

Transitional forms of Energy: Mechanical → work } work is the more valuable commodity.
 Thermal → Heat }

- Work can be completely and continuously converted to heat.
- Opposite is not true.
- Portion of heat that cannot be converted to work is unavailable energy.
 That "unavailable energy" has to be rejected as "low-grade" heat.

⊕ Energy is conserved, but "availability" is not.

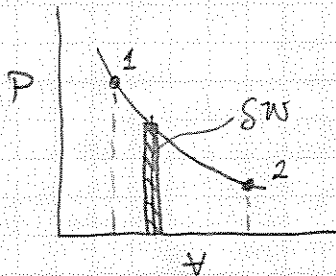
Entropy

- introduced & named by Clausius in 1865
- entropy is a property
- units of Btu/°R, J/K
 Btu/°R lbm, J/kg K for specific enthalpy

Consider Non-Flow Work (gas expansion against a piston)

$${}_1W_2 = \int_1^2 P dV$$

graphically, the total work is the area under the curve.



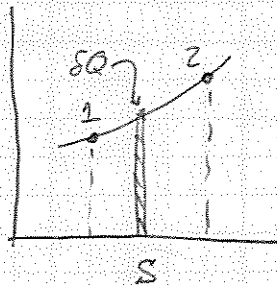
It would be convenient to have a similar graph where the area under the curve represents the heat transfer during a process.

For the work above, the pressure ^{difference} is the driving potential.

For heat, temperature ^{difference} is the driving potential.

$${}_1Q_2|_{rev} = \int_1^2 T dS$$

$${}_1Q_2 \leq \int_1^2 T dS$$



graphically, the total heat transfer for the process is the area under the curve
 Caveats: the process must be reversible

Reversible, Adiabatic Process : $Q = 0$

$$Q_2 = \int_1^2 T ds = 0$$

since $T \neq 0$, $\int_1^2 ds = s_2 - s_1 = 0 \rightarrow s \equiv \text{constant} \equiv \text{isentropic}$

isentropic ($\Delta s = 0$) \Leftrightarrow reversible, adiabatic process

Definitions:

entropy \equiv the property which remains constant in an adiabatic, reversible process

temperature \equiv the property which remains constant in an isothermal process.

TdS - Gibbs Relationships

The change in entropy, S , during an energy conversion process can be a useful measure of the efficiency of the process. While the First Law of Thermodynamics observes that energy is not created or destroyed during conversion, there is nearly always a portion of energy converted that becomes unusable; otherwise known as unavailable. The fraction of available energy to unavailable energy during a conversion process is a measure of conversion efficiency. No energy is lost even though the term lost work is often used to define efficiency. We merely observe that a fraction of energy converted appears impossible to recover. Therefore, determining the entropy change during a conversion process can assist with measuring total efficiency and optimizing energy conversion systems. In order to determine a change in entropy, we must be able to relate ΔS to other measurable properties.

Consider a closed system on which reversible heat transfer and reversible work are applied to affect an energy change.

Applying the 1st Law to this process:

$$\delta Q_{REV} - \delta W_{REV} = dE$$

• for a closed system, $dE = dU$

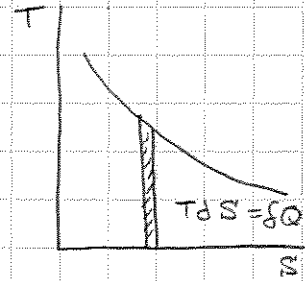
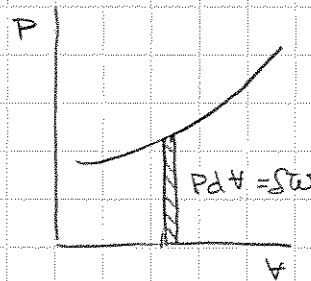
$$TdS - PdV = dU$$

• on a per mass basis: $Tds = du + PdV$ ← Gibbs Equation

Integrating the Gibbs equation will relate changes in entropy, ΔS , to pressure, temperature and volume; provided a property relationship exists for internal energy. The Gibbs equation can also be expressed in terms of enthalpy changes.

$$h = u + PV \rightarrow dh = d(u + PV) = \{ du + PdV \} + VdP$$

$$Tds = dh - VdP$$



Entropy Change in an Incompressible Substance (liquid or solid)

Incompressibility implies that $dV=0$, or $\Delta V \approx 0$. Pressure and Temperature changes do not affect specific volume or density. Also, the change in internal energy of an incompressible substance is $du = c dT$. Therefore, the Gibbs equation can be simplified to

$$T ds = du + P dV = c dT$$

-OR-

$$ds = c \frac{dT}{T}$$

which when integrated becomes: $S_2 - S_1 = c \ln\left(\frac{T_2}{T_1}\right)$ [liquid, solid]

If the temperature of an incompressible substance remains constant during an energy conversion process, then that process is isentropic ($\Delta S=0$).

Entropy Change in an Ideal Gas

For an ideal gas, the property relationships of interest are $PV = RT$ and $du = c_v dT$. Substituting these into the Gibbs equation,

$$ds = \frac{du}{T} + \frac{P dV}{T} = c_v \frac{dT}{T} + R \frac{dV}{V}$$

Integration, assuming constant specific heats, results in:

$$S_2 - S_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{V_2}{V_1}\right)$$
 [ideal gas]

When integrating the enthalpy form of the Gibbs equation:

$$S_2 - S_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$
 [ideal gas]

Now changes in entropy of an ideal gas can be related to changes in pressure, temperature and volume.

Entropy Change in a Vapor or Liquid near Saturation (i.e., steam)

There is no simple algebraic expression that relates properties of vapor and liquids residing in energy states at or near saturation. Liquids are not completely incompressible and may evaporate during pressure, temperature or volume changes. Vapors do not behave as an ideal gas nor as a non-ideal gas and may condense during an energy conversion process. As such relating changes in entropy to other measurable properties requires the use of the Gibbs equation and tabulated values of T, P, V, u and h .

Reference State for Entropy

Although we can only determine a change in entropy during an energy conversion process, the use of entropy involves a single, state value. For example, the entropy of saturated steam (vapor) at 340°F and 64 psia is tabulated as $1.6687\text{ Btu/lbm}^{\circ}\text{R}$. In actuality, this value of entropy is the difference in entropy at $T=340^{\circ}\text{F}$, $P=64\text{ psia}$ and some reference state, S° . Most often the reference state is taken to be at an absolute temperature of $0\text{ [K or }^{\circ}\text{R}]$. And the value of entropy at $T=0\text{ [K, }^{\circ}\text{R}]$ is arbitrarily set to $0\text{ [} \frac{\text{kJ}}{\text{kg K}}, \frac{\text{Btu}}{\text{lbm}^{\circ}\text{R}} \text{]}$.

Thus, $S_2 - S_1$ is really $(S_2 - S^{\circ}) - (S_1 - S^{\circ})$. As long as the reference state is the same for both S_1 and S_2 , the difference can be determined. Caution should be exercised if values of entropy are found from different sources or tables since the reference state may not be the same. This caution is also true for changes in internal energy and enthalpy.

Isentropic Process with a liquid (incompressible)

For a liquid, $\Delta S = c \ln(T_2/T_1)$. For $\Delta S = 0$, $T_1 = T_2$. Therefore, the temperature of an incompressible substance remains constant during an isentropic process.

Isentropic Process with an ideal gas

$$S_2 - S_1 = 0 = c_v \ln(T_2/T_1) + R \ln(N_2/N_1)$$

• rearranging,

$$\ln(T_2/T_1) = -\frac{R}{c_v} \ln(N_2/N_1) = \ln(N_1/N_2)^{R/c_v}$$

• recall that $R = c_p - c_v$ and $k = c_p/c_v$; substitution of the definitions results in:

$$\left. \frac{T_2}{T_1} \right|_{\Delta S=0} = \left(\frac{N_1}{N_2} \right)^{k-1} \quad \text{[ideal gas]}$$

• similarly,

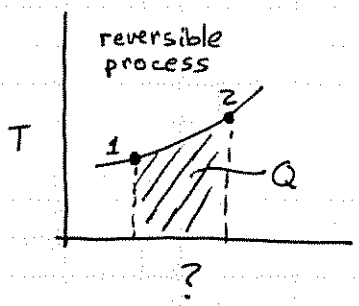
$$\left. \frac{T_2}{T_1} \right|_{\Delta S=0} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}} \quad \text{[ideal gas]}$$

$$\left. \frac{P_2}{P_1} \right|_{\Delta S=0} = \left(\frac{N_1}{N_2} \right)^k \quad \text{[ideal gas]}$$

These three isentropic process relationships for an ideal gas can be rewritten as:

$$\left. \begin{aligned} T N^{k-1} &= \text{constant} \\ T P^{\frac{1-k}{k}} &= \text{constant} \\ P N^k &= \text{constant} \end{aligned} \right\} \text{ideal gas}$$

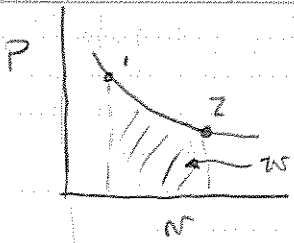
The last expression, $P N^k = \text{constant}$, is in form of a polytropic process where the polytropic coefficient is equal to the specific heat ratio. This represents the most efficient possible polytropic process with an ideal gas because the entropy change is zero.



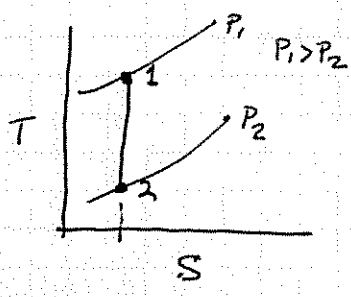
$$Q_2 = \int_1^2 T dV$$

$$\delta Q = T dS$$

$$d? = \frac{\delta Q}{T}$$



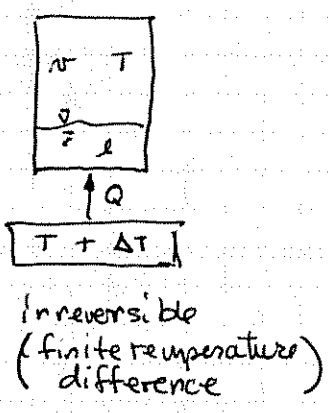
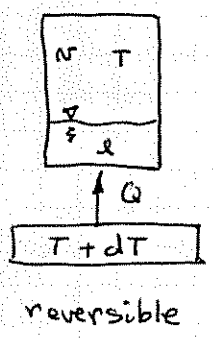
↑ property; ~~did not~~ ^{classical} named this entropy and gave it the symbol S .



Entropy \equiv the property which remains constant during an adiabatic, reversible process

$$\delta Q \leq T dS$$

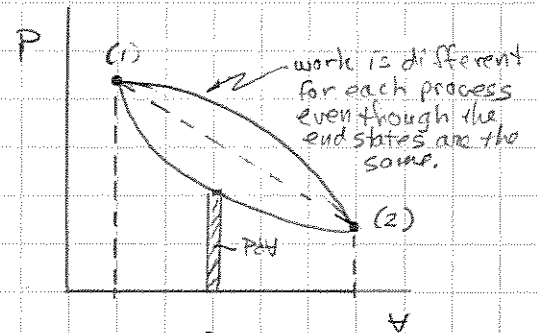
Reversible Heat Transfer



Polytropic & Isentropic Processes with an Ideal Gas

A polytropic process is described by $PV^n = \text{constant}$ and is derived by linear interpolation of a pressure-volume path on a log-log plot. A polytropic process is particularly useful for studying work from expansion or to compression of a gas.

For example, the process that generates the maximum work in the $P-V$ diagram shown is obvious. But which process has the least irreversibility. In other words, which process is closest to being isentropic ($\Delta S=0$)?



If the process is polytropic, which is often an accurate approximation for compression or expansion of a gas, then the work from the process is found by substituting C/V^n for P in the work integral.

$${}_1W_2 = \int_1^2 P dV$$

$${}_1W_2 = \int_1^2 P dV = \int_1^2 C \frac{dV}{V^n} = C \left(\frac{1}{1-n} \right) \left[V_2^{1-n} - V_1^{1-n} \right]$$

Since $P_1 V_1^n = P_2 V_2^n = C$, the work from a polytropic process is:

$${}_1W_2 = \frac{P_2 V_2 - P_1 V_1}{1-n} \quad ; \text{ provided the polytropic coefficient } n \neq 1.$$

When $n=1$, the work is a special case of an isothermal ideal gas.

$${}_1W_2 = \int_1^2 P dV = \int_1^2 \frac{C dV}{V} = C \ln \left(\frac{V_2}{V_1} \right)$$

Since $P_1 V_1 = P_2 V_2 = C$, the work from a polytropic process with $n=1$ is:

$${}_1W_2 = P_1 V_1 \ln \left(\frac{V_2}{V_1} \right) = P_2 V_2 \ln \left(\frac{V_2}{V_1} \right)$$

Isoprocesses for an Ideal Gas

Process	$PV^n = c; n$	Δu	Δh	ΔS
constant volume; isochoric	∞	$c_v \Delta T$	$c_p \Delta T$	$c_v \ln(T_2/T_1)$
constant pressure; isobaric	0	$c_v \Delta T$	$c_p \Delta T$	$c_p \ln(T_2/T_1)$
constant temperature; isothermal	1	0	0	$R \ln(V_2/V_1)$
constant entropy; isentropic (adiabatic, reversible)	$k = \frac{c_p}{c_v}$	$c_v \Delta T$	$c_p \Delta T$	0
polytropic	$0 < n < \infty$	$c_v \Delta T$	$c_p \Delta T$	$c_v \ln(T_2/T_1) + R \ln(V_2/V_1)$

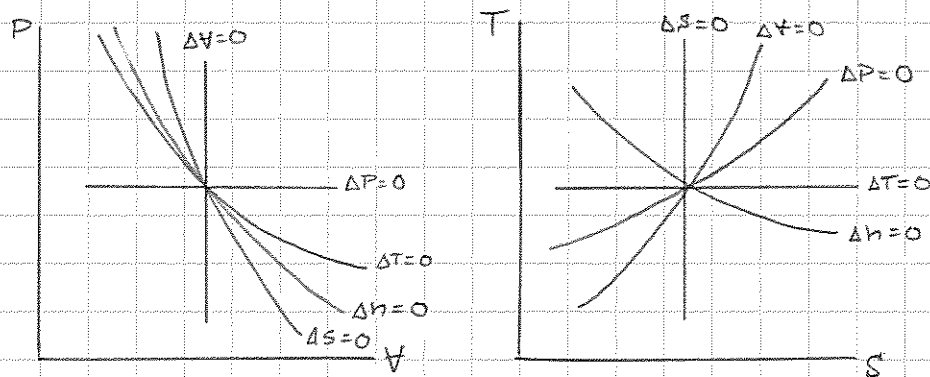
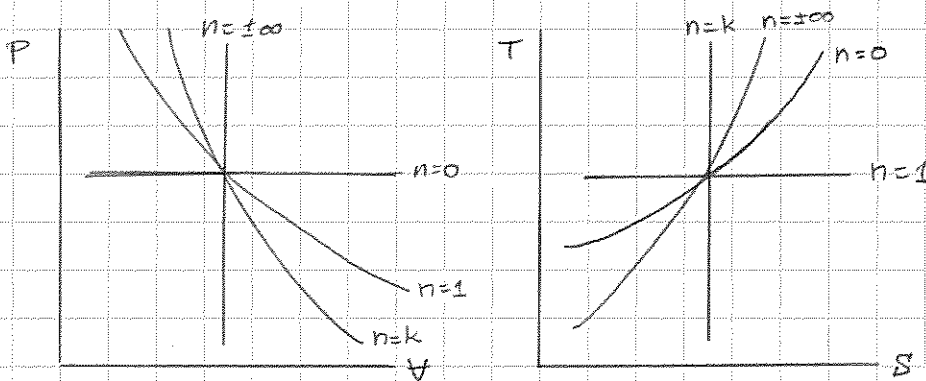
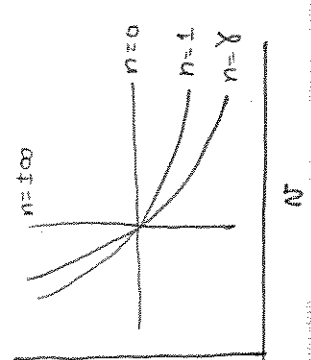
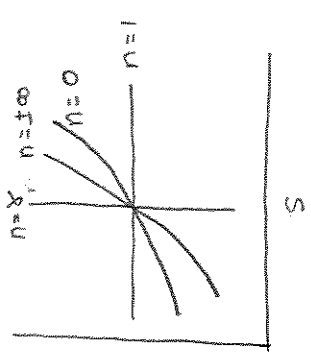


Table 1-3 Perfect-gas relationships (constant specific heats)

Process	P, v, T relationships	$u_2 - u_1$	$h_2 - h_1$	$s_2 - s_1$	w (nonflow)	w (flow)	Q
Isothermal $T = \text{constant}$ $P_1/P_2 = v_2/v_1$		0	0	$(R/D)\ln(v_2/v_1)$	$(P_1 v_1/D)\ln(v_2/v_1)$	$(P_1 v_1/D)\ln(v_2/v_1)$	$(P_1 v_1/D)\ln(v_2/v_1)$
Constant pressure <i>(isobaric)</i> $P = \text{constant}$ $T_2/T_1 = v_2/v_1$		$c_v(T_2 - T_1)$	$c_p(T_2 - T_1)$	$c_p \ln(T_2/T_1)$	$P(v_2 - v_1)/J$	0	$c_p(T_2 - T_1)$
Constant volume <i>(isochoric)</i> $v = \text{constant}$ $T_2/T_1 = P_2/P_1$		$c_v(T_2 - T_1)$	$c_p(T_2 - T_1)$	$c_v \ln(T_2/T_1)$	0	$v(P_2 - P_1)/J$	$c_v(T_2 - T_1)$
Isentropic <i>(adiabatic reversible)</i> $s = \text{constant}$ $P_1 v_1^k = P_2 v_2^k$ $T_2/T_1 = (v_1/v_2)^{k-1}$ $T_2/T_1 = (P_2/P_1)^{(k-1)/k}$		$c_v(T_2 - T_1)$	$c_p(T_2 - T_1)$	0	$\frac{(P_2 v_2 - P_1 v_1)}{J(1-k)}$	$\frac{k(P_2 v_2 - P_1 v_1)}{J(1-k)}$	0
Throttling $h = \text{constant}$ $T = \text{constant}$ $P_1/P_2 = v_2/v_1$		0	0	$(R/D)\ln(v_2/v_1)$	0	0	0
Polytropic $P_1 v_1^n = P_2 v_2^n$ $T_2/T_1 = (v_1/v_2)^{n-1}$ $T_2/T_1 = (P_2/P_1)^{(n-1)/n}$		$c_v(T_2 - T_1)$	$c_p(T_2 - T_1)$	$c_v \ln(P_2/P_1) + c_p \ln(v_2/v_1)$	$\frac{(P_2 v_2 - P_1 v_1)}{J(1-n)}$	$\frac{n(P_2 v_2 - P_1 v_1)}{J(1-n)}$	$c_v \left(\frac{k-n}{1-n} \right) (T_2 - T_1)$

$J = \text{Energy Conversion Factor} = 778.16 \text{ ft} \cdot \text{lb}_f / \text{Btu} = 1.0 \text{ Nm/J}$

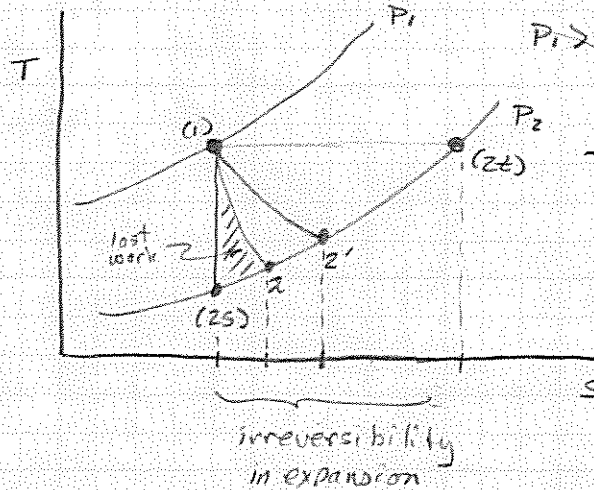
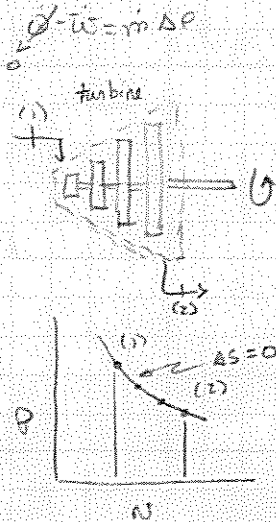


-El-Wakil (1984)

Polytropic Processes

Expansion of an Ideal Gas

Adiabatic Expansion, $\dot{Q} = 0$



$$-\dot{w}_t = h_1 - h_2$$

$$P_1 > P_2$$

$$-\dot{w}_{t,ideal} = h_1 - h_{2s}$$

$$\Delta h = c_p \Delta T \text{ (ideal gas)}$$

← The pressure curves from the ideal gas relationships

$$\Delta S = c_p \ln\left(\frac{T_2}{T_1}\right)$$

$$T_2 = T_1 e^{\Delta S/c_p}$$

1-2s: Reversible, Adiabatic ($\Delta S = 0$) \equiv isentropic

$$\frac{T_1}{T_{2s}} = \left(\frac{P_2}{P_1}\right)^{\frac{k-1}{k}} \text{ (ideal gas)}$$

1-2t: Unrestrained Expansion (throttling) ← most irreversible process

- energy dissipated in fluid friction
- T remains constant

1-2: Irreversible Expansion, but not as bad as throttling

1-2': slightly greater irreversibilities than 1-2

1st Law for a turbine:

$$-\dot{W}_t = \dot{m} (e_{out} - e_{in})$$

$$e = \underbrace{\tilde{u}}_{Pv} + \frac{1}{\rho} P + \frac{1}{2} V^2 + g z$$

small change

$$-\dot{W}_t = \dot{m} [(\tilde{u} + Pv)_{out} - (\tilde{u} + Pv)_{in}] = \dot{m} (h_{out} - h_{in})$$

$$-\dot{W}_t = h_{out} - h_{in}$$

1-2s: $\dot{W}_{2s} = h_1 - h_{2s}$ ← reversible

1-2: $\dot{W}_2 = h_1 - h_2$ ← irreversible

The degree of irreversibility is given by the ^{isentropic} adiabatic turbine efficiency.

• also known as: polytropic turbine efficiency

^{isentropic}
adiabatic turbine efficiency

$$\eta_t = \frac{\text{energy sought}}{\text{energy cost}} = \frac{\text{actual work}}{\underbrace{(\text{actual work}) + (\text{lost work})}_{\text{irreversibilities}}} = \frac{\text{actual work}}{\text{ideal work}} = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

— 9 —

• for an ideal gas with constant specific heats, $h_1 - h_2 = c_p(T_1 - T_2)$

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{T_1 - T_2}{T_1 - T_{2s}} \quad \delta h = c_p \delta T$$

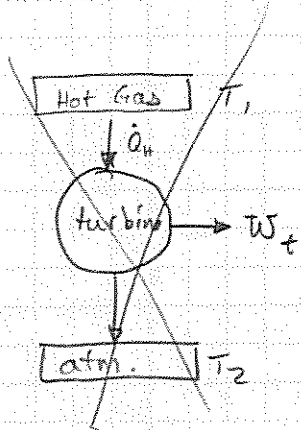
↖ only for ideal gas with constant c_p

! { "we can now state that the change in entropy ^{can be} is a measure of the unavailable energy." (E.I. Wajid)

↖ that is, the portions of ^{thermal energy?} ~~heat~~ that cannot be converted to work.

Example (Short Version)

Air expands in a gas turbine from 10 atm and 2000°F to 1 atm and 1050°F at the exhaust. Assuming a constant specific heat of 0.240 Btu/lbm°F, find the turbine work (in Btu/lbm) and the adiabatic turbine efficiency.



$$W_{\text{turbine}} = h_1 - h_2 \quad (\text{adiabatic expansion } q=0)$$

• for an ideal gas, $h_1 - h_2 = c_p (T_1 - T_2)$

$$\underline{W_{\text{turbine}}} = (0.240 \frac{\text{Btu}}{\text{lbm} \cdot ^\circ\text{F}}) (2000^\circ\text{F} - 1050^\circ\text{F}) = \underline{228 \frac{\text{Btu}}{\text{lbm}}}$$

• the adiabatic turbine efficiency is: $\eta_T = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{T_1 - T_2}{T_1 - T_{2s}}$

↳ reversible, adiabatic expansion

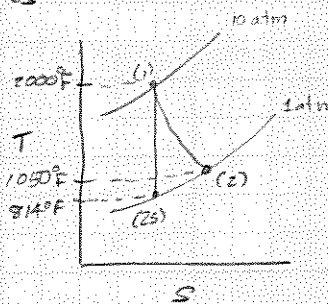
• need to find adiabatic reversible expansion temperature, T_{2s}

• for an adiabatic process & ideal gas,

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1} \right)^{\frac{k-1}{k}}, \quad \text{where } k = \frac{c_p}{c_v}$$

• for air, $k = 1.4$

$$T_{2s} = (2000 + 460 \text{ } ^\circ\text{R}) \left(\frac{1 \text{ atm}}{10 \text{ atm}} \right)^{(1.4-1)/1.4} = 1274 \text{ } ^\circ\text{R} = 814 \text{ } ^\circ\text{F}$$

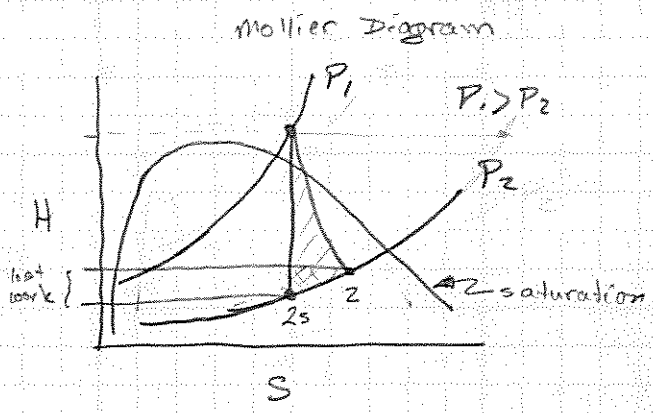
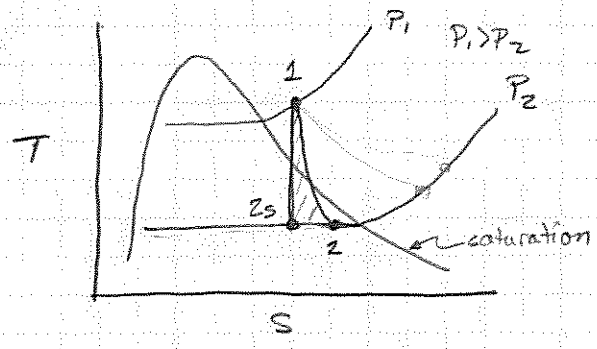


• Therefore,

$$\underline{\eta_T} = \frac{(2000^\circ\text{F} - 1050^\circ\text{F})}{(2000^\circ\text{F} - 814^\circ\text{F})} = 0.801 = \underline{80.1\%}$$

-or lookup in tables-

Case of Vapor Expansion



$T_{2s} = T_2$ } because we are
in the two-phase
region.

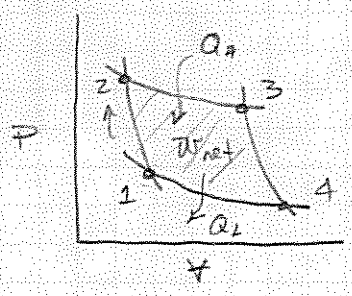
$h_2 > h_{2s}$, exit enthalpy is greater
in the irreversible case

Quality increases with
irreversibilities, but temp.
does not.

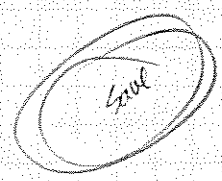
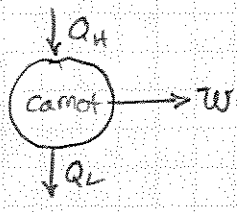
degree of irreversibility is

$$\eta_T = \frac{h_1 - h_2}{h_1 - h_{2s}}$$

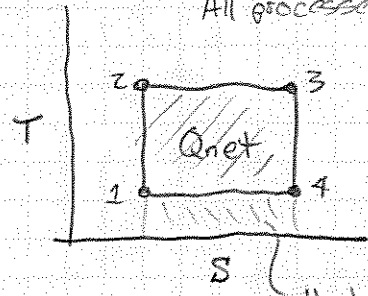
Revisiting Carnot Cycle



(Clausius)



All processes are reversible



(Gibbs)

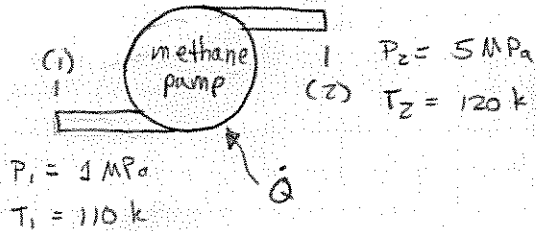
Heat which is not
available for conversion
to work.

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
GARRETT

Example - Cryogenic Methane Pump*

From *Thermodynamics: Concepts and Applications*, 4th ed.

- Liquid methane used in cryogenic applications.
- Determine the entropy change in the liquid methane as it passes through a pump:
 - from property data
 - assuming totally incompressible liquid, Error using this?



Critical temperature of methane is 191 K (-82°C); temperature below which methane remains liquid

(a) state 1: $P_1 = 1 \text{ MPa}$, $T_1 = 110 \text{ K}$ } $S_1 = 4.875 \text{ kJ/kg K}$
 $C_{p1} = 3.471 \text{ kJ/kg K}$

state 2: $P_2 = 5 \text{ MPa}$, $T_2 = 120 \text{ K}$ } $S_2 = 5.145 \text{ kJ/kg K}$
 $C_{p2} = 3.486 \text{ kJ/kg K}$

$S_2 - S_1 = 0.270 \frac{\text{kJ}}{\text{kg K}}$

0.43% change in C_p

(b) $S_2 - S_1 = C \ln(T_2/T_1)$
 $C_{\text{avg}} = 3.4785 \text{ kJ/kg K}$

$$S_2 - S_1 = (3.4785 \frac{\text{kJ}}{\text{kg K}}) \ln\left(\frac{120 \text{ K}}{110 \text{ K}}\right) = 0.303 \frac{\text{kJ}}{\text{kg K}}$$

$$\text{error} = \frac{\Delta S_{\text{actual}} - \Delta S_{\text{ideal}}}{\Delta S_{\text{actual}}} = \underline{\underline{0.122 (12.2\%)}}$$

* Adapted from Cengel & Boles, 4th ed

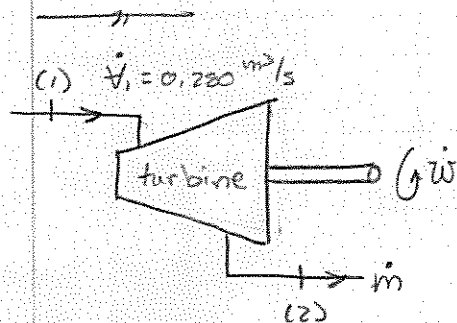
Example - Replacing Throttling Valve with Turbine*

A cryogenic manufacturing facility handles liquid methane at 115 K and 5 MPa at a rate of $0.280 \text{ m}^3/\text{s}$. A process requires dropping the pressure of the liquid methane to 1 MPa, which is accomplished by throttling the liquid methane by passing it through a valve (flow restrictor).

A recently hired engineer proposes to replace the throttling valve with a turbine in order to produce power while dropping the pressure to 1 MPa.

Determine the maximum power that the turbine can produce.

Determine how much this turbine could save the facility in electricity costs per year if the turbine operates continuously and the facility pays \$0.075/kWh on average for electricity.



Assumptions:

- adiabatic
 - reversible
 - steady flow
 - uniform flow
 - negligible change in kinetic & potential energy
- } maximum work out

$$\begin{aligned} (1) \quad & \left. \begin{aligned} P_1 &= 5 \text{ MPa} \\ T_1 &= 115 \text{ K} \end{aligned} \right\} \begin{aligned} h_1 &= 232.2 \text{ kJ/kg} \\ s_1 &= 4.9945 \text{ kJ/kgK} \\ \rho_1 &= 422.15 \text{ kg/m}^3 \end{aligned} \end{aligned}$$

$$(2) \quad \left. \begin{aligned} P_2 &= 1 \text{ MPa} \\ s_2 &= s_1 \end{aligned} \right\} \begin{aligned} h_2 &= 222.8 \text{ kJ/kg} \end{aligned}$$

$$\dot{Q}_2 - \dot{W}_2 = \dot{m} (e_2 - e_1)$$

$\uparrow h_2 - h_1$

$$\dot{m} = \rho_1 \dot{V}_1 = 118.2 \text{ kg/s}$$

$$-\dot{W}_{\text{turbine}} = (118.2 \text{ kg/s}) [222.8 \text{ kJ/kg} - 232.2 \text{ kJ/kg}]$$

$$\boxed{\dot{W}_{\text{turbine}} = 1123 \text{ kW}}$$

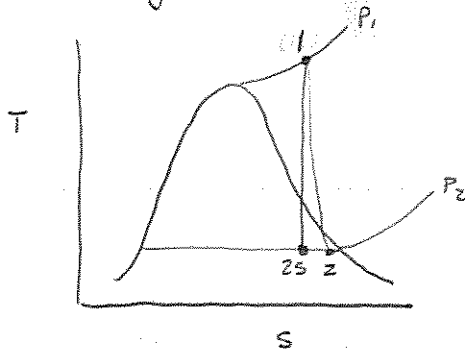
$$\begin{aligned} \text{Annual Power Production} &= \dot{W}_{\text{turbine}} \times \Delta t = (1123 \text{ kW})(8760 \text{ hr/year}) \\ &= 9.837 \cdot 10^6 \text{ kW}\cdot\text{hr}/\text{yr} \end{aligned}$$

$$\text{At } \$0.075/\text{kWh}, \text{ Annual Savings} = \$737,800/\text{yr}$$

$$\text{Actual turbine} \sim 80\% \text{ efficient} \rightarrow 900 \text{ kW} \text{ \& save } \$600,000/\text{yr}$$

* Adapted from Cengel & Boles, 4th ed.

A steam turbine is used to drive a feedwater pump of a large utility boiler. A 17.78 kg/s flow of Supercritical steam enters the turbine at 808.3 k and 23.26 MPa. The steam exits the turbine at 5,249 kPa with a quality of 0.9566. Determine the power produced by the turbine and the turbine isentropic efficiency.



State 1 - Supercritical Steam

$$T_1 = 808.3 \text{ K}$$

$$P_1 = 23.26 \text{ MPa}$$

$$h_1 = 3312.1 \text{ kJ/kg}$$

$$s_1 = 6.1762 \text{ kJ/kg K}$$

State 2 - Saturated liquid & vapor

$$x_2 = 0.9566$$

$$P_2 = 5,249 \text{ kPa}$$

$$h_2 = (1 - x_2)h_{f2} + x_2h_{g2}$$

$$= (1 - 0.9566)(141.38 \frac{\text{kJ}}{\text{kg}}) + (0.9566)(2562.3 \frac{\text{kJ}}{\text{kg}})$$

$$= 2457.2 \text{ kJ/kg}$$

$$s_2 = (1 - x_2)s_{f2} + x_2s_{g2}$$

$$= (1 - 0.9566)(0.48803 \frac{\text{kJ}}{\text{kg K}}) + (0.9566)(8.3765 \frac{\text{kJ}}{\text{kg K}})$$

$$= 8.0341 \text{ kJ/kg K}$$

$$s_{2s} = s_1 = 6.1762$$

$$x_{2s} = \frac{s_{2s} - s_{f2}}{s_{g2} - s_{f2}} = 0.721$$

$$h_{2s} = (1 - x_{2s})h_{f2} + x_{2s}h_{g2} = 1887.0 \frac{\text{kJ}}{\text{kg}}$$

Specific work:

$$w_t = h_1 - h_2 = 3312.1 \text{ kJ/kg} - 2457.2 \text{ kJ/kg} = 854.9 \text{ kJ/kg}$$

Power:

$$\dot{W}_t = \dot{m} w_t = (17.78 \frac{\text{kg}}{\text{s}})(854.9 \frac{\text{kJ}}{\text{kg}}) = 15,200 \text{ kW}$$

Isentropic Efficiency:

$$\eta_t = \frac{h_1 - h_2}{h_1 - h_{2s}} = \frac{3312.1 \text{ kJ/kg} - 2457.2 \text{ kJ/kg}}{3312.1 \text{ kJ/kg} - 1887.0 \text{ kJ/kg}} = 0.60 = 60\%$$

Process - a system transitions from one equilibrium state to another

Path - series of states that a system passes through during a process

quasi-equilibrium - process during which the system remains nearly in equilibrium

iso-processes - process during which one property is constant

isochoric \rightarrow Volume is constant

isobaric \rightarrow Pressure is constant, also isopiestic

isothermal \rightarrow Temperature is constant

isentropic \rightarrow Entropy is constant

adiabatic \rightarrow no transitional thermal energy conversion

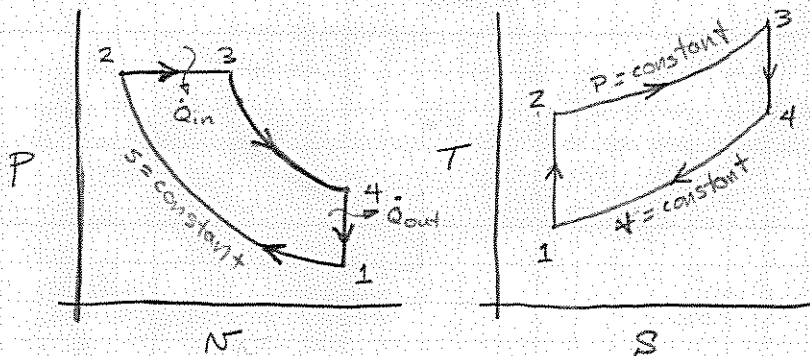
cycle - succession of states in which the system ultimately returns to the initial state

Cycle

"In order to convert forms of energy, particularly heat, to work on an extended or continuous basis, one needs to operate on a cycle"

- "series of processes that begins and ends at the same state, and thus can repeat indefinitely, or as long as needed."

Ideal Diesel Cycle (no losses)



- 1-2: adiabatic compression
- 2-3: isobaric heat addition
- 3-4: adiabatic expansion
- 4-1: isochoric heat rejection

For the Cycle: $\int \delta Q - \int \delta W = 0 \Rightarrow \Delta Q_{net} = Q_{in} - |Q_{out}| = \Delta W_{net}$

⊙ Alternative form of the first law:

$$\int \delta Q = \int \delta W$$

↑ units conversion factor; $J = 778.16 \frac{\text{ft} \cdot \text{lb}_f}{\text{BTU}} = 1.0 \frac{\text{N} \cdot \text{m}}{\text{J}}$

Gas Cycles

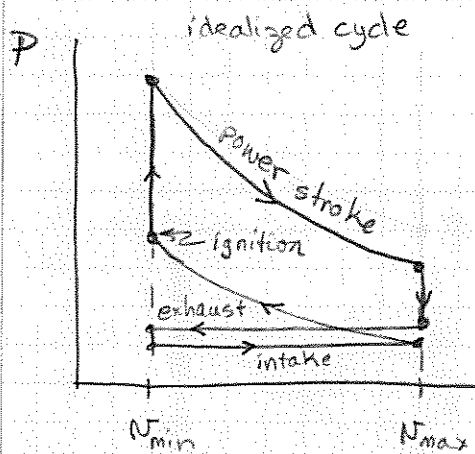
$$\text{Engine Efficiency, } \eta_{th} = \frac{W_{net}}{Q_{in}}$$

Gas Cycle Heat Engines:

Internal Combustion, Spark Ignition (SI)
Internal Combustion, Compression Ignition (CI)
External Combustion, Piston
External Combustion, Turbine

OTTO Cycle
Diesel Cycle
Stirling Cycle
Ericsson, Brayton

OTTO Cycle



Gas Cycles

S.I. Engine (Otto)

C.I. Engine (Diesel)

Gas Turbine (Brayton & Ericsson)

External Combustion (Stirling)

} "open" Cycle → working fluid does not go through a complete cycle

- Advantageous to devise a "closed" cycle that approximates the "open" cycle.

Air Standard Cycle

- start w/ otto cycle
 - description
 - image of engine from Motors manual
 - EES ~~is~~ example
 - indicator from cabinet

Gas Cycles

Air Standard Cycles

S.I. Engine
 Diesel Engine
 Gas Turbine } "open" cycle → working fluid does not go through a complete cycle
 -advantageous to devise "closed" cycles that approximate the "open" cycle

Air Standard Cycle

- fixed mass of air throughout the entire cycle; ~~air is~~ ← no inlet/exhaust processes
- air is an ideal gas
- combustion process is replaced by heat transfer from an external source
- cycle is completed by heat transfer to surroundings
- all processes internally reversible
- air has constant specific heat

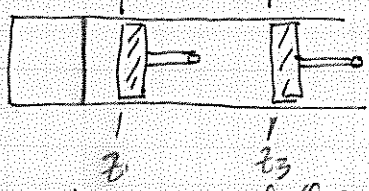
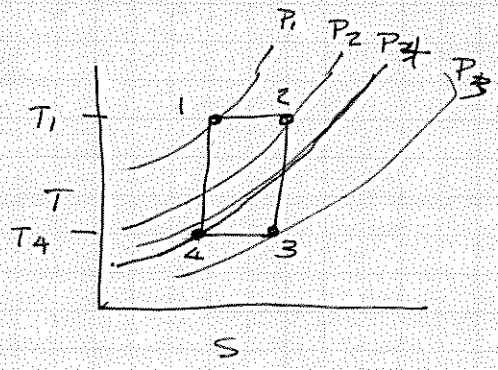
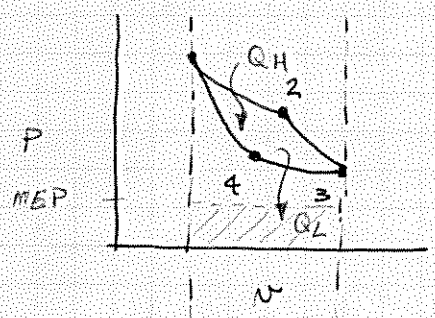
ASC → qualitative study of the influence of a number of variables
 • efficiency, mean effective pressure, etc. don't match actual systems

mean effective pressure → the pressure which, if it acted on the piston during the entire power stroke, would do the same work as actually done on the piston

FIG 9-12
 PG. 215 (499)
 § 8.13.

$$\frac{\text{Work}}{\text{cycle}} = \frac{(\text{MEP}) \cdot A_{\text{piston}} \cdot \text{stroke}}{\text{cycle}}$$

Air Standard Carnot Cycle



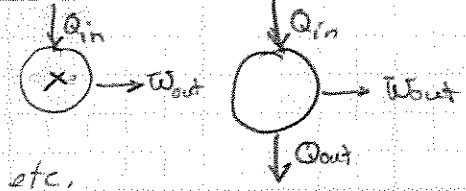
$$\eta_{th} = 1 - \frac{T_2}{T_1} = 1 - \frac{T_4}{T_3} = 1 - \frac{T_2}{T_3}$$

other measures of efficiency: isentropic pressure ratio $\equiv r_{ps} \equiv \frac{P_1}{P_4} = \frac{P_2}{P_3} = \left(\frac{T_2}{T_1}\right)^{\frac{\gamma}{\gamma-1}}$

isentropic compression ratio $\equiv r_{cs} \equiv \frac{v_4}{v_1} = \frac{v_3}{v_2} = \left(\frac{T_1}{T_2}\right)^{\frac{1}{\gamma-1}}$

Second Law

1st Law for a cycle: $\Delta Q = \Delta W$

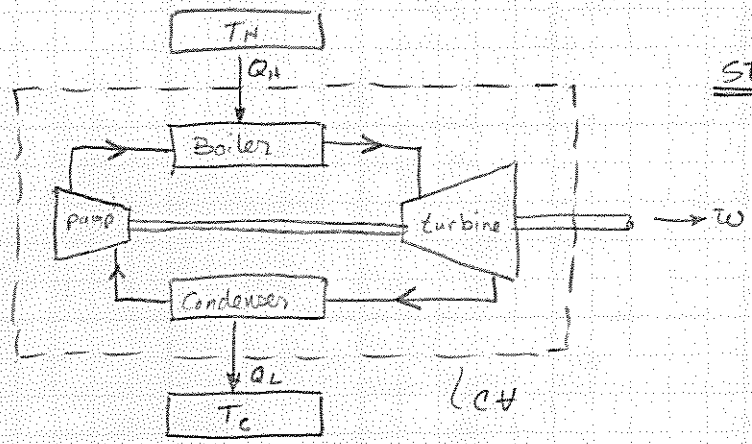


- precludes heat rejection, friction, etc.
- given a conversion efficiency, can analyze a real system

2nd Law is based on the heat engine \rightarrow really only applies to a heat engine

Heat Engine conversion of transitional thermal energy (heat) to/from transitional mechanical energy (work) using two thermal reservoirs

Thermal Reservoir - constant temperature source/sink - source/sink of thermal energy (not true, but ^(stored) heat exchange)
 - heat transfer to/from thermal reservoir does not change its temperature

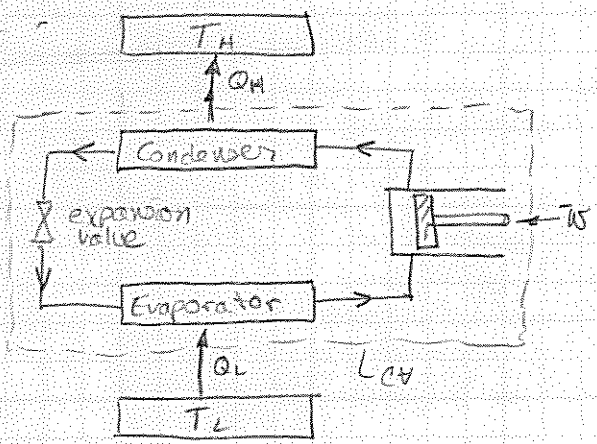


Steam Turbine (Rankine Cycle)

Thermal Efficiency: $\eta_{thermal} \equiv \frac{\text{energy sought}}{\text{energy cost}} = \frac{W}{Q_H}$

$W = Q_H - Q_L$ (1st Law)
 $Q_L \neq 0$ (2nd Law)

$\eta_{thermal} = \frac{Q_H - Q_L}{Q_H} = 1 - \frac{Q_L}{Q_H}$



Refrigeration Cycle

Coefficient of Performance (COP):

$\beta \equiv \frac{\text{energy sought}}{\text{energy cost}} = \frac{Q_L}{W} = \frac{Q_L}{Q_H - Q_L}$

$\beta = \frac{1}{\frac{Q_H}{Q_L} - 1}$

22-141 50 SHEETS
 22-142 100 SHEETS
 22-144 200 SHEETS



Second Law Statements

(A) Kelvin-Planck: Impossible to build a 100% efficient heat engine.

$$W \neq Q_H \quad (W < Q_H)$$

(B) Clausius: Impossible to transfer heat from a cold body to a hot body without work input. (cyclically)

$$Q_C \neq Q_H \text{ in a cycle}$$

• cannot prove "negative" postulates; proof rests on experimental evidence.

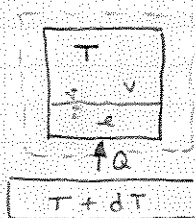
Reversible Processes → deviation from equilibrium is infinitesimal and occurs at an infinitesimal rate

- work expressions from Wed. lecture

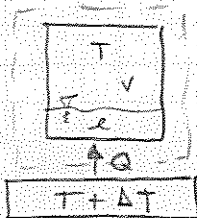
- Irreversibilities:
- friction
 - unrestrained expansion → work required to restore state
 - heat transfer through ΔT (as opposed to dT)
 - refrigeration required to restore state which rejects heat to surroundings
 - mixing of two substances → work required to separate
 - Joule heating (i^2R)
 - combustion

- External vs. Internal Irreversibilities

Internal → no irreversibility occurs within boundary during process
 External → no irreversibility occurs outside boundary during process



(a) Externally Irreversible
Internally Reversible



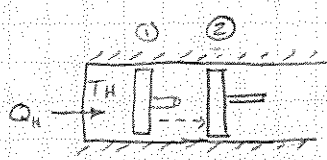
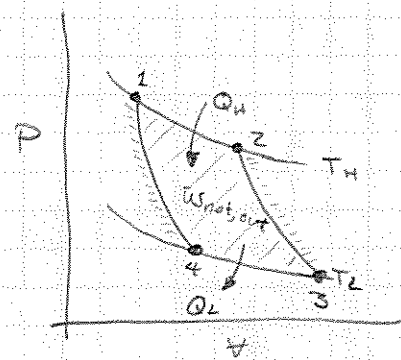
(b) Internally Reversible
Externally Irreversible

system passes through exact same states, but (a) is ^{totally} reversible & (b) is only internally reversible.

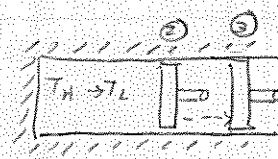
Canot Cycle (proposed in 1824 by Sadi Canot (1796-1832))

Canot Heat Engine - cycle composed of 4 reversible processes } closed or steady flow system
 (2) isothermal
 (2) adiabatic

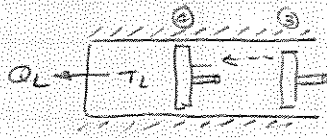
- 1-2: Reversible Isothermal Expansion
- 2-3: Reversible Adiabatic Expansion
- 3-4: Reversible Isothermal Compression
- 4-1: Reversible Adiabatic Compression



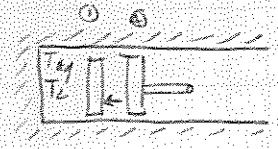
T_H remains constant between 1 & 2



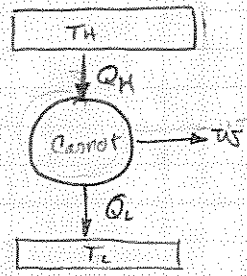
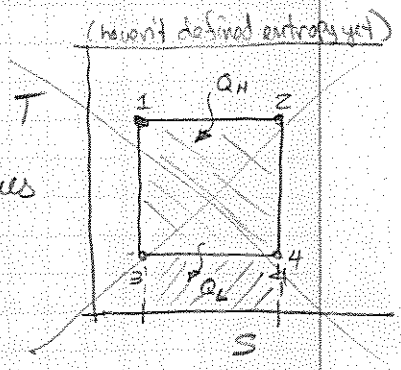
$Q=0$, $T_H \rightarrow T_L$ as expansion continues



T_L remains constant between 3 & 4



$Q=0$, $T_L \rightarrow T_H$ as compression continues



- cycle reversed is a refrigerator
- cycle is independent of the working fluid, it is only dependent heat input & output

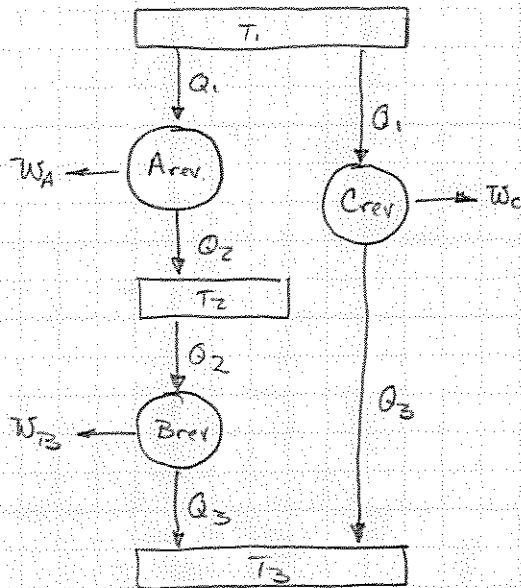
1st Proposition: It is impossible to construct a heat engine that is more efficient than a reversible engine.

2nd Proposition: All engines that operate on a Carnot cycle between two given constant-temperature reservoirs have the same efficiency.

22-141 50 SHEETS
22-142 100 SHEETS
22-144 200 SHEETS
SAMPAL

Thermodynamic Temperature Scale

The dependence of Carnot cycle upon temperatures and not working fluid is basis for the thermodynamic temperature scale.



$$\eta_{\text{thermal}} = \frac{\dot{w}_{\text{net}}}{-\dot{Q}_{\text{in}}} = \frac{\dot{Q}_{\text{in}} - \dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}} = 1 - \frac{\dot{Q}_{\text{out}}}{\dot{Q}_{\text{in}}}$$

$$\frac{Q_L}{Q_H} = f(T_H, T_L) \text{ for a Carnot cycle}$$

$$\frac{Q_1}{Q_2} = f(T_1, T_2)$$

$$\frac{Q_2}{Q_3} = f(T_2, T_3)$$

$$\frac{Q_1}{Q_3} = f(T_1, T_3)$$

$$\frac{Q_1}{Q_2} \cdot \frac{Q_2}{Q_3} = \frac{Q_1}{Q_3}$$

$$f(T_1, T_2) f(T_2, T_3) = f(T_1, T_3)$$

only a function of T_1 & T_3

$$\text{Therefore, } f(T_1, T_2) = \frac{\Phi(T_1)}{\Phi(T_2)}$$

$$\frac{Q_L}{Q_H} = \frac{\Phi(T_L)}{\Phi(T_H)} \quad (*)$$

Many functional relationships will satisfy eqn (*). Lord Kelvin proposed two different relationships:

$$(1) \left. \begin{aligned} \frac{Q_H}{Q_L} &= \frac{T_H}{T_L} \\ \frac{Q_L}{Q_H} &= \frac{T_L}{T_H} \end{aligned} \right\} \text{Kelvin Scale } 0 \leq T < \infty \text{ (ratio)}$$

• Triple point of water was assigned the value of 273.16 K.

$$(2) \frac{Q_H}{Q_L} = \frac{e^{\tilde{T}_H}}{e^{\tilde{T}_L}}$$

$$\log_{10} (Q_H/Q_L) = \tilde{T}_H - \tilde{T}_L$$

relation between (1) & (2):

$$\tilde{T} = \log_{10} T + L$$

where $L \equiv$ temp. that corresponds to 0 on the logarithmic scale

$$-\infty < \tilde{T} < \infty$$

$$\text{Carnot Efficiency: } \eta_c = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \left. \begin{aligned} &\text{maximum} \\ &\text{efficiency} \\ &\text{of a thermal} \\ &\text{power cycle} \end{aligned} \right\}$$

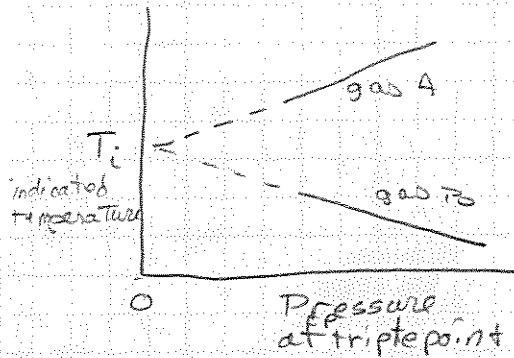
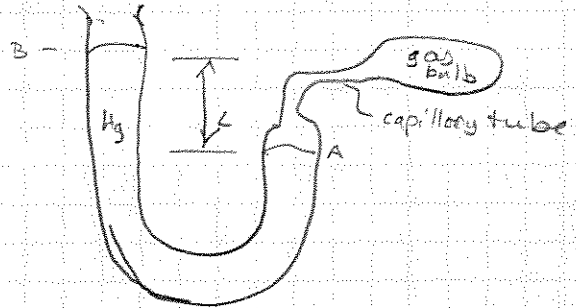
Ideal-Gas Temperature Scale

$$PV = RT$$

$$T = 273.16 \left(\frac{P}{P_{tp}} \right)$$

↑ triple point of water

- No gas is ideal; however, as $P \rightarrow 0$, the behavior of all gases approach that of an ideal gas.



varying amounts of gas

turns out this is the ideal-gas temperature scale is equivalent to the thermodynamic temp scale.

can be shown by using an ideal gas as the working fluid in a Carnot cycle

22-141 50 SHEETS
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22-144 200 SHEETS
GAMPAL

VALUES OF THE GAS CONSTANT, R

Coleman J. Major

The numerical value of the gas constant, R, defined by the equation $PV = nRT$, depends upon the units of P, V, n, and T. A large number of values of the constant may be calculated. The accompanying table gives 84 values of R in a convenient form using the most common units of pressure and volume. It also incorporates both the pound and gram mole and both Rankine and Kelvin temperature scales. Various combinations of metric and English units may, therefore, be used without the necessity of converting each variable to a common system of units. Conversion factors and constants used for computing the values of R are listed at the bottom of the table. The following example illustrates the use of the table:

Calculate: The volume in ft^3 occupied by 2 lb. moles of a gas at 15°C at a pressure of 32.2 ft. of water, assuming the ideal gas law.

Solution: $15^\circ\text{C} + 273.2 = 288.2^\circ\text{K}$

Enter the top of the table under the column headed "ft H₂O" and proceed downward to the value of 44.6 for R. (Note that this lines up horizontally with the desired units of ft^3 , $^\circ\text{K}$, and lb. moles shown on the left side of the table)

$$V = \frac{nRT}{P} = \frac{2 \times 44.6 \times 288.2}{32.2} = 798 \text{ ft}^3$$

$$\text{Values of Gas Constant, } R = \frac{PV}{nT}$$

Absolute Pressure

Volume	Temp.	moles	Atm	psia	mm Hg	cm Hg	in Hg	in H ₂ O	ft H ₂ O
ft^3	$^\circ\text{K}$	gm	0.00290	0.0426	2.20	0.220	0.0867	1.18	0.0982
		lb	1.31	19.31	999	99.9	39.3	535	44.6
	$^\circ\text{R}$	gm	0.00161	0.02366	1.22	0.122	0.0482	0.655	0.0546
		lb	0.730	10.73	555	55.5	21.8	297	24.8
cm ³	$^\circ\text{K}$	gm	82.05	1206	62,400	6240	2450	33,400	2780
		lb	37,200	547,000	2.83×10^7	2.83×10^6	1.11×10^6	1.51×10^7	1.26×10^6
	$^\circ\text{R}$	gm	45.6	670	34,600	3460	1360	18,500	1550
		lb	20,700	304,000	1.57×10^7	1.57×10^6	619,000	8.41×10^6	701,000
liters	$^\circ\text{K}$	gm	0.08205	1.206	62.4	6.24	2.45	33.4	2.78
		lb	37.2	547	28,300	2830	1113	15,140	1262
	$^\circ\text{R}$	gm	0.0456	0.670	34.6	3.46	1.36	18.5	1.55
		lb	20.7	304	15,700	1570	619	8410	701

Conversion Factors and Constants

1 lb. = 453.59 gm	359.0 $\text{ft}^3/\text{lb mole}$
1 atm = 14.696 psia	22.414 $\text{cm}^3/\text{gm mole}$
1 atm = 760 mm Hg	1 inch = 2.54 cm
1 atm = 76 cm Hg	Std. temp. = 273.16°K or 491.69°R
1 atm = 29.921 in Hg	28.31605 liters = 1 ft^3
1 atm = 406.79 in H ₂ O	$R = 8.31432 \pm 0.00034 \times 10^7 \text{ erg } ^\circ\text{K}^{-1} \text{ mol}^{-1}$
1 atm = 33.90 ft H ₂ O	