# Principles of Energy Conversion

## Part 11A. Solar Energy Insolation

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Department of Mechanical Engineering – Engineering Mechanics
Michigan Technological University

# Article 19

# Solar Radiation

## 19.1 Overview and Background

The overall objective of this article is to determine the electromagnetic energy available at the ground level, then determine how much of the electromagnetic energy is converted into thermal energy. The resulting thermal energy may be desired for space heating or power generation, but it may also be undesired by creating an unwanted heat load on a building. Failure to account for solar energy loadings will result in inefficient, energy-wasting HVAC systems or uncomfortable environments. The challenges in achieving this objective are many.

*Challenge 1:* The amount of electromagnetic energy available varies with the wavelength of light. For example, blue light is more energetic than red light. The solar spectrum is non-uniformly distributed across wavelengths. Some wavelengths pass through the Earth's atmosphere easily; other wavelengths do not.

*Challenge 2:* The amount of solar energy available at ground level varies with location on the Earth, time of year, and time of day. A building located in Houghton, Michigan receives a different amount of solar energy than one located in Tuscon, Arizona. The solar energy received by a building in either location in January is significantly different that received in July.

*Challenge 3:* The amount of solar energy absorbed by a surface varies with materials used and surface configuration. The size and shape of the surface will affect the amount of solar energy that can be absorbed. And the materials used (brick vs wood vs glass) will affect the amount of energy absorbed, reflected or transmitted.

*Challenge 4:* The amount of solar energy impinging on a surface will vary with the immediate surroundings. A large glass building adjacent to the surface of interest may block solar energy or double the heating load due to reflection. A line of trees on one side of a building may affect the solar energy loading in the winter and have no affect in the summer.

*Challenge 5:* The amount of solar energy transmitted through a surface varies with what is behind the surface. For example, shades, curtains, and screens will affect the transmittance of solar radiation into the interior of a building.

To address these challenges, we will break down the complexity of this objective into the following subtopics:

- discussion of the nature electromagnetic energy and the solar spectrum;
- determination of how much solar energy gets through the atmosphere, which varies during the day and season;

- determination of how much solar energy is absorbed by a surface; and
- determination of how much solar energy is transmitted through windows and openings in a structure.
- discussion on regulating solar energy transmission through external and internal structures and materials.

### 19.1.1 Translation of Nomenclature

The nomenclature used in solar engineering vary widely with no accepted standard for symbols and units. The nomenclature used in these notes is not the same as in the HVAC textbook. Translation of symbols with those used in the textbook are periodically located throughout the notes.

**Table 19.1.** Translation of nomenclature to that used in McQuiston et al. [2].

| **This Article** | | | | **McQuiston et al. [2]** |
|---|---|---|---|---|
| Beam Insolation on a clear day [W/m$^2$] | $I_B$ | $\equiv$ | $G_{ND}$ | Direct Irradiation on a clear day [W/m$^2$] |
| Beam Insolation normal to a flat surface; $I_{BN} = I_B \cos\theta$ | $I_{BN}$ | $\equiv$ | $G_D$ | Direct Irradiation on a surface of arbitrary orientation; $G_D = G_{ND} \cos\theta$ |
| Diffuse Scattered Insolation | $I_{DS}$ | $\equiv$ | $G_d$ | Diffuse Irradiation |
| Reflected Insolation | $I_R$ | $\equiv$ | $G_R$ | Reflected Irradiation |
| Apparent Extraterrestrial Solar Insolation | $A_S$ | $\equiv$ | $A$ | Apparent Solar Irradiation ($m_a = 0$) |
| Atmospheric Extinction Coefficient | $B_S$ | $\equiv$ | $B$ | Atmospheric Extinction Coefficient |
| Clearness Number | $C_S$ | $\equiv$ | $C$ | Clearness Number |
| Shape Factor; angle between surface and sky for diffusion radiation | $F_{ss}$ | $\equiv$ | $F_{ws}$ | Configuration Factor; angle between surface and sky for diffusion radiation |
| Angle Factor; angle between surface and ground for ground-reflected radiation | $F_{sg}$ | $\equiv$ | $F_{wg}$ | Configuration Factor; angle between surface and ground for ground-reflected radiation |
| Incident Angle; angle between surface normal and Sun's rays | $\theta$ | $\equiv$ | $\theta$ | Angle of Incidence |
| Altitude Angle of Sun; incident angle between beam insolation and horizontal | $\beta_1$ | $\equiv$ | $\beta$ | Solar Altitude Angle; incident angle between beam insolation and horizontal |
| | | $\equiv$ | $\theta_z$ | Solar Zenith Angle; incident angle between beam insolation and vertical $\theta_z = 90 - \beta$ |

**Table 19.1.** (continued)

| **These Notes** | | **McQuiston et al. [2]** |
|---|---|---|
| Solar Azimuth Angle; relative to South, west of due south are positive angles, east of due south are negative angles | $\alpha_1 \equiv \phi$ | Solar Azimuth Angle; relative to North. This choice of defining the solar azimuth angle can cause confusion since it results in $\cos\alpha_1 = -\cos\phi$. The computed value of $\theta$ is the same since the dependency is $\cos(\alpha_1 - \alpha_2)$. |
| Tilt Angle of Surface relative to horizontal | $\beta_2 \equiv \alpha$ | Tilt Angle of Surface |
| Azimuth Angle of Surface Normal relative to South, angles east of south are negative | $\alpha_2 \equiv \psi$ | Surface Azimuth Angle relative to North; $\psi = \alpha_2 + 180$; as with $\phi$, the definition of $\psi$ should be relative to South |
| | $\gamma$ | surface solar azimuth; $\gamma = \lvert\phi - \psi\rvert$ |
| Declination Angle; angle between Earth's equatorial plane and beam insolation | $\delta \equiv \delta$ | Declination Angle |
| Latitude Angle | $L \equiv \ell$ | Latitude Angle |
| Hour Angle; angles east of due south (morning) are negative | $H \equiv h$ | Hour Angle |
| Coordinated Universal Time; 0° longitude (Greenwich Mean Time) | UTC ≡ GCT | Greenwich Civil Time |
| Local Standard Time; depends upon time zone of interest, EST: eastern standard time | EST ≡ EST | Local Standard Time |
| Daylight Savings Time; depends upon time zone of interest, EDT: eastern daylight time | EDT ≡ DST | Daylight Savings Time |
| Mean Solar Time; constant velocity solar time, based on location relative to local meridian; | MST ≡ LCT | Local Civil Time |
| Apparent Solar Time; corrected solar time for variation in velocity of Earth; AST = MST + EOT | AST ≡ LST | Local Solar Time; LST = LCT + EOT |
| Equation of Time; correction to MST; various relationships can be used or look up tabulated values | EOT ≡ EOT | Equation of Time |

## 19.2 Solar Energy

Solar energy is categorized as electromagnetic, which can be described as the kinetic energy of a particle (photon) and as the energy of a wave. The energy of a single photon can be expressed in terms of wavelength or frequency of the electromagnetic wave.

$$E_{\text{photon}} = h\nu_p = \frac{ch}{\lambda_p}, \tag{19.1}$$

where $h$ is Planck's constant ($6.625 \cdot 10^{-34}$ J·s), $\nu_p$ is frequency ($s^{-1}$) and $\lambda_p$ is wavelength (km, m, mm, $\mu$m, nm, or Å). Frequency and wavelength are related by the speed of light $c$ in the material of interest.

$$c = \nu\lambda \tag{19.2}$$

The range of electromagnetic energy is illustrated in Figure 19.1. The upper scale expresses energy as wavelength ($\lambda$) and the bottom scale expresses energy as frequency ($\nu$).

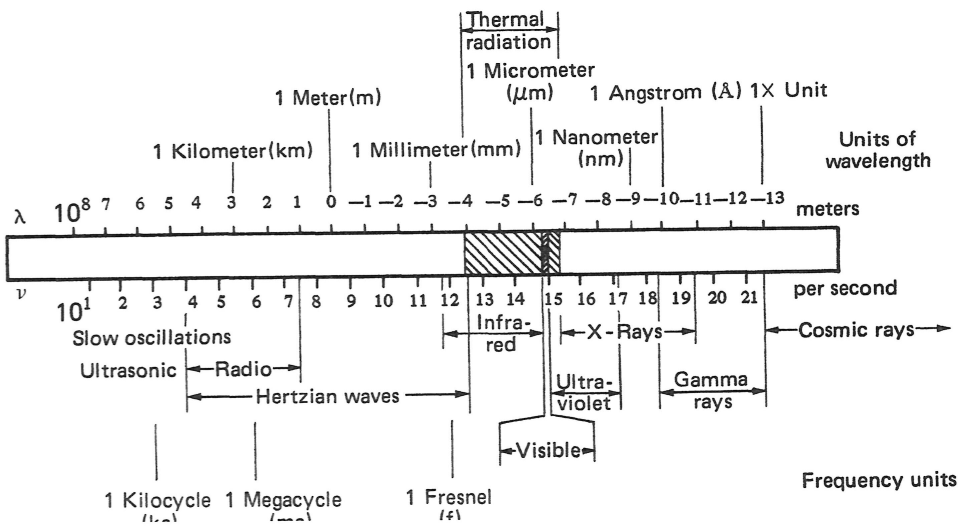



**Figure 19.1.** Electromagnetic spectrum in units of wavelength and frequency. [3]

### 19.2.1 Solar Energy Spectrum

Solar energy received by the Earth does not cover the entire spectrum shown in Figure 19.1. The variation of extraterrestrial solar energy with wavelength is shown in Figure 19.2. The higher energy is now on the left side of the plot; the wavelength axis is opposite that shown in Figure 19.1. The energy flux per wavelength shown in Figure 19.2 is the wavelength-specific energy flux impinging on the atmosphere of the earth. The flux of solar energy [energy/area] is known by a variety of terms: irradiance, radiance, irradiation, insolation, and energy flux.

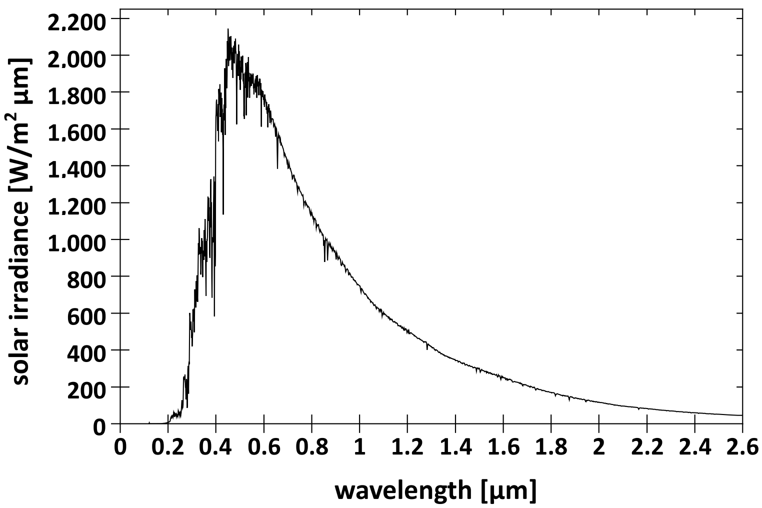



**Figure 19.2.** Extraterrestrial solar spectra. Data from ASTM E-490-00 Standard Extraterrestrial Reference [4].

### 19.2.2 Solar Constant

All of the solar energy lies in a narrow band of the electromagnetic spectrum; from approximately 0.2 $\mu$m to 2.5 $\mu$m. The total extraterrestrial solar energy flux is found by integrating under the wavelength-specific energy flux curve; which is known as the solar constant $S$.

There is no consensus on the value of the solar constant. The annual average value of the solar constant reported by NASA is $S = 1353 \pm 1.6\%\,\mathrm{W/m^2}$; other values reported are 1367 $\mathrm{W/m^2}$ and 1377 $\mathrm{W/m^2}$ [2, 3].

Extraterrestrial solar insolation, $A_S$, varies by approximately ±3.3% during the year as a result of the Earth's elliptical orbit around the Sun. Sunspots and solar bursts also result in variations in extraterrestrial solar insolation. Figure 19.3 plots the average variation in extraterrestrial solar energy with time of year. Tabulated values of $A_S$ are available. Alternatively, a simple engineering relationship for the variation in extraterrestrial solar insolation is:

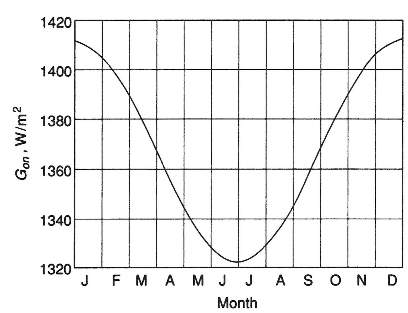



**Figure 19.3.** Variation in the solar constant with time of year [5].

$$A_S(n) = S\left[1 + 0.034\cos\left(\frac{360n}{365}\right)\right], \tag{19.3}$$

where $A_s$ is the daily extraterrestrial solar insolation and $n$ is the day of the year beginning at January 1st.

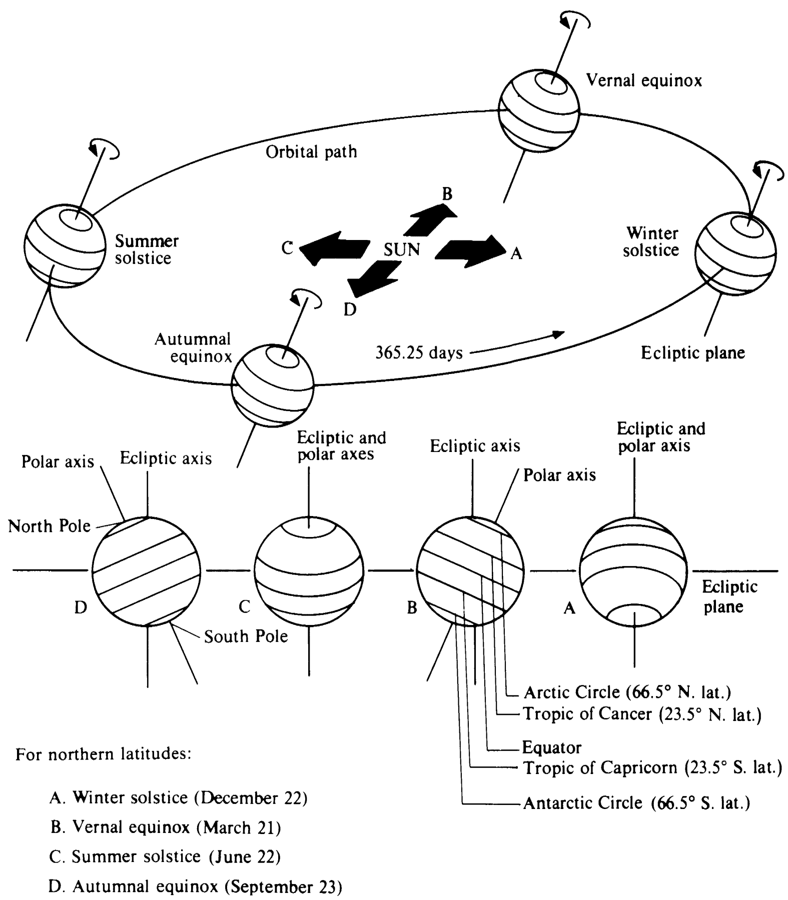



**Figure 19.4.** Earth's orientation with respect to sun [1].

### 19.2.3 Solar Energy and the Atmosphere

The next step is to determine how much of the extraterrestrial solar energy gets to the Earth's surface. As the electromagnetic energy (radiation) passes through the atmosphere a portion of the energy is scattered or absorbed by particles or water drops. Scattering occurs due to air molecules, water vapor, water drops and dust. The absorption and scattering is wavelength dependent.

The spectral transmittance of solar energy through the atmosphere can be seen Figure 19.5. Absorption of solar radiation is due primarily to ozone ($O_3$) in the ultraviolet (0.3 - 0.4 $\mu$m). Below 0.29 $\mu$m there is nearly complete absorption by ozone. Water vapor and carbon dioxide absorb energy in the infrared portion of the spectrum. Water has very strong absorption at 1.0, 1.4, and 1.8 $\mu$m.

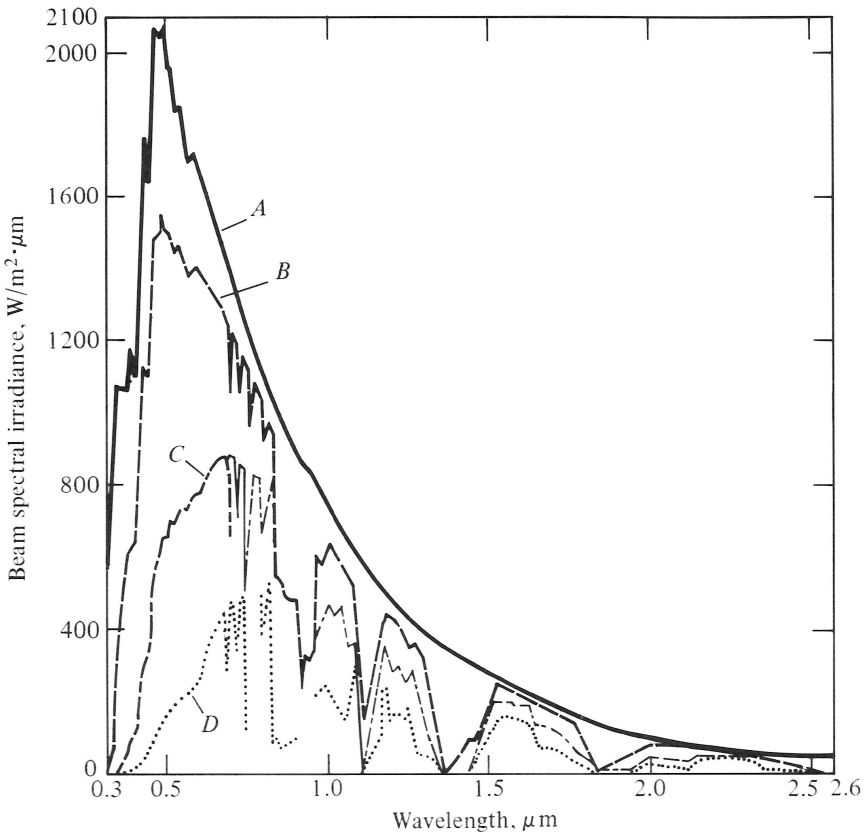



**Figure 19.5.** Solar irradiance attenuation in Earth's atmosphere. (A) Extraterrestrial spectral energy flux [1353 W/m$^2$]. (B) Terrestrial spectral energy flux with $m_a = 1$ [924.9 W/m$^2$] (C) Terrestrial with $m_a = 4$ [528.9 W/m$^2$], (D) Terrestrial $m_a = 10$ [234.5 W/m$^2$]. Curves computed for 20-mm water vapor, 3.4-mm ozone, and clear atmosphere. [6]

### 19.2.4 Transmittance

Multiple curves are shown in Figure 19.5 corresponding to different air mass ratios, which is the thickness of the atmosphere that the solar energy passes through. At solar noon, the Sun is directly south and the atmospheric thickness will be at the minimum for that day. In the early morning and late afternoon the Sun is low on the east and west horizons, respectively, and the thickness of the atmosphere that the solar radiation passes through is at a maximum because of the angle between the Sun's rays and the normal to the Earth's surface. An air mass ratio equal to 1 corresponds to solar noon.

The ratio of solar radiation that passes through the atmosphere to the total extraterrestrial available is known as the *Transmittance*.

$$T = \frac{I_{\text{terrestrial}}}{I_{0,\text{extraterrestrial}}} \tag{19.4}$$

The amount of solar energy flux lost due to reflection, scattering and absorption is proportional to the extraterrestrial solar energy flux and the thickness of the air.

$$I_{\text{lost}} \sim I_0 \Delta x \tag{19.5}$$

At some wavelengths, such as $1.4\,\mu$m, virtually no energy gets through while at other wavelengths nearly all of the energy gets through. Therefore, the amount of energy flux lost is expressed in terms of spectral (wavelength dependent) values.

$$I_{\text{lost},\lambda} = I_{0,\lambda} - I_\lambda(x) = k_\lambda I_{0,\lambda} \Delta x\,, \tag{19.6}$$

where the proportionality constant $k_\lambda$ is known as the *monochromatic extinction coefficient*. The value of $k_\lambda$ depends upon scattering and absorption. For an infinitesimal thickness the transmission of solar energy becomes

$$\frac{dI_\lambda}{I_\lambda} = -k_\lambda dx \quad \Longrightarrow \quad I_\lambda(x) = I_{0,\lambda} e^{-k_\lambda x} \tag{19.7}$$

Equation 19.7 is known as the Beer-Lambert law. The monochromatic transmittance is

$$T_\lambda = e^{-k_\lambda x} \tag{19.8}$$

The total transmittance is found by integrating over all wavelengths.

$$T = \int_0^\lambda T_\lambda d\lambda \tag{19.9}$$

## 19.3 Terrestrial Insolation & Solar Angles

Terrestrial insolation received by a structure or building is generally divided into three components. The first is *normal beam insolation* $I_{BN}$, which is the irradiation directly (line-of-sight) from the sun relative to the surface normal. The second component is *diffuse scattered insolation* $I_{DS}$, which is the irradiation felt when in the shade. The third component is *reflected insolation* $I_R$ due to surrounding structures and the ground.

$$I_{\text{total}} = I_{BN} + I_{DS} + I_R \tag{19.10}$$

All three components are related to the *terrestrial beam insolation* $I_B$; i.e., the solar irradiation that made it through the atmosphere. The first step in determining $I_{\text{total}}$ is to calculate $I_B$.

### 19.3.1 Beam Insolation on Clear Days

Beam Insolation, $I_B$, is the amount of direct solar radiation received through the atmosphere. $I_B$ is determined using the Beer-Lambert law (equation 19.7), $I_B = A_S e^{-B_s m_a}$ once three parameters are established.

1. Extraterrestrial solar insolation, $A_S$, which varies with day of the year due to the elliptical orbit of the earth and cyclic variations in solar activity. Values of $A_S$ are tabulated in §19.9. Alternatively, equation 19.3 can be used to approximate $A_S$.
2. Extinction coefficient, $B_S$, which varies with seasonal changes in water content in the atmosphere. For the purposes here, the extinction coefficient has been integrated across all wavelengths in the solar spectrum. Values of $B_S$ are tabulated in §19.9.
3. Atmospheric path length through which the solar radiation must pass. The atmospheric path length, $m_a$, will vary from dawn to dusk on each day and has to be computed for a specific time.

On any given day the minimum thickness of the atmosphere through which solar radiation passes occurs at solar noon. At this time the air mass ratio is one $m_a = 1$. In the morning and evening, the atmospheric thickness is relatively large $(m_a > 1)$ resulting in greater attenuation of solar irradiation.

Determining $m_a$ is traditionally based on the Ptolemaic view that the sun moves relative to a fixed coordinate system based on the observer's location. The sun's position is measured relative the horizontal plane at the observer's location. This angle is known as the *Altitude Angle* denoted as $\beta_1$. Using analytical geometry the air mass ratio can be related to the altitude angle; $m_a = 1/\sin\beta_1$. The resulting expression for terrestrial beam insolation is

$$I_B = A_S e^{-B_s/\sin\beta_1} \tag{19.11}$$

### 19.3.2 Altitude Angle

The altitude angle $\beta_1$ is the angle between the sun's rays and horizontal at the observer's location. In addition to time of day, this angle varies with location and day of year. For example, at 2:00 eastern standard time the angle between the sun's rays in Houghton, Michigan is not the same as the angle in Chicago, Illinois due to the curvature of the earth. The tilt of Earth's axis relative to the sun changes from January to July which results in the daily altitude angle varying. Therefore, three new angles are necessary to calculate $\beta_1$. These three angles, illustrated in Figure 19.7, are

*Declination Angle*, $\delta$: angle between the earth's equatorial plane and the sun's rays.

*Latitude Angle*, $L$: angle between location and earth's equatorial plane. This angle is the Latitude of the location.

*Hour Angle*, $H$: angle between sun's rays projected on a horizontal plane and due south. In the morning when the sun is east of due south the hour angle is negative. When the sun is west of due south the hour angle is positive.

The geometric relationship between these three angles and the altitude angle is:

$$\sin\beta_1 = \cos L \cos\delta \cos H + \sin L \sin\delta \tag{19.12}$$

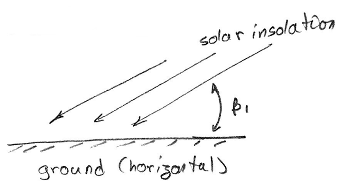



**Figure 19.6.** Altitude Angle, $\beta_1$.

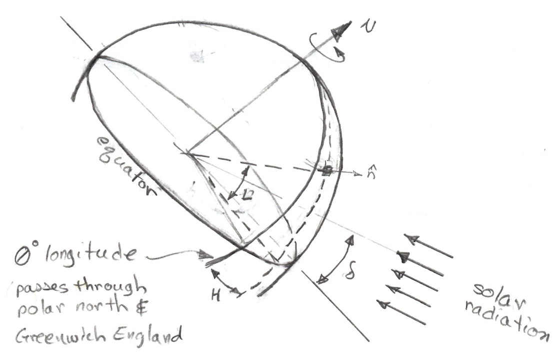



**Figure 19.7.** Declination, Latitude and Hour Angles.

### 19.3.3 Declination Angle

The declination angle is the angle between the sun's rays and the equatorial plane of the earth. Due to the tilt of the earth's axis and the elliptical orbit, the declination angle varies between ±23.45°. The value of the declination angle is tabulated by date in §19.9. A simple engineering approximation is

$$\delta = 23.45^\circ \sin\left[\frac{360}{365}\left(284 + n\right)\right] \tag{19.13}$$

where $1 \leq n \leq 365$.

### 19.3.4 Hour Angle

The hour angle $H$ is measured relative to solar noon; that is when the sun is at the zenith for that particular day. When the sun is east of solar noon the hour angle is negative. In the afternoon when the sun is west of solar noon the hour angle is positive. Modern society works on time measured relative to meridians or time zones and not solar position relative to the zenith. Therefore, local time (i.e., clock time) will need to be converted into solar time.

The earth revolves 360 degrees in a 24-hour period. Each hour increment of rotation is equivalent to 15 degrees, which is used to define meridians. Meridians are imaginary lines running from pole to pole around the earth's surface on 15 degree increments in longitude. Meridians are used to define time zones and are nominally in the center of the geographic time zone. The standard meridian runs from the north pole through Greenwich, England to the south pole and defines Greenwich Mean Time (GMT), which is now known as Coordinated Universal Time (UTC). [1]

| **US Time Zone** | **Meridians** |
|---|---|
| Atlantic | 60° W longitude |
| Eastern US | 75° W |
| Central US | 90° W |
| Mountain | 105° W |
| Pacific | 120° W |
| Yukon | 135° W |
| Alaska-Hawaii | 150° W |

The relation between mean solar time (MST) and local standard time is based on the difference in location and meridian longitudes. There are 4 minutes for each degree of longitude. Thus,

$$\text{MST} = \left\{\begin{matrix}\text{local}\\\text{standard}\\\text{time}\end{matrix}\right\} + \left(\frac{4\text{ min}}{^\circ\text{ longitude}}\right)\left\{\begin{matrix}\text{standard}\\\text{meridian}\\\text{longitude}\end{matrix} - \begin{matrix}\text{local}\\\text{longitude}\end{matrix}\right\} \tag{19.14}$$

During Daylight Savings Time clocks are advanced by one hour so the local standard time must be adjusted.

$$\left\{\begin{matrix}\text{local}\\\text{standard}\\\text{time}\end{matrix}\right\} = \left\{\begin{matrix}\text{local daylight}\\\text{savings time}\end{matrix}\right\} - \left\{\text{ 1 hour }\right\} \tag{19.15}$$

Mean Solar Time (MST) is based on the average velocity of the earth around the sun, but the earth's velocity is not constant due to the orbit and the "wobble" of the earth's axis. A correction to the MST must be applied that provides the *Apparent Solar Time* (AST). The correction is known as the *Equation of Time* (EOT).

$$AST = MST + EOT \tag{19.16}$$

The correction (EOT) varies from +16.3 minutes in November to -14.4 minutes in February. Daily variation in EOT is tabulated in multiple sources. Engineering approximations are also available [3].

[1]Houghton, Michigan is part of the Eastern US time zone with longitude 88.57° W, very close the Central US Meridian.

#### 19.3.4.1 Example: Mean and Apparent Solar Times

Calculate the mean and apparent solar times for Houghton, MI at 12:00 pm (noon) on June 21. On this date, daylight savings time is in effect. Houghton is located at 88.57° W longitude and 47.90° N latitude. On this date the equation of time is EOT = -1.5 min.

$$\text{MST} = \underbrace{\left(12:00\,\text{pm} - 1\,\text{hr}\right)}_{\text{local standard time}} + \underbrace{\left(\frac{4\,\text{min}}{\text{deg. long.}}\right)\left(75^\circ\,\text{W} - 88.57^\circ\,\text{W}\right)}_{\text{time from eastern meridian = -54.28 min}} = 10.095\,\text{am} = 10:05:42\,\text{am}$$

$$\text{AST} = \text{MST} + \text{EOT} = 10:09\,\text{am} + 1.5\,\text{min}\left(\frac{1\,\text{hr}}{60\,\text{min}}\right) = 10.065\,\text{am} = 10:04\,\text{am}$$

Solar noon occurs two hours before noon local daylight savings time.

#### 19.3.4.2 Example: Hour Angle

Calculate the Hour Angle for the example in §19.3.4.1. The apparent solar time (AST) is 10:04 am, which is -116 minutes before east of zenith. The hour angle will be negative.

$$H = -116\,\text{min}\left(\frac{\text{deg. long.}}{4\,\text{min}}\right) = -28.5^\circ$$

#### 19.3.4.3 Example: Altitude Angle

Calculate the Altitude Angle for the example in §19.3.4.1.

Latitude Angle: $L = 47.90^\circ$ (Houghton, MI)
Hour Angle: $H = -28.5^\circ$ (noon EDT, June 21)
Declination Angle: $\delta = +23.45^\circ$ (June 21, table lookup)

$$\sin\beta_1 = \cos L \cos\delta \cos H + \sin L \sin\delta = 0.8358$$

$$\text{Altitude Angle: } \beta_1 = 56.7^\circ$$

#### 19.3.4.4 Example: Terrestrial Beam Insolation

Calculate the Terrestrial Beam Insolation for the location and time in §19.3.4.1. Using a lookup table for June 21.

Apparent Extraterrestrial Solar Insolation: $A_S = 1087\,\text{W/m}^2$
Extinction Coefficient: $B_S = 0.207$

$$I_B = A_S e^{-B_S/\sin\beta_1} = 848.5\,\text{W/m}^2$$

## 19.4 Beam Normal Insolation

Beam insolation, $I_B$, is the amount of solar energy available at ground level, but this is not the amount that will be absorbed by or transmitted through a structure. The amount of solar energy available at a surface if found by integrating the beam insolation over an area:

$$\text{direct solar energy: } I_{BN} = \int \mathbf{I_B} \cdot \mathbf{dA}$$

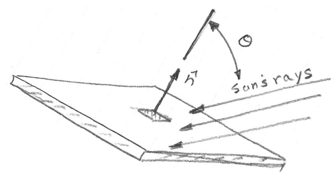



**Figure 19.8.** Relative angle between surface normal and sun's rays.

where $I_{BN}$ is the beam normal insolation. $\mathbf{dA} = \hat{n} dA$ and $\hat{n}$ is the unit vector normal to the surface at incremental area dA as shown in Figure 19.8. Over a flat surface the direction of $\hat{n}$ does not vary and the integral of the dot product reduces to:

$$I_{BN} = I_B \cos\theta \tag{19.17}$$

The angle $\theta$ between the beam insolation (sun's rays) and the surface normal is *Incident Angle*.

In order to determine the incident angle four angles will need to be calculate. Two of the angles, $\alpha_1$ and $\alpha_2$, are related to the orientation with respect to zenith of the sun's rays and surface normal, respectfully. The other two angles, $\beta_1$ and $\beta_2$, are related to the orientation relative to the horizontal plane of the sun and the surface, respectfully. The altitude angle of the sun $\beta_1$ has already been defined in Figure 19.6 and §19.3.2. Using analytical geometry, the incident angle is related to these four angles as

$$\cos\theta = \sin\beta_1 \cos\beta_2 + \cos\beta_1 \sin\beta_2 \cos(\alpha_1 - \alpha_2) \tag{19.18}$$

### 19.4.1 Surface Tilt Angle

The *Tilt Angle* $\beta_2$ is the angle between the surface and horizontal as shown in Figure 19.9. Equivalently, $\beta_2$ is also the angle between the surface normal and vertical. When the surface is horizontal $\beta_2 = 0°$ and $\cos\theta = \sin\beta_1$. This orientation is equivalent to $\theta = 90 - \beta_1$.

### 19.4.2 Solar Azimuth Angle

The sun's position is measured relative to the zenith, which can be measured by projecting the sun's rays onto a horizontal plane at the observer's location. The angle between this projection and due south is the *Solar Azimuth Angle*, illustrated in Figure 19.9 as $\alpha_1$. This angle can be computed using either of the following relationships.

$$\sin\alpha_1 = \frac{\cos\delta \sin H}{\cos\beta_1} \tag{19.19}$$

$$\cos\alpha_1 = \frac{\sin\delta \cos L - \cos\delta \sin L \cos H}{\cos\beta_1} \tag{19.20}$$

### 19.4.3 Surface Azimuth Angle

The angle between the sun's zenith (due south) and the projection of the surface normal onto the horizontal plane is the *Surface Azimuth Angle* denoted as $\alpha_2$. A south-facing wall would have a surface azimuth angle of 0°. Azimuth angles east of south are negative while west of south are positive. The slanted roof shown in Figure 19.9 is pointed towards the southeast which results in $\alpha_2 = -45°$. When the surface is vertical and facing south then $\beta_2 = 90°$ and $\alpha_1 = \alpha_2$ resulting in $\cos\theta = \cos\beta_1$ or $\theta = \beta_1$.

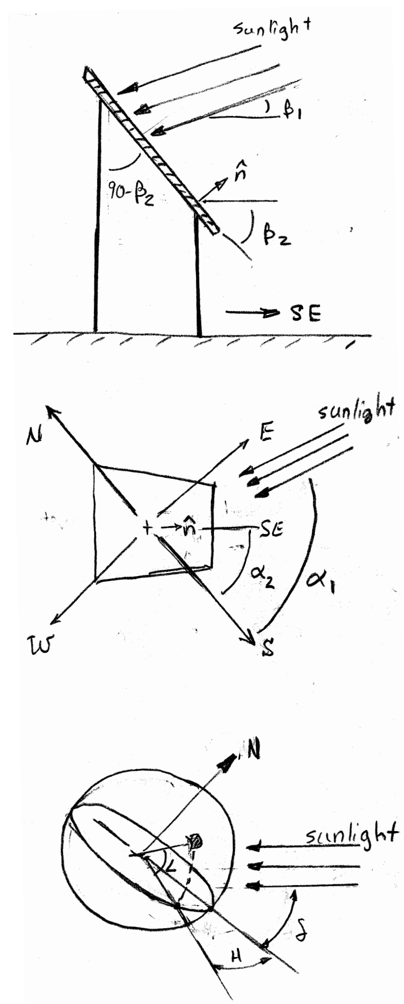



**Figure 19.9.** Solar angles. Top: Side view. Middle: Birds-eye view. Bottom: Global view.

#### 19.4.3.1 Example: Incidence Angle on a Slanted Surface

Calculate the incidence angle $\theta$ on an old, galvanized roof. The roof is inclined at 70° from vertical and facing southeast. The building is located at 38° N latitude, 67° W longitude. It is 9:00 am on July 7.

The latitude angle is $L = 38°$.

To find the hour angle, first determine the mean solar time MST. The nearest standard meridian is 60° so we will assume this building is in the Atlantic time zone. On July 7, the location is assumed to be in daylight savings time.

$$\text{MST} = \left\{ 9{:}00 \text{ am} - 1 \text{ hr} \right\} + \left( \frac{4\,\text{min}}{\text{deg}} \right) \left[ 60° - 67° \right] = 7:32\,\text{am}$$

On this date the equation of time correction is $\text{EOT} = -4.7\,\text{min}$ per §19.9. The apparent solar time is:

$$\text{AST} = \text{MST} + \text{EOT} = 7{:}27\,\text{am}$$

The hour angle $H$ is the number of degrees relative to the zenith. The number of minutes before zenith are:

$$\Delta t = 7:27\,\text{am} - 12:00 = -273\,\text{min}$$

with a corresponding hour angle of:

$$H = \left( \frac{\text{deg. long.}}{4\,\text{min}} \right) \Delta t = -68.25°$$

The declination angle is $\delta = +22.5°$ per §19.9.

Now the solar altitude and azimuth angles can be calculated.

$$\sin\beta_1 = \cos L \cos\delta \cos H + \sin L \sin\delta = 0.5054 \quad \Longrightarrow \quad \beta_1 = 30.36°$$

$$\sin\alpha_1 = \frac{\cos\delta \sin H}{\cos\beta_1} = -0.9945 \quad \Longrightarrow \quad \alpha_1 = -84°$$

The azimuth and tilt angles of the roof are:

$$\beta_2 = 20° \quad \text{(tilt from horizontal)} \qquad \alpha_2 = -45°$$

The incidence angle between the sun's rays and this surface at this time is:

$$\cos\theta = \sin\beta_1 \cos\beta_2 + \cos\beta_1 \sin\beta_2 \cos(\alpha_1 - \alpha_2) = 0.7043 \qquad \theta = 45.7°$$

Thus, the angle between the sun's rays and the surface normal is 45.7°.

## 19.5 Diffuse-Scattered Insolation

There are three components of insolation to consider when determining the heat load on a structure or building due to solar radiation. The first is the beam normal insolation $I_{BN}$. The second is insolation due to the sky radiation; that is, the solar radiation scattered in the atmosphere referred to as *diffuse-scatted insolation* $I_{DS}$. This is the radiation felt when standing in the shade.

Diffuse-scattered insolation is linearly proportional to the sky radiation. The constant of proportionality is a shape factor between the surface and the sky. The shape factor can be approximated as:

$$\text{shape factor: } F_{ss} \approx \frac{1 + \cos\beta_2}{2} \tag{19.21}$$

where $\beta_2$ is the tilt angle of the surface. Sky radiation is proportional to the beam insolation through a *Clearness Number* $C_S$, which varies seasonally. The clearness number is the ratio of measured diffuse-scattered insolation to beam insolation on a horizontal surface where $F_{ss} = 1$; $C_S = I_{DS}/I_B$. Tabulated values of $C_S$ are provided in §19.9.

For an arbitrary surface orientation, therefore, diffuse-scatted insolation can be computed as

$$I_{DS} = I_B C_S F_{ss} \tag{19.22}$$

## 19.6 Reflected Insolation

The ground-reflected insolation occurs due to the total insolation on a horizontal surface multiplied by a ground reflectivity $\rho_g$ and an *Angle Factor* between the surface and the ground $F_{sg}$.

$$I_R = (I_B + I_{DS})\,\rho_g F_{sg} \tag{19.23}$$

The angle factor can be approximated as

$$\text{angle factor: } F_{sg} \approx \frac{1 - \cos\beta_2}{2} \tag{19.24}$$

where $\beta_2$ is the tilt angle of the surface. Reflectivity for a grassy surface is approximately $\rho_g = 0.2$ and for snow covered ground is approximately $\rho_g = 0.8$ [3].

## 19.7 Total Insolation

The total insolation on a surface is the sum of the beam normal, diffuse-scatted and reflected insolation.

$$I_{\text{total}} = I_{BN} + I_{DS} + I_R \tag{19.25}$$

All three components are dependent on the beam insolation $I_B$. Combining equations 19.11, 19.22, and 19.23:

$$I_{\text{total}} = I_B \left\{ \cos\theta \frac{1}{2} \left[ C_S \left( 1 + \cos\beta_2 \right) + \rho_g \left( 1 - \cos\beta_2 \right) \right] + \frac{1}{4} C_S \rho_g \sin^2\beta_2 \right\} \tag{19.26}$$

#### 19.7.0.1 Example: Total Insolation

Calculate the total insolation on a clear day for the previous example.

First, determine the beam insolation. The solar constants for July 7 are (see §19.9):

Extraterrestrial Insolation: $A_S = 1086.6\,\mathrm{W/m^2}$
Extinction Coefficient: $B_S = 0.206$
Clearness Number: $C_S = 0.135$

Beam Insolation: $I_B = A_S e^{-B_S/\sin\beta_1} = 722.9\,\mathrm{W/m^2}$

Beam Normal Insolation: $I_{BN} = I_B \cos\theta = 504.9\,\mathrm{W/m^2}$

The shape factor for the surface-to-sky view is $F_{ss} = \dfrac{1+\cos\beta_2}{2} = 0.97$.

Diffuse-Scattered Insolation: $I_{DS} = I_B C_S F_{ss} = 94.6\,\mathrm{W/m^2}$

Assume the surrounding ground is grassy with a reflectance of $\rho_g = 0.2$.

The angle factor for the surface-to-ground view is $F_{sg} = \dfrac{1-\cos\beta_2}{2} = 0.03$.

Reflected Insolation: $I_R = (I_B + I_{DS})\,\rho_g F_{sg} = 4.9\,\mathrm{W/m^2}$. The reflected insolation is very small for this surface. The reflected component would be much greater for a vertical surface.

Thus, the total insolation on the roof is:

$$I_{\text{total}} = I_{BN} + I_{DS} + I_R = 504.9 + 94.6 + 4.9 = 604.4\,\mathrm{W/m^2}$$

#### 19.7.0.2 Example: Heat Load

In the previous example, the total insolation on an old galvanized roof was computed. Not all of the insolation will be absorbed by the roof. Some will be reflected. The electromagnetic energy absorbed is equal to:

$$I_{\text{absorbed}} = \alpha I_{\text{total}}\,,$$

where $\alpha$ is the absorptivity. For most surfaces, the absorptivity $\alpha$ is the same as the emissivity $\epsilon$. These values are tabulated in numerous sources. For dirty, galvanized iron the emissivity is between 0.87 and 0.91 [1]. Using a value of $\epsilon = 0.89$, the total energy flux absorbed (rate of energy accumulation) by the roof is 537.9 W/m$^2$.

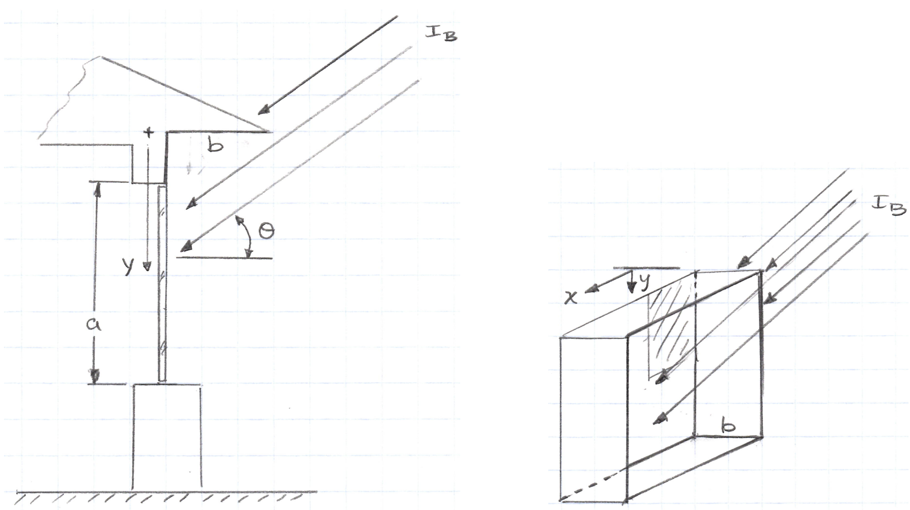
## 19.8 Shading

An old practice for managing heating loads on buildings is to design the roof overhang such that the windows are blocked from the Sun's rays during summer months, but allow the sunlight to enter in the winter due to the smaller altitude angle. The portion of the window that is shaded depends on the altitude and azimuthal angles. Based on the schematic shown in Figure 19.10a, the vertical distanced shaded is:

$$y = b\frac{\tan\beta_1}{\cos(\alpha_1 - \alpha_2)} \tag{19.27}$$

Similarly, if the window is inset with side blockages similar to the overhang, then the horizontal distance shaded is:

$$x = b\tan(\alpha_1 - \alpha_2) \tag{19.28}$$

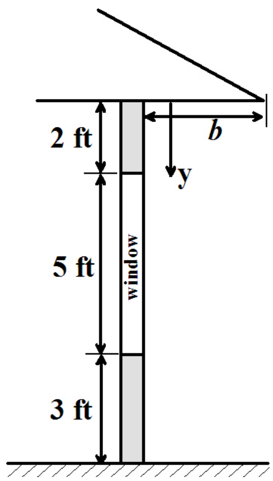



**(a)** Vertical Shading **(b)** Horizontal Shading

**Figure 19.10.** Shading of a window from an overhang.

#### 19.8.0.1 Example: Sizing Window Overhang

Using the ASHRAE Clear Sky Model, compute the following for a south facing window in Minneapolis, MN. The time is solar noon on June 21. A paved parking surface is located just south of the building with a ground reflectivity of 0.35.




(a) The altitude and incident angles of the solar insolation.
(b) The size of the overhang, $b$, so that 90% of the window is shaded vertically.
(c) The total insolation ($I_{BN} + I_{DS} + I_R$).

The following data (§19.9) is for June 21 in Minneapolis, MN.

| | |
|---|---|
| $L = 45°$ | $A_s = 1087\,\text{W/m}^2$ |
| $\delta = 23.45°$ | $B_s = 0.205$ |
| EOT $=$ -1.5 min | $C_s = 0.134$ |
| $H = 0°$ (solar noon) | $n_{\text{day}} = 172$ |

(a) The solar altitude angle, $\beta_1$, is equal to:

$$\sin\beta_1 = \cos L \cos\delta\cos H + \sin L \sin\delta = 0.93 \qquad \Longrightarrow \qquad \beta_1 = 68.45°$$

solar azimuth, $\alpha_1$: $\sin\alpha_1 = \dfrac{\cos\delta\sin H}{\cos\beta_1} = 0$ (solar noon)

surface tilt: $\beta_2 = 90°$ (vertical surface)

surface azimuth: $\alpha_2 = 0°$ (facing south)

The solar incident angle, $\theta$, is:

$$\cos\theta = \sin\beta_1 \underbrace{\cos\beta_2}_{=0} + \underbrace{\sin\beta_2}_{=1} \cos\beta_1 \underbrace{\cos(\alpha_1 - \alpha_2)}_{=1}$$

$$\text{Thus, } \cos\theta = \cos\beta_1 \qquad \Longrightarrow \qquad \theta = 68.45°$$

(b) The vertical shaded distance is 90% of the window, which corresponds to:

$$y = 2\,\text{ft} + 0.90 \cdot 5\,\text{ft} = 7.5\,\text{ft}$$

$$y = b\,\frac{\tan\beta_1}{\cos(\alpha_1 - \alpha_2)} \qquad \Longrightarrow \qquad b = 2.6\,\text{ft}$$

(c) The total insolation is the sum of the beam normal, diffuse scattered, and reflected components:

beam, $I_B$:

$$I_B = A_S e^{-B_s/\sin\beta_1} = 872.0\,\mathrm{W/m^2}$$

beam normal, $I_{BN}$:

$$I_{BN} = I_B \cos\theta = 320.3\,\mathrm{W/m^2}$$

diffuse scattered, $I_{DS}$:

$$I_{DS} = I_\mathrm{B} C_s F_{ss} \quad \text{and} \quad F_{ss} = \frac{1}{2}(1 + \cos\beta_2) = \frac{1}{2} \quad \Longrightarrow \quad I_{DS} = 58.4\,\mathrm{W/m^2}$$

reflected, $I_R$:

$$I_R = (I_B + I_{DS})\,\rho_g F_{sg} \quad \text{and} \quad F_{sg} = \frac{1}{2}(1 - \cos\beta_2) = \frac{1}{2} \Longrightarrow I_R = 162.8\,\mathrm{W/m^2}$$

total insolation on vertical surface, $I_{\text{total}}$:

$$I_{\text{total}} = I_{BN} + I_{DS} + I_R = 541.5\,\mathrm{W/m^2}$$

## 19.9 Solar Constants for Northern Latitudes [1]

| Date | Angle of declination $\delta$ | Equation of time, min | $A_S$,† kW/m²‡ | $B_S$,† am§ | $C_S$† | Day of year |
|---|---|---|---|---|---|---|
| January | | | | | | |
| 1 | −23.0 | −3.6 | 1231 | 0.142 | 0.057 | 1 |
| 11 | −21.7 | −8.0 | 1230 | 0.142 | 0.058 | 11 |
| 21 | −19.6 | −11.4 | 1229 | 0.142 | 0.058 | 21 |
| 31 | −17.3 | −13.5 | 1224 | 0.142 | 0.058 | 31 |
| February | | | | | | |
| 1 | −17.0 | −13.6 | 1223 | 0.143 | 0.059 | 32 |
| 11 | −13.9 | −14.4 | 1218 | 0.143 | 0.059 | 42 |
| 21 | −10.4 | −13.8 | 1213 | 0.144 | 0.060 | 52 |
| 28 | −7.6 | −12.6 | 1206 | 0.147 | 0.063 | 59 |
| March | | | | | | |
| 1 | −7.4 | −12.5 | 1205 | 0.148 | 0.063 | 60 |
| 11 | −3.5 | −10.2 | 1195 | 0.152 | 0.067 | 70 |
| 21 | 0.0 | −7.4 | 1185 | 0.156 | 0.071 | 80 |
| 31 | 4.3 | −4.3 | 1169 | 0.164 | 0.079 | 90 |
| April | | | | | | |
| 1 | 4.7 | −4.0 | 1167 | 0.165 | 0.080 | 91 |
| 11 | 8.5 | −1.1 | 1151 | 0.172 | 0.088 | 101 |
| 21 | 12.0 | 1.2 | 1135 | 0.180 | 0.096 | 111 |
| 30 | 14.9 | 2.8 | 1126 | 0.185 | 0.104 | 120 |
| May | | | | | | |
| 1 | 15.2 | 2.9 | 1125 | 0.186 | 0.105 | 121 |
| 11 | 17.9 | 3.7 | 1114 | 0.191 | 0.113 | 131 |
| 21 | 20.3 | 3.6 | 1103 | 0.196 | 0.121 | 141 |
| 31 | 22.0 | 2.5 | 1098 | 0.199 | 0.125 | 151 |
| June | | | | | | |
| 1 | 22.1 | 2.4 | 1097 | 0.199 | 0.126 | 152 |
| 11 | 23.1 | 0.6 | 1092 | 0.202 | 0.130 | 162 |
| 21 | 23.45 | −1.5 | 1087 | 0.205 | 0.134 | 172 |
| 30 | 23.1 | −3.4 | 1086 | 0.206 | 0.135 | 181 |
| July | | | | | | |
| 1 | 23.1 | −3.4 | 1086 | 0.206 | 0.135 | 182 |
| 11 | 22.1 | −5.6 | 1085 | 0.206 | 0.135 | 192 |
| 21 | 20.6 | −6.2 | 1084 | 0.207 | 0.136 | 202 |
| 31 | 18.1 | −6.2 | 1091 | 0.205 | 0.132 | 212 |
| August | | | | | | |
| 1 | 17.9 | −6.2 | 1092 | 0.205 | 0.132 | 213 |
| 11 | 15.2 | −5.1 | 1099 | 0.203 | 0.126 | 223 |
| 21 | 12.0 | −3.1 | 1106 | 0.201 | 0.122 | 233 |
| 31 | 8.7 | −1.5 | 1120 | 0.193 | 0.113 | 243 |
| September | | | | | | |
| 1 | 8.2 | 0.0 | 1122 | 0.192 | 0.112 | 244 |
| 11 | 4.4 | 3.3 | 1136 | 0.185 | 0.097 | 254 |
| 21 | 0.0 | 6.8 | 1150 | 0.177 | 0.092 | 264 |
| 30 | −2.9 | 9.9 | 1162 | 0.172 | 0.086 | 273 |
| October | | | | | | |
| 1 | −3.3 | 10.2 | 1164 | 0.171 | 0.086 | 274 |
| 11 | −7.2 | 13.1 | 1177 | 0.166 | 0.079 | 284 |
| 21 | −10.8 | 15.3 | 1191 | 0.160 | 0.073 | 294 |
| 31 | −14.3 | 16.2 | 1200 | 0.157 | 0.070 | 304 |
| November | | | | | | |
| 1 | −14.6 | 16.3 | 1201 | 0.156 | 0.069 | 305 |
| 11 | −17.5 | 15.9 | 1211 | 0.153 | 0.066 | 315 |
| 21 | −20.0 | 15.1 | 1220 | 0.149 | 0.063 | 325 |
| 30 | −21.7 | 11.3 | 1224 | 0.147 | 0.061 | 334 |
| December | | | | | | |
| 1 | −21.9 | 11.0 | 1224 | 0.147 | 0.061 | 335 |
| 11 | −23.0 | 6.8 | 1228 | 0.144 | 0.059 | 345 |
| 21 | −23.45 | 2.0 | 1232 | 0.142 | 0.057 | 355 |
| 31 | −23.1 | −3.1 | 1231 | 0.142 | 0.057 | 365 |

† $A$ is the apparent solar irradiation at air mass zero (outside the earth's atmosphere).
$B$ is the atmospheric extinction coefficient, in units of (standard atmosphere)$^{-1}$.
$C$ is the ratio of the diffuse to direct normal irradiation on a horizontal surface.
‡ 1 W/m² = 0.3173 Btu/h · ft².
§ am = units of standard air mass.
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