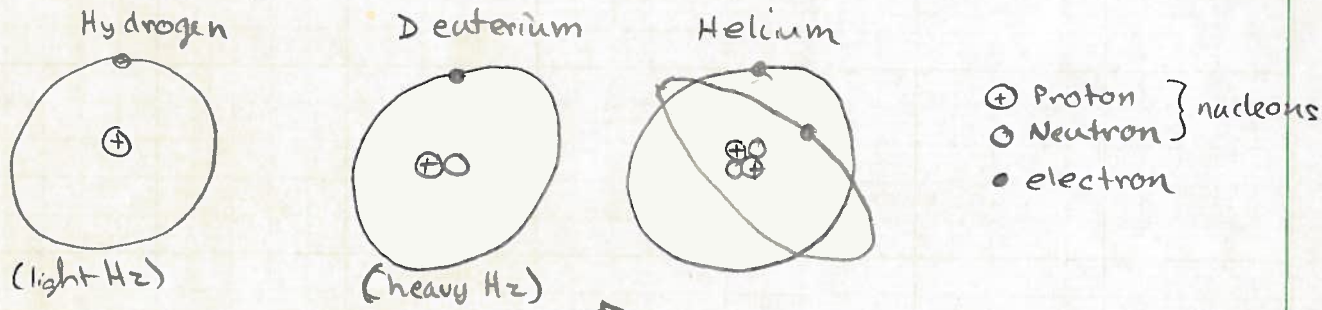
Mass-Energy Conversion Factors (from EW, Table 9-1)

Mass	MeV	J	Btu	kWh	MW·day
amu	931.478	$1.4924 \cdot 10^{-10}$	$1.4145 \cdot 10^{-13}$	$4.1456 \cdot 10^{-17}$	$9.9494 \cdot 10^{-13}$
kg	$5.6094 \cdot 10^{29}$	$8.9873 \cdot 10^{16}$	$8.5184 \cdot 10^{13}$	$2.4965 \cdot 10^{10}$	$5.9916 \cdot 10^{14}$
lbm	$2.5444 \cdot 10^{29}$	$4.0766 \cdot 10^{16}$	$3.8639 \cdot 10^{13}$	$1.1324 \cdot 10^{10}$	$2.7177 \cdot 10^{14}$

example: $5.6094 \cdot 10^{29}$ MeV/kg

Nuclear Energy

$\Delta E = \frac{1}{g_c} \Delta m c^2$



Neutron Mass: 1.008665 amu

Proton Mass: ~~1.007277 amu~~ or

Electron Mass: 0.0005486 ($_1H \equiv 1.007825$

exists in 1 : 6660 H_2

Atomic Mass Unit, amu $\equiv 1.66 \cdot 10^{-27}$ kg $= 3.66 \cdot 10^{-27}$ lbm $= \frac{1}{12} \cdot$ mass of C-12 atom

1 amu $\equiv$ 931.5 MeV $(mc^2) = \frac{1}{AV\#}$

$Z \equiv$ Atomic Number $\rightarrow$ # of protons

$A \equiv$ Mass Number $\rightarrow$ # of nucleons (protons + neutrons)

Nuclear Symbol: $^A_Z X$ ← chemical symbol

Hydrogen: $^1_1 H$

Deuterium: $^2_1 H$

Helium: $^4_2 He$

Z is often dropped since the chemical symbol & the number of protons represent the same information

$^4_2 He \rightarrow$ helium-4, He-4, $_2He^4$

- Chemical & Physical Properties dictated by numbers of protons
- Nuclear Properties dictated by numbers of nucleons $\rightarrow$ isotopes

- Natural Uranium:

99.282 mass % $_{92}U^{238}$
0.712 mass % $_{92}U^{235}$
0.006 mass % $_{92}U^{234}$

} Atomic # is 92

- many isotopes that do not naturally occur in nature appear in the lab or during nuclear reactions:

Uranium has 14 known isotopes ranging from 227-240

Nuclear Particles without any protons:

electron: ${}_{-1}e^0 \rightarrow e^-, \beta^-$ ← Beta particle

nuetron: ${}_0n^1$

positron: ${}_{+1}e^0 \rightarrow e^+, \beta^+$

nuetrino (little neutron): ν ⟶ carries 5% of total energy produced in fission

- mass of ${}_1H^1$ not quite equal to the sum of individual particle masses

isotope	mass
electron, ${}_{-1}e^0$	0.000549
proton, ${}_1p^1$	1.007277
neutron, ${}_0n^1$	1.008665
hydrogen, ${}_1H^1$	1.007825 ⟷ $m_e + m_p = 1.007826$
deuterium, ${}_1H^2$	2.01410 ⟷ $m_e + m_p + m_n = 2.016491$
helium, ${}_2He^4$	4.00260

- mass of ${}_1H^2$ (deuterium) is significantly less than sum of individual particle masses
- difference in mass is known as mass defect (MD)

↑ negative mass "glue" that prevent the Coulomb forces associated with protons from tearing the nucleus apart

- in order to break the nucleus, a minimum energy, equivalent to MD, has to be added to the nucleus

$$MD = Z \cdot m_p + (A-Z)\, m_n - \text{nucleus mass}$$

↑ proton mass ↑ neutron mass

- nucleus mass difficult to measure
- atomic mass is easier to measure, but it includes the orbiting electrons

$$\therefore\ MD = Z \cdot m_{H\text{-}1} + (A-Z)\, m_n - \text{atomic mass}$$

↑ mass of light H2, ${}_1H^1$

Energy Equivalent (mc^2) of MD is the total Binding Energy

- BE ≡ absolute minimum energy required to break a nucleus into Z-protons & (A-Z) neutrons, or A nucleons

Calculate the Binding Energy per nucleon for the following isotopes:

(a) heavy hydrogen, $^{2}_{1}H$

atomic mass = 2.0141 amu (Appendix k)

$Z \equiv$ # of protons = 1

$A \equiv$ # of nucleons = 2

Mass Defect = $(1)\underbrace{1.007825\ \text{amu}}_{\substack{\text{mass of } ^{1}_{1}H \\ \text{(proton + neutron)}}} + (2-1)\underbrace{1.0086625\ \text{amu}}_{\text{mass of neutron}} - \underbrace{2.01411\ \text{amu}}_{\text{mass of } ^{2}_{1}H}$

MD = 0.00239 amu

Binding Energy = 0.00239 amu · 931.5 MeV/amu = 2.226 MeV

BE/nucleon = 1.113 MeV/nucleon

(b) Nickel-59, $^{59}_{28}Ni$ atomic mass = 58.9342 amu

Mass Defect = $1.007825 \frac{\text{amu}}{^{1}_{1}H}(28\ ^{1}_{1}H) + 1.008665 \frac{\text{amu}}{^{1}n}(59-28\ ^{1}n) - 58.9342$ amu

= 0.5535 amu

Binding Energy = 0.5535 amu · 931.5 MeV/amu = 515.60 MeV

BE/nucleon = 8.739 MeV/nucleon

(c) Iron-56, $^{56}_{26}Fe$ atomic mass = 55.934934 amu

MD = $1.007825 \frac{\text{amu}}{^{1}_{1}H}(26\ ^{1}_{1}H) + 1.008665 \frac{\text{amu}}{^{1}n}(56-26\ ^{1}n) - 55.934934$ amu = 0.528461 amu

BE = 492.2614 MeV

BE/nucleon = 8.79 MeV/nucleon

(d) Uranium-238, $^{238}_{92}U$ atomic mass = 238.0289 amu

MD = 1.956090 amu

BE = 1822.0978 MeV

BE/nucleon = 7.6559 MeV/amu

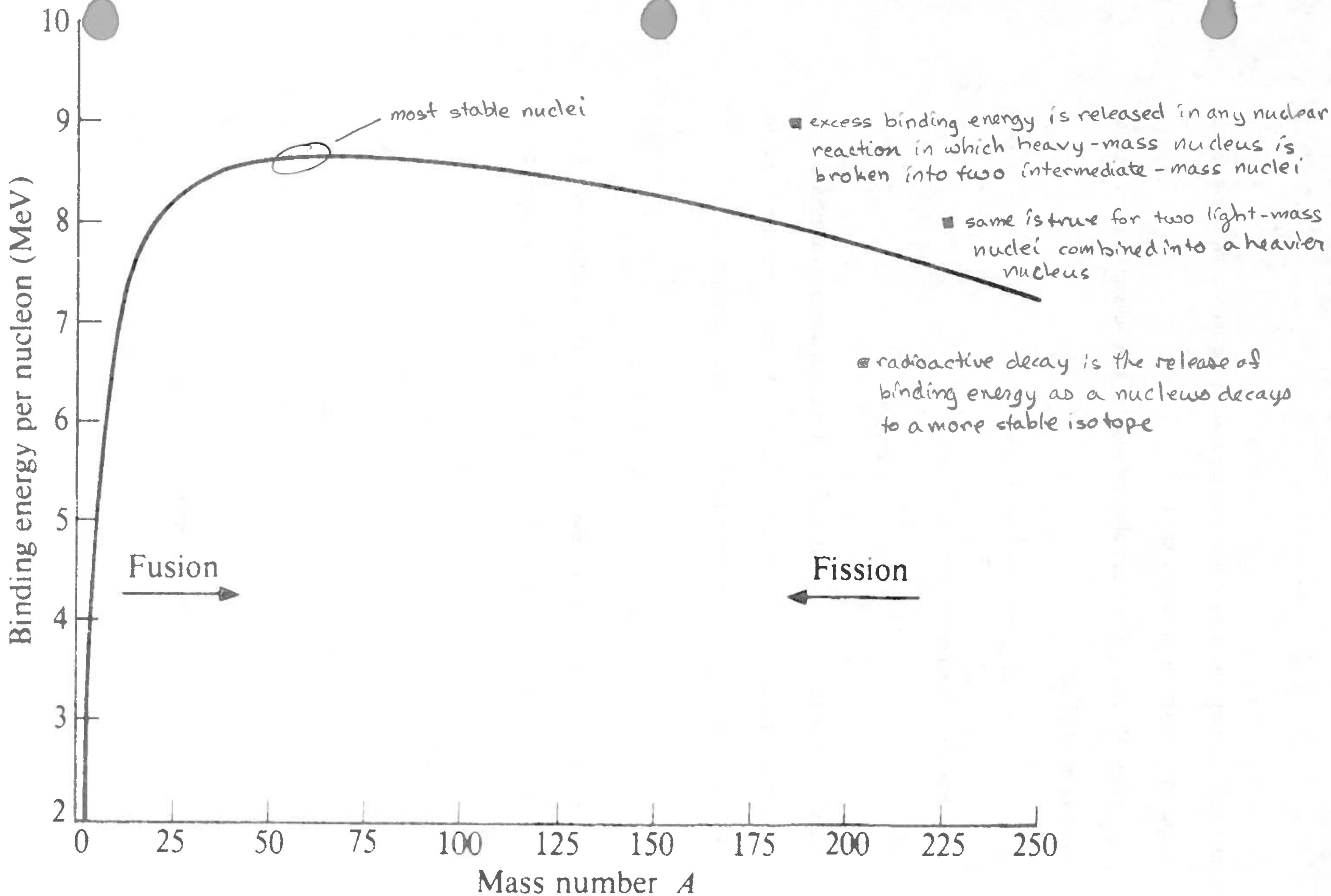


FIGURE 2.7
Variation of the binding energy per nucleon with the atomic mass. (*From Steam: Its Generation and Use, 1972.*)

El-Wakil, Example 9-1

One exothermic reaction occurs when common aluminum is bombarded with high energy α-particles resulting in ^{30}Si (a heavy isotope of silicon; most common isotope is ^{28}Si). During the reaction, a small particle is emitted. Write the complete reaction and calculate the change in mass,

$$^{27}_{13}Al + ^{4}_{2}\alpha \longrightarrow ^{30}_{14}Si + ^{A}_{Z}X$$; X is unknown particle

- mass numbers are conserved in nuclear reactions, mass is not
- sum of mass and energy is conserved in nuclear reactions

$$\left.\begin{aligned} Z &= 13+2-14 = 1 \\ A &= 27+4-30 = 1 \end{aligned}\right\}$$ only particle satisfying this is a proton, $^{1}_{1}H$

Isotope Masses:

Reactants		Products	
^{27}Al	26.98153 amu	^{30}Si	29.97376 amu
^{4}He	4.00260 amu	^{1}H	1.007825 amu
	30.98413 amu		30.98159 amu

- There is a decrease in Mass, $\Delta m = -0.00254$ amu (exothermic)

Radioactive Decay (Radioactivity)

- unstable isotopes are radioactive – radioisotopes
- most naturally occuring isotopes are stable
- radioactivity → spontaneous disintigration
  - ↳ emission of one or more small particles from the "parent" nucleus which changes into the "daughter" nucleus
- radioactivity results in a net mass decrease → exothermic
  - energy released is in the form of
    - (1) kinetic energy
    - (2) γ radiation
    - (3) both (1) & (2)
- naturally occuring radioisotopes emit α, β, and γ particles and radiation.
- artificially generated radioisotopes may emit or undergo α, β, and γ as well as positrons, orbital electron absorption, K-capture, and neutrons, and neutrinos.

- alpha decay, α

α-particle $\rightarrow {}^{4}_{2}\alpha \rightarrow$ Helium-4 nucleus

- 150 isotopes which emit α-particles
- important with respect to power production

$${}^{238}_{92}U \rightarrow {}^{234}_{90}Th + {}^{4}_{2}\alpha + \gamma_d \qquad t_{1/2} = 4.51 \cdot 10^{9} \text{ years}$$

$\uparrow$ γ-radiation

- daughter nucleus has a mass number 4 less than parent nucleus
- half-life, $t_{1/2}$, is time required for half or radioactive atoms to decay
- resulting nucleus may be stable or unstable (radioactive)
- α-particles are monoenergetic, 4-6 MeV $\leftarrow$ very high kinetic energy
- low penetration power & not a biological hazart unless injested

- beta decay, β (negatron) ${}^{0}_{-1}\beta$, ${}^{-}\beta$, β^{-}

β-particle $\rightarrow {}^{0}_{-1}\beta \rightarrow$ electron Total Energy $\sim \frac{1}{3} E_{k\beta} + \frac{2}{3} E_{k\nu}$

- 450 isotopes which emit β-particles
- commonly used in medicine

$${}^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + {}^{0}_{-1}\beta + \gamma_d + \nu \qquad t_{1/2} = 5.3 \text{ years}$$

$\uparrow$ neutrino

- neutron in nucleus changes into a proton and an electron & neutrino are emitted

- gamma decay, γ_d

electromagnetic radiation (photon)

- very short wavelength $\leftrightarrow$ high frequency
- high energy
- similar to x-rays,
  - x-rays $\longrightarrow$ orbital change of electrons
  - γ-rays $\longrightarrow$ from nucleus
- usually accompanies α- and β-decay

- positron decay, β^+ $\quad {}^{0}_{+1}\beta$, ${}^{+}\beta$
  - antielectron

$${}^{55}_{27}Co \rightarrow {}^{55}_{26}Fe + {}^{0}_{+1}\beta + \gamma_d + \nu \qquad t_{1/2} = 18 \text{ days}$$

  - proton changes into a neutron and emits a positron and neutrino
  - annihilation with β^- results in 2 γ_a, each having a <u>rest mass of an electron</u>
    - γ_a travel in opposite direction to conserve momentum
    - annihilation γ-rays
    - 0.51 MeV
    - mass is complety converted into energy
  - reaction is reversible;
    - If a high energy γ-ray ($E_\gamma > 1.02$ MeV) passes near a nucleus, it can be converted into an electron and positron in a γ-ray reaction known as <u>pair</u> <u>production</u>

- k-capture (electron capture)
  - nucleus absorbs one of two nearest orbiting electrons (k-shell) and a proton is converted into a neutron
  - nucleus generally left in an unstable (excited) state

$${}^{57}_{27}Co + {}^{0}_{-1}e \longrightarrow {}^{57}_{26}Fe + \gamma_d + \text{x-rays} \qquad t_{1/2} = 270 \text{ days}$$

produced from orbital electrons falling into lower energy shells

---

<u>Units of Radioactivity</u>

— following Radioactivity —

becquerel (Bq) $\equiv$ 1 disintigration per second

curie (Ci) $\equiv$ decay rate of 1 g of pure radium-266

$= 3.7 \cdot 10^{+10}$ Bq

rutherford $\equiv 10^6$ Bq

Radioactivity

Decay Rate, $\frac{dN}{dt} \sim N$

$N \equiv$ # of radioactive nuclei

$\lambda \equiv$ constant of proportionality
- decay constant

$$-\frac{dN}{dt} = \lambda N$$

-or-

$$\lambda t = \ln\left(\frac{N_0}{N}\right)$$

Half-Life, $t_{1/2}$

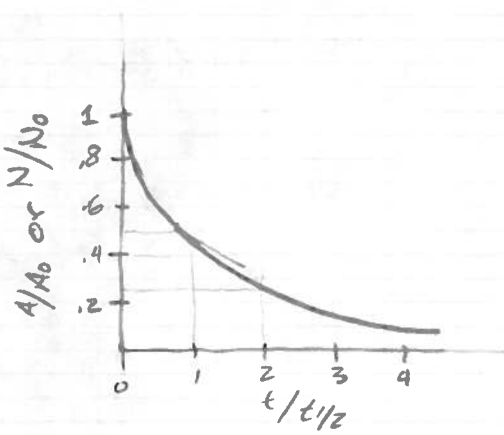
- time required for half of radioactive nuclei to decay

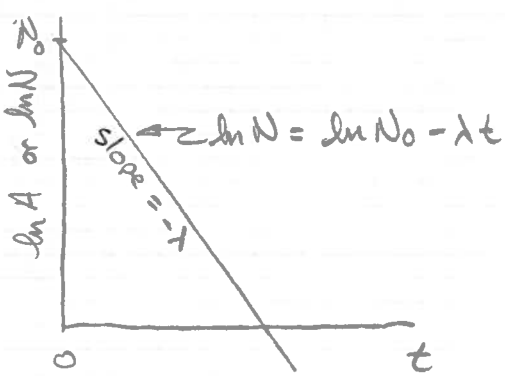
$$N = \frac{N_0}{2} = N_0 e^{-\lambda t_{1/2}} \longleftrightarrow \lambda t_{1/2} = \ln(2)$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Activity $\equiv$ Decay Rate $= -\frac{dN}{dt}$

Activity $\propto N \longrightarrow A = A_0 e^{-\lambda t}$





EW
ex 9.2

Radium-226 decays into Radon gas. Compute (i) the decay constant and (ii) the initial activity of 1 g of Radium-226.

$^{226}_{88}Ra$ 226.0254 amu $t_{1/2} = 1600$ yr α-decay (from Appendix K)

$$\text{Activity} \equiv \text{decay rate} = -\frac{dN}{dt} = \lambda N$$

proportional to number of radioactive nuclei, N, disintigrating in time

decreasing in time

constant of proportionality } decay rate

$$\frac{dN}{dt} = -\lambda N \rightarrow \int \frac{dN}{N} = -\int \lambda dt \rightarrow \ln\left(\frac{N}{N_0}\right) = -\lambda(t - t_0) \rightarrow N = N_0 e^{-\lambda t}$$

0

$$\text{Activity}, A = -\frac{dN}{dt} = \lambda N$$

$\therefore$ A will have the same constant of proportionality ; $A = A_0 e^{-\lambda t}$

$$-\lambda t_{1/2} = \ln\left(\frac{\frac{1}{2}N_0}{N_0}\right) \longrightarrow \lambda = \frac{\ln 2}{t_{1/2}}$$

(i) $\lambda_{Ra\text{-}226} = \frac{\ln 2}{1600\,y} \cdot \frac{1\,yr}{31{,}536{,}000\,s} = 1.37372 \cdot 10^{-11}\ s^{-1}$

(from Appendix B)

The number of radioactive nuclei,

$$\text{mass} = \frac{N_{Ra\text{-}226} \cdot M_{R\text{-}226}}{N_A} = \frac{N_{Ra\text{-}226} \cdot 226.0254\ \frac{g}{mol}}{6.0225 \times 10^{23}\ \text{nuclei/mol}}$$

$$m = 1\,g\ Ra\text{-}226 = N_{Ra\text{-}226}\left(3.753 \times 10^{-22}\ \frac{g\ Ra\text{-}226}{\text{nuclei } Ra\text{-}226}\right)$$

$$N_{Ra\text{-}226} = 2.6645 \times 10^{21}\ \frac{\text{nuclei}}{\text{gram}}$$

The initial Activity,

$$A_0 = \lambda N_0 = \frac{\text{disintigrations}}{s}$$

$$A_0 = \left(1.37372 \cdot 10^{-11}\ s^{-1}\right)\left(2.6645 \cdot 10^{21}\ \frac{\text{nuclei } Ra\text{-}226}{g\ Ra\text{-}226}\right)\left(1\,g\ Ra\text{-}226\right)$$

(ii) $A_0 = 3.6603 \cdot 10^{10}\ \frac{\text{disintigrations } Ra\text{-}226 \text{ nuclei}}{s}$

Determine the activity, in Curies, of the three uranium isotopes found in 100 kg of uranium nitride (U_3N_4) when natural uranium is used.

Activity, $A = \lambda N$

- three naturally-occuring uranium isotopes

$^{234}_{92}U$ 0.006% $t_{1/2} = 2.4\times10^{5}$ y 234.0409 amu

$^{235}_{92}U$ 0.720% $t_{1/2} = 7.1\times10^{8}$ y 235.0439 amu

$^{238}_{92}U$ 99.274% $t_{1/2} = 4.51\times10^{9}$ y 238.0508 amu

} from Appendix K

$$\left.\begin{matrix}\text{molecular mass}\\ \text{of compound}\end{matrix}\right\} \approx \left(3\ \frac{\text{kg mol U}}{\text{kg mol }U_3N_4}\right)\left(238\ \frac{\text{kg U-238}}{\text{kgmol U-238}}\right) + \left(2\ \frac{\text{kg mol }N_2}{\text{kgmol }U_3N_4}\right)\left(28\ \frac{\text{kg }N_2}{\text{kgmol }N_2}\right) = 770\ \frac{\text{kg }U_3N_4}{\text{kg mol }U_3N_4}$$

- If the isotopic masses are used with the mass fractions, then $M_{U_3N_4} = 770.0867\ \frac{\text{kg }U_3N_4}{\text{kg mol }U_3N_4}$

- Number of radioactive nuclei per 100 kg U_3N_4,

$$N = f\cdot\left\{\text{mass }U_3N_4\right\}\left\{\frac{\text{nuclei}}{\text{kgmol}}\right\}\left\{\frac{\text{kgmol U}}{\text{kgmol }U_3N_4}\right\}\Big/\left\{\begin{matrix}\text{molecular mass}\\ U_3N_4\end{matrix}\right\}$$

fraction of each isotope; e.g. $f_{U\text{-}235} = 0.0072\ \frac{\text{kg U-235}}{\text{kg U}}$ ← important because activities of each isotope are very different

$$N = f\left\{\frac{(100\text{ kg }U_3N_4)(6.0225\cdot10^{26}\ \frac{\text{nuclei}}{\text{kg mol}})(3\ \frac{\text{kg mol U}}{\text{kg mol }U_3N_4})}{(770\text{ kg }U_3N_4/\text{ kgmol }U_3N_4)}\right\}$$

$$N = f\cdot(2.336\times10^{26}\text{ nuclei U}) \quad \text{in 100 kg } U_3N_4$$

- Activity, $A = \lambda N$, $\lambda = \frac{\ln 2}{t_{1/2}}$ $\Big\}$ $A = \frac{N\ln 2}{t_{1/2}} = \frac{(2.336\times10^{26}\text{ nuclei U})\cdot f\cdot \ln 2}{t_{1/2}\ (8766\ \frac{hr}{y})(3600\ \frac{s}{hr})}$

$t_{1/2}$ in years

$$A = \left(5.1529\cdot10^{18}\ \frac{\text{disintigrations U}\cdot y}{s}\right)\frac{(f\ {}^{z}U/U)}{t_{1/2}}$$

$^{234}_{92}U$: $A = \left(5.1529\times10^{18}\ \frac{\text{dis U}}{s}\ y\right)\frac{(0.00006\text{ U-234/U})}{(2.4\times10^{5}\ y)} = 1.2517\cdot10^{9}\ \frac{\text{U-234 dis}}{s}$

$^{235}_{92}U$: $A = 0.0523\times10^{9}$ Bq

$^{238}_{92}U$: $A = 1.1342\times10^{9}$ Bq

* $^{234}_{92}U$ only appears because it is in the decay chain of $^{238}_{92}U$. Therefore, the activity of U-234 should be identical to U-238. We donot have enough significant digits in f_{U234} to show this.

* $A_{TOTAL} = (1.2517 + 0.0523 + 1.342)\times10^{9}$ Bq $= 2.4389\cdot10^{9}$ Bq $= 65.92$ mCi

- For $A_{U234} = A_{U\text{-}238}$, $A_{TOTAL} = (2(1.342) + 0.0523)\cdot10^{9}$ Bq $= 2.321\cdot10^{9}$ Bq $= 62.73$ mCi

The activity of 1-g Ra-226 is very small compared to the total number of nuclei (10^{10} disintegrations/s versus 10^{21} nuclei). Therefore, the activity can be treated as a constant. This is true for any isotope with a long half-life.

Early measurements indicated the activity of Ra-226 to be $3.7 \cdot 10^{10} \frac{dis}{s}$.

This value was adopted as a measure of radioactivity.

1 curie, $Ci = 3.7 \cdot 10^{10}$ dis/s

$$\left.\begin{array}{l} 1\ mCi = 10^{-3}\ Ci \\ \text{becquerel, } Bq = 1\ dis/s \end{array}\right\} 1\ Ci = 3.7 \cdot 10^{10}\ Bq$$

rutherford $= 10^6$ Bq

During radioactive decay, the sum of mass and energy must be conserved.
The energy distribution among the product particles can be determined using conservation of momentum.

$$E_{TOTAL} = 931.5 \frac{MeV}{amu} \left\{ \begin{bmatrix} \text{parent} \\ \text{nuclear} \\ \text{mass} \end{bmatrix} - \begin{bmatrix} \text{particle} \\ \text{mass} \end{bmatrix} - \begin{bmatrix} \text{daughter} \\ \text{nuclear} \\ \text{mass} \end{bmatrix} \right\}$$

$$\text{Total kinetic Energy} \equiv E_{KE,\,TOTAL} = E_{TOTAL} - \underbrace{E_{\gamma_0}}_{\substack{\text{total emitted} \\ \underbrace{\gamma\text{-ray energy}}_{\text{experimentally determined}}}} = \underbrace{\tfrac{1}{2} m u^2}_{\text{light nucleus}} + \underbrace{\tfrac{1}{2} M \upsilon^2}_{\text{heavy nucleus}}$$

- neglecting E_{γ_0}
- assume initial $E_{KE} = 0 \longrightarrow$ conservation of momentum requires $mu = M\upsilon$

therefore, $E_{KE} = \frac{1}{2} M \upsilon^2 + \frac{1}{2}\left(\frac{m^2 u^2}{M}\right) = \frac{1}{2} m u^2 \left(1 + \frac{m}{M}\right)$

- similarly,

$$E_{KE} = \frac{1}{2} M \upsilon^2 \left(1 + \frac{M}{m}\right)$$

$$E_{KE,\,heavy} = \frac{1}{2} M \upsilon^2 = \frac{E_{KE}}{1 + \frac{M}{m}}$$

$$E_{KE,\,light} = \frac{1}{2} m u^2 = \frac{E_{KE}}{1 + \frac{m}{M}}$$

$m << M$; thus, the light nucleus carries most of the momentum!

$$\frac{1}{2} M \upsilon^2 \left\{ 1 + \frac{m u^2}{M \upsilon^2} \right\} = \frac{1}{2} M \upsilon^2 \left\{ 1 + \frac{u}{\upsilon} \right\}$$

$$\frac{u}{\upsilon} \sim \frac{M}{m}$$

Uranium undergoes α-decay with emission of a 0.17 MeV gamma ray. What is the kinetic energy of the product nucleus and the α-particle.

$$^{235}_{92}U \longrightarrow \,^{231}_{90}Th + \,^{4}_{2}\alpha + \gamma_d\,(0.17\,MeV)$$

$$\left.\begin{aligned} M_{U\text{-}235} &= 235.0439 \text{ amu} \\ M_{Th\text{-}231} &= 231.0347 \text{ amu} \\ M_{He\text{-}4} &= 4.0026 \text{ amu} \end{aligned}\right\} \text{from Appendix K}$$

Total Kinetic Energy,

$$E_{K_{TOTAL}} = \left(931.5\,\frac{MeV}{amu}\right)\underbrace{\left\{\left(M_{U\text{-}235} - 92m_{e^-}\right) - \left(M_{Th\text{-}231} - 90m_{e^-}\right) - \left(M_{He\text{-}4} - 2m_e\right)\right\}} - \gamma_d$$

Mass Defect; notice the mass of the electrons cancels so we do not have to factor m_{e^-} into the energy balance.

$$E_{K_{TOTAL}} = \left(931.5\,\frac{MeV}{amu}\right)\left(6.6\cdot 10^{-3}\text{ amu}\right) - \left(0.17\,MeV\right) = 5.9779\,MeV$$

$$E_{K_{TOTAL}} = E_{K_{Th\text{-}231}} + E_{K_\alpha} = \tfrac{1}{2}m_{Th231}V_{Th231}^2 + \tfrac{1}{2}m_\alpha V_\alpha^2$$

- Conservation of momentum requires $m_{Th231}V_{Th231} = m_\alpha V_\alpha$

$$E_{K_{TOTAL}} = \frac{1}{2}m_{Th231}V_{Th231}^2\left\{1 + \frac{m_\alpha V_\alpha^2}{m_{Th231}V_{231}^2}\right\} = \frac{1}{2}m_{Th231}V_{231}^2\left\{1 + \frac{m_{Th231}}{m_\alpha}\right\}$$

— OR —

$$E_{K_{TOTAL}} = \frac{1}{2}m_\alpha V_\alpha^2\left\{1 + \frac{m_{Th231}V_{Th231}^2}{m_\alpha V_\alpha^2}\right\} = \frac{1}{2}m_\alpha V_\alpha^2\left\{1 + \frac{m_\alpha}{m_{Th231}}\right\}$$

- Thus, the kinetic energy of each particle can be written in terms of the total kinetic energy,

$$E_{K,Th\text{-}231} = \frac{1}{2}m_{Th231}V_{Th231}^2 = \frac{E_{K_{TOTAL}}}{1 + m_{Th231}/m_\alpha} = \frac{5.9779\,MeV}{1 + 231/4} = 0.102\,MeV$$

$$E_{K,He\text{-}4} = \frac{1}{2}m_\alpha V_\alpha^2 = \frac{E_{K,TOTAL}}{1 + m_\alpha/m_{231}} = \frac{5.9779\,MeV}{1 + 4/231} = 5.876\,MeV$$

inclusion of the electron mass results in a very small error.

Sources of Radioisotopes

2 sources → naturally occuring; including that produced by cosmic radiation
→ manufactured ~~from~~ in fission reactors & particle accelerators

- Long-lived isotopes have existed for billions of years,

$^{87}_{37}Ru$, $t_{1/2} = 5 \cdot 10^{10}$ y, β^- decay

$^{40}_{19}K$, $t_{1/2} = 1.28 \cdot 10^{9}$ y, k/β^- decay ← note 2 decay mechanisms

$^{50}_{23}V$, $t_{1/2} = 6.0 \cdot 10^{15}$ y, k capture

$^{96}_{40}Zr$, $t_{1/2} = 3.6 \cdot 10^{17}$ y, β^- decay

$^{115}_{49}In$, $t_{1/2} = 6.0 \cdot 10^{14}$ y, β^- decay

$^{138}_{57}La$, $t_{1/2} = 1.1 \cdot 10^{11}$ y, $K/\beta^+/\beta^-$

$^{142}_{58}Ce$, $t_{1/2} = 5.0 \cdot 10^{16}$ y, β^-

$^{148}_{62}Sm$, $t_{1/2} = 8.0 \cdot 10^{15}$ y, α

$^{209}_{83}Bi$, $t_{1/2} = 2 \times 10^{18}$ y, α ← only naturally occuring Bi isotope

$^{232}_{90}Th$, $t_{1/2} = 1.41 \cdot 10^{10}$ y, α ← only naturally occuring Th isotope

$^{238}_{92}U$, $t_{1/2} = 4.51 \cdot 10^{9}$ y α

- Most naturally occurring isotopes are produced by the decay of 3 parent isotopes,

$$^{232}_{90}Th \longrightarrow {}^{208}_{82}Pb + 6\left({}^{4}_{2}\alpha\right) + 4\left({}^{0}_{-1}\beta\right)$$

$$^{235}_{92}U \longrightarrow {}^{207}_{82}Pb + 7\left({}^{4}_{2}\alpha\right) + 4\left({}^{0}_{-1}\beta\right)$$

$$^{238}_{92}U \longrightarrow \underbrace{{}^{206}_{82}Pb}_{\text{stable isotopes of lead}} + 8\left({}^{4}_{2}\alpha\right) + 6\left({}^{0}_{-1}\beta\right)$$

← U-238 goes through 14 stages of decay to reach a stable nuclide.
- included in the decay chain are:
  - radium-266, $t_{1/2} = 1690$ y; isolated by M. Curie in 1902
  - radon-222, $t_{1/2} = 3.82$ d; noble gas

- Cosmic radiation ≡ high energy neutrons, protons, nuclei
  - produces isotopes from atoms in atmosphere
    carbon-14 (radiocarbon)
    hydrogen-3 (tritium)

- Manufactured isotopes → from accelerators & fission reactors

$$^{59}_{27}Co + {}^{1}_{0}n \longrightarrow \underbrace{{}^{60}_{27}Co^{*}}_{\text{excited state}} \longrightarrow {}^{60}_{27}Co + \underbrace{\gamma_e}_{\text{capture } \gamma}$$

Radioisotope Thermal Generators (RTG)

$$\underbrace{\begin{Bmatrix}\text{nuclear}\\\text{decay}\end{Bmatrix} \rightarrow \begin{Bmatrix}\text{kinetic}\\\text{energy}\end{Bmatrix}} \rightarrow \begin{Bmatrix}\text{thermal}\\\text{energy}\end{Bmatrix} \xrightarrow{\text{(TEC)}} \begin{Bmatrix}\text{electrical}\\\text{energy}\end{Bmatrix}$$

$$\begin{Bmatrix}\text{thermal}\\\text{energy}\end{Bmatrix} \xrightarrow{\text{stirling engine}} \begin{Bmatrix}\text{mechanical}\\\text{energy}\end{Bmatrix} \rightarrow \begin{Bmatrix}\text{electrical}\\\text{energy}\end{Bmatrix}$$

- energy release in form of
  - decay-gamma (γ_d) energy ← γ_d is very penetrating & probably not captured in RTG; typically neglected in calculations
  - &
  - kinetic energy of particle (α or β^-)

- US: Systems for Auxiliary Power (SNAP)
  - odd #'s → RTG → SNAP-7b
  - even #'s → reactors → SNAP-10a
  - used for:
    - remote weather stations
    - undersea monitoring instruments
    - Apollo lunar missions
    - coast ~~Guard~~ Guard buoys
    - deep-space satellites

- isotopes should have specific power > 1 kW/kg
  - high melting temp.
  - low γ_d emission

  } Appendix L has most viable isotope candidates

  - ~~long opt~~ $10 \le t_{1/2} \le$ ~~300y~~ 300 yr (long lasting & decent activity)

~~Sap~~ SNAP-7b → ^{90}Sr-titanate, $t_{1/2} = 29.1\,yr$

<images>

Specific Power includes daughter decay.

$$A \xrightarrow{\alpha} B \xrightarrow{\alpha} C \xrightarrow{\beta^-} D \xrightarrow{\alpha} E$$

$$\left.\begin{aligned} P_A &= P_{0A} e^{-\lambda_A t} \\ &+ P_B \\ &+ P_C + \dots \end{aligned}\right\} \text{Total Power per kg}$$

Culp Example 2.7

Find the initial mass of plutonium-238 metal (80% Pu-238) that must be used to supply a minimum thermal power of 0.1 kW_{th} for a period of 30 years. Also, calculate the initial excess power that must be "dumped" and the helium-expansion volume that will limit the helium gas pressure to 5 MPa at 400°C.

Pu-238 238.0495 amu $t_{1/2} = 86.0\,y$ α-emitter } Appendix K - Partial List of Isotopes

600°C m.p. $\underbrace{0.45\ kW/kg}_{\text{specific power}}$ $\underbrace{6.80\ kW/liter}_{\text{power density}}$ } Appendix L - Radioisotope Fuels

specific power = 0.45 kW/kg = $\dot{w}$

activity coefficient (decay constant); $\lambda = \frac{\ln 2}{t_{1/2}} = 185.42 \cdot 10^{-12}\ \frac{1}{s}$

- initial thermal power, → find the "present value" power required to have 0.1 kW_{th} after 30 years.

$$P = P_0 e^{-\lambda t} = P_0 e^{-\ln 2 \cdot \frac{t}{t_{1/2}}}$$

$$\hookrightarrow P_0 = P e^{\ln 2 \cdot \frac{30y}{86y}} = (0.1\,kW_{th})(1.2735) = \underline{\underline{0.1274\,kW_{th}}}$$

Therefore, $\boxed{0.0274\,kW_{th} \text{ must be "dumped" initially}}$

- Initial mass of Pu-238 metal required to achieve this power.

$$m_{Pu\text{-}238} = \frac{P_0}{\dot{w}} = \frac{0.1274\,kW_{th}}{0.45\,kW/kg\ Pu\text{-}238} = 0.28301\ kg\ Pu\text{-}238$$

$\boxed{m_{metal} = 0.35376\ kg}$

- Pu-238 is an α-particle emitter as are some of the ~~decay~~ daughter nuclei. The α-particle will pick up 2 electrons and become a He atom (gas). The decay chain for Pu-238 eventually becomes Pb-206, which is stable.

$$^{238}_{94}Pu \longrightarrow {}^{206}_{82}Pb + 8\,{}^{4}_{2}\alpha + 4\,{}^{0}_{-1}\beta + 4\nu + \gamma_d$$

Therefore, each Pu-238 atom produces 8 Helium atoms.

$$m_{He}\ (\text{after 30 years}) = \frac{N_{He} \cdot M_{He}}{N_A}$$

$M_{He} = 4.00260\ \frac{kg}{kg\,mol}$ ← (Appendix K)

$N_A = 6.0225 \times 10^{26}\ \frac{atoms}{kg\,mol}$

$$N_{He} = 8 N_{Pu\text{-}238} = 8\,\frac{(m_{Pu\text{-}238})\,N_A}{M_{Pu\text{-}238}}$$

Assuming all of the Pu-238 decays (worst case scenario),

$$N_{0,Pu\text{-}238} = \frac{(0.28301 \text{ kg Pu-238})(6.0225\times10^{26} \text{ atoms/kgmol})}{(238 \text{ kg/kgmol Pu-238})} = 716.15\times10^{21} \text{ atoms}$$

$$N_{He} = 8\, N_{0,Pu\text{-}238} = 5.729\times10^{24} \text{ atoms He}$$

$$\text{mass of He} = \frac{N_{He}\cdot M_{He}}{N_A} = \frac{(5.729\cdot10^{24} \text{ atoms He})(4.0026 \text{ kg/kgmol He})}{(6.0225\cdot10^{26} \text{ atoms/kgmol})} = 0.03808 \text{ kg He}$$

Using the ideal gas law,

$$V_{He} = \frac{m R_{He} T}{P} = \frac{(0.03808 \text{ kg})(2.0769 \text{ kJ/kg K})(400+273 \text{ K})}{(5000 \text{ kPa})}$$

$$\boxed{V_{He} = 10.645\cdot10^{-3} \text{ m}^3 = 10{,}650 \text{ cm}^3}$$ required to insure $P_{He} \le 5.0$ MPa

Using this volume for He, the pressure after 30 years should be (at 400°C):

$$P = \frac{m R_{He} T}{V_{He}}$$

$$\text{mass of He} = \frac{N_{He} M_{He}}{N_A} = \frac{(8\Delta N_{Pu\text{-}238})\, M_{He}}{N_A}$$

$$N_{Pu\text{-}238} = N_0\, e^{-\ln 2\left(\frac{30}{88}\right)} = (716.15\cdot10^{21} \text{ atoms})(0.78522) = 562.34\cdot10^{21} \text{ atoms}$$

$$\Delta N_{Pu\text{-}238} = 153.81\cdot10^{21} \text{ atoms}$$

$$\text{mass of He after 30 years} = 8.178\cdot10^{-3} \text{ kg He}$$

$$P_{30\text{ years},\,400°C} = \frac{(8.178\cdot10^{-3} \text{ kg})(2.0769 \text{ kJ/kg K})(400+273 \text{ K})}{(10.645\cdot10^{-3} \text{ m}^3)} = \underline{1.074 \text{ MPa}}$$