Principles of Energy Conversion

Part 4. Introduction to Energy Economics

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Article 7

Energy Economics

Energy economics is a specialized field used to make decisions on energy purchases, selection of competing energy generation technologies, and financing of energy technologies. A thorough study of this subject is beyond the scope of this course, but every engineer should have a basic understanding of energy economics in order to bridge the gap between engineering decision analysis and economic decision analysis.

7.1 Energy Costs

Energy costs can generally be divided into two categories both of which are called by many different names:

1. **Capital**
   startup
   investment
   initial
   ⋮
2. **Recurring**
   ongoing
   operational
   operations and maintenance (O&E)
   income
   ⋮

Examples of **capital costs** include cash or borrowed money used for construction of facilities, equipment purchase, and/or equipment installation. Capital costs are usually one-time expenses incurred at the beginning of a project.

Examples of **recurring costs** are numerous; including salaries, taxes, annuity payments, maintenance costs, losses due to scheduled shutdowns for maintenance, loan payments and fuel costs. These costs may be uniform payments in time (regular) or sporadic (irregular).

The difference in the two categories has to do with time. Capital costs are always in today's value of money (present value) whereas recurring costs occur at some future time when the value of money will have changed. This poses difficulties when comparing competing energy systems. For example, an economic comparison of a conventional gas-fired to a solar home hot water heater requires evaluation of money at different times. The conventional hot water heater is very low capital costs, but with recurring future fuel costs. A solar hot water heater has relatively high capital costs, but minimal recurring costs since fuel is not being consumed. Energy economic analysis methods attempt to convert all monetary costs to a common basis so as to enable quantitative comparisons of competing technologies.

7.2 Time Value of Money

The premise of *time value of money* is that a dollar received today is perceived to be worth more than a dollar received in the future. There are several factors that drive this perception. The first is *time preference*.

> *Time preference* of humans is the reason that present consumption (money) is preferred to future consumption. Time preference is naturally different for each person in a society. It is difficult to measure for individuals and even more difficult to aggregate for a society. [1]

A second factor affecting the time value of money is the "rental rate", or interest rate, on funds. There is a future cost associated with the present value of the rented funds. Finally, a third factor is the possibility of currency inflation, which reduces the future purchasing power of present funds.

Economic comparison of competing energy technologies requires a common time basis for money. The common basis may be in terms of today's dollar, the value at some time in the future, or the value at some time in the past.

7.3 Discount Rate

The value of money has historically declined with time. The effect of a dollar purchasing less today than 20 years ago is known as inflation. There are many factors that can cause inflation, not the least of which is the perception that "a dollar in hand today is more valuable than one to be received at some time in the future" [2]. In order to compare energy system benefits and costs that occur at different points in time, all monetary factors should be converted to a common time basis. This conversion is known as discounting.

Discount Rate is the fractional decline in the value of money used for comparing present and future costs on a common basis. The discount rate may also be thought of as the rental rate on funds needed for the investment that could be undertaken. This investment may be for an energy system, or a potential alternative investment used as a comparison.

The value used for a discount rate is industry specific, but some common means for determining a discount rate are:

- rate higher than a U.S. Treasury Bond
- anticipated rate of inflation
- rate of interest that balances costs and benefits (savings and/or revenues)
- rate of financing available
- rate of return on alternative investment with similar risk

The choice of discount rate depends on the specific scenario being analyzed. Low discount rates tend to be favorable towards projects with long-deferred benefits. High discount rates tend to be favorable towards projects with quick paybacks. The discount rate should reflect the 'opportunity cost' of the capital to be invested.

7.4 Economic Analysis Methodologies

There are many, varied approaches to energy economic decision analysis. All of these variations include some type of analysis that includes both capital and recurring costs. The scope of the analysis can vary significantly and the most appropriate choice of analysis method depends on the desired basis for comparison. These various analysis methods can be generally divided into subsets of three general approaches:[1]

1. determine largest possible savings for a fixed budget,
2. determine the minimum budget required, or
3. determine return-on-investment.

An example of the first is retrofitting an existing facility to be more energy efficient, and the person/department charged with retrofitting has a fixed budget. An example of the second scenario could be implementation of a government or corporate regulation to cut electrical usage by X% with minimum expenditures. The third scenario looks for the optimum energy technology that results in the lowest energy usage/cost or largest profit. For all three methods, the most economically efficient choice may not be the most energy efficient choice.

The type of analysis chosen has much to do with type of energy project being considered. For instance, a short-lived project may not be affected by the future value of money, but a project which is expected to take decades to complete, such as a power plant, will certainly be affected by future costs. The cost effectiveness of the short-lived project might be accomplished using a *simple payback method*. The long-lived project may be better assessed through some form of a *life cycle analysis* (LCA).

Simple Payback Method determines the time period to recover capital costs. Typical considerations are accumulation of savings, no future value of money, no interest on debt, and no comparison to fuel costs. The Simple Payback Method penalizes projects with long life potentials in part because any savings beyond payback period are ignored. There is no accounting for inflation or for escalation of future savings in fuel costs that historically have increased at a faster rate than inflation.

Life Cycle Analysis, also known as Engineering Economic Analysis, considers the total cost over anticipated useful life, where useful life is the lesser of lifetime or obsolescence. Analysis may some or all of the following: capital costs, operating costs, maintenance costs and contracts, interest on investment, fuel cost, salaries, insurance, salvage value, lost future value of money not invested, and taxes or tax incentive.

Life Cycle Analysis (LCA) might include indirect costs paid by society but not reflected as cash flow. An example would be health and environmental costs associated with pollution due to electric power generation from coal; a cost not directly paid by the

[1] see Krieth and Goswami [3, Chapter 3. Economic Methods] for more detailed explanation

power generating utility. The difficulty with life cycle analysis is that many of the costs are in the future and can only be estimated with some unknown uncertainty. New technologies may also result in unanticipated obsolescence that, in hindsight, will turn a 'cost effective' decision into an investment loss.

All of the economic evaluation analysis methods are attempting to do two things. The first is to manipulate costs and savings to a common basis in time. The second is to assess these costs against a comparative objective; i.e., (i) which energy system has the lowest total expense, (ii) which system maximized return on investment, or (iii) which system will maximize savings in energy costs. Some common LCA methodologies[2] are:

- *Life-Cycle Cost Method (LCC):* all future costs are brought to present values for a comparison to a base case. The base case may be a conventional energy system, design variations in alternative energy systems, or the alternative of not making the investment. LCC is commonly used to determine the 'cost-minimizing' option that will achieve a common objective.

- *Levelized Cost of Energy (LCOE):* seeks to convert all costs (capital and recurring) to a value per energy unit that must be collected (or saved) to ensure expenses are met and reasonable profits collected. Future revenues are discounted at a rate that equals the rate of return that might be gained on an investment of similar risk; often called the 'opportunity cost of capital'. LCOE is often used to compare competing energy producing technologies.

- *Net Present Value (NPV):* (also known as Net Benefits, Net Present Worth, Net Savings Methods) determines the difference between benefits and expenses with everything discounted to present value. NPV is used for determining long-term profitability.

- *Benefit-to-Cost Ratio (BCR):* (also known as Savings-to-Investment Ratio) is similar to NPV, but utilizes a ratio instead of a difference. Benefits usually implies savings in energy cost. What to include in the numerator (benefits) and denominator (costs) varies and care should be taken when assessing a reported benefit-to-cost ratio. This method is often used when setting priorities amongst competing projects with a limited budget. Projects with the largest ratio get the highest priority.

- *Overall Rate-of-Return (ORR):* determines the discount rate for which savings in energy costs are equal to total expenditures. This is equivalent to determining the discount rate that results in a zero NPV. This method enables cash flow to be expressed in terms of the future value at the end of the analysis period. Previous methods require specification of a discount rate; this method solves for the discount rate.

- *Discounted Payback Method (DPM):* determines the time period required to offset the initial investment (capital cost) by energy savings or benefits. Unlike

[2] These economic evaluation analyses are based on Krieth and Goswami [3, Chapter3. Economic Methods]. There are numerous variations on these names and purposes in the Energy Economics field.

the simple payback method, the time value of money is considered. DPM is often used when the useful life of the project or technology is not known.

The number of analysis methods and the associated jargon (CRF, ACC, LCOE, MARR, DCFROI, PPM, IRR, TER, FVF, PWF, NPV, BCR, SIR . . .) can be overwhelming. As stated previously, all of the economic evaluation analysis methods are attempting two objectives:

1. manipulate costs and savings to a common time basis, and
2. assess these costs against some comparative objective.

Fortunately, the various forms of analysis and vocabulary are generally constructed around two simple arithmetic concepts: (i) compounding and (ii) uniform series.

Article 8

Discounting Tools

8.1 Discounting Tool - Future Value

A basic measure of the time value of money is the future value obtainable through compounding. Compounding is a fractional growth rate based on finite time intervals; similar to exponential growth, which is fractional growth at infinitesimal time intervals.

8.1.1 Compounding Formula

Compounding can be described mathematically in terms of a growth rate, a time period over which compounding occurs, and the number of compounding periods being considered.

P: quantity which increases by a fractional rate at fixed time intervals
τ: time interval; hr, day, month, year, quarter, ...
j: growth rate, fractional increase in value per time interval [%/τ]
n: number of time intervals

The compounding formula can be derived by examining the increase in P after each time interval. At the end of the first time interval, the initial value, P_0, has increased by the fractional amount $(j\tau)P_0$. Similarly, at the end of the second time interval, the starting value, P_1 has increased by a fractional amount.[1]

$$\begin{aligned}
\text{at the end of period 1: } P_1 &= P_0 + (j\tau)\,P_0 = P_0\,(1+j\tau) \\
\text{end of period 2: } P_2 &= P_1 + (j\tau)\,P_1 = P_1\,(1+j\tau) = P_0\,(1+j\tau)^2 \\
\text{period 3: } P_3 &= P_2 + (j\tau)\,P_2 = P_2\,(1+j\tau) = P_1\,(1+j\tau)^2 = P_0\,(1+j\tau)^3 \\
&\vdots \\
\text{period } n\text{: } P_n &= P_0\,(1+j\tau)^n
\end{aligned}$$

Compound Interest Formula: $P_n = P_0\,(1+j\tau)^n$

[1] The growth rate, j, and time interval, τ, are usually combined into a single fraction, $i = j\tau$ with an implied time interval of one year. We will maintain the distinction between growth rate and time interval since the time interval of concern may not always be the same during an analysis of competing energy conversion technologies or a comparison of forms of energy, i.e., solar energy versus chemical energy. The symbols F and P are commonly used for P_n and P_0, respectively.

Typically, the compound interest formula is expressed as an annual growth and the time interval is implicit; $P_n = P_0\,(1 + j)^m$, where i is the percent growth per year and m is the number of years.

8.1.1.1 Relation Between Compounding and Exponential Growth

As alluded to previously, compounding is essentially the same as exponential growth but with finite time intervals for fractional growth.

$$\left.\begin{array}{c} P_1 - P_0 = (j\tau)P_0 \\ \tau \equiv \Delta t \end{array}\right\} \frac{\Delta P}{\Delta t} = jP_0 \longrightarrow \lim_{\Delta t \to 0} \frac{dP}{dt} = jP \longrightarrow P = P_0 e^{jt}$$

8.1.1.2 Multiple Compound Rates

When dealing with energy, the escalation rate due to inflation is often insufficient to describe future increase in costs. Historically, fuel prices have increased at a rate higher than inflation. Therefore, if recurring costs include fuel then two rates should be considered. In the simplest terms, the two factors are multiplied. If i is the rate of inflation and j is the escalation rate of fuel cost, then from the compound interest formula,

$$\frac{P_n}{P_0} = (1 + i\tau)^n\,(1 + j\tau)^n = \left[(1 + i\tau)\,(1 + j\tau)\right]^n$$

The growth factor (right side of equation) may be substituted with by a Total Escalation Rate growth factor, $P_n = P_0(1 + \mathrm{TER})^n$, where

$$\mathrm{TER} = (1 + \text{inflation rate})\,(1 + \text{escalation rate}) - 1\,.$$

8.1.2 Effective Interest Rate (Short-Term Interest Rate)

Many financial institutions calculate interest payments on an annual basis even though the time interval for compounding is less than a year. The annual percentage rate (APR) is not necessarily the annual growth rate because of the multiple compounding periods which occur per annum. For example, an APR of 18% compounded monthly is in actuality 1.5% interest on a balance applied monthly with an effective annual interest rate of 19.56%. The effective annual interest rate can be determined by equating compounding formulas for which the beginning and ending balances should be the same; i.e., P_n must be the same regardless of how growth rate and compounding period are calculated.

$$\left.\begin{aligned}
&j \equiv \text{monthly interest rate} = \left(\tfrac{18\%}{\text{year}}\right)\left(\tfrac{1\text{ year}}{12\text{ months}}\right) = 1.5\%\text{ per month}\\
&\tau \equiv \text{compounding period} = \text{month}\\
&n \equiv \text{number of time intervals} = 12
\end{aligned}\right\} j = (\text{APR})\left(\tfrac{1\text{ year}}{n\tau}\right)$$

$$j' \equiv \text{effective annual interest rate (not APR)}$$

$$\tau' \equiv \text{compounding period} = \text{year}$$

$$n' \equiv \text{number of time intervals} = 1$$

The effective annual interest rate can be derived by equating the monthly and yearly growths and rearranging to solve for j':

$$\frac{P_n}{P_0} = (1 + j\tau)^n = (1 + j'\tau')^{n'}$$

$$n \ln(1 + j\tau) = n' \ln(1 + j'\tau')$$

$$(1 + j\tau)^{n/n'} = (1 + j'\tau')$$

$$\text{Effective Interest Rate: } j' = \frac{1}{\tau'}\left[(1 + j\tau)^{n/n'} - 1\right]$$

Thus, for 18% APR compounded monthly,

$$j' = \frac{1}{1\text{ yr}}\left[\left(1 + \frac{0.015}{\text{mo}} \cdot \text{mo}\right)^{12/1} - 1\right] = 19.56\%\text{ per year.}$$

An alternative formulation for the effective annual interest rate is:

$$j' = \frac{1}{\tau'}\left[\left(1 + \frac{\text{APR}}{n\tau}\right)^{n/n'} - 1\right]$$

8.1.2.1 Example 4-1. APR, Compounded Quarterly

Determine the effective annual interest rate on a loan with 6% APR compounded quarterly.

compounding time interval, $\tau \equiv$ quarter (3 months)

growth rate per τ, $j = \left(\dfrac{0.06\text{ APR}}{\text{year}}\right)\left(\dfrac{1\text{ year}}{4\text{ quarters}}\right) = 0.015$ per quarter

number of time intervals per year, $n = 4$

$$\frac{P_n}{P_0} = \underbrace{(1 + j\tau)^n}_{\text{per quarter}} = \underbrace{(1 + j'\tau')^{n'}}_{\text{per year}}$$

$$j' = \frac{1}{\tau'}\left[(1+j\tau)^{n/n'} - 1\right] = \frac{1}{\text{year}}\left[\left(1 + \frac{0.015}{\text{quarter}} \cdot \text{quarter}\right)^{4/1} - 1\right] = \frac{0.0614}{\text{year}}$$

The effective annual interest rate is 6.14%.

8.1.2.2 Example 4-2. APR, Compounded Daily

Determine the effective annual interest rate on a loan with 17.23% APR compounded daily.

compounding time interval, $\tau \equiv$ day

number of time intervals per year, $n = 365$

growth rate per τ, $j = \left(\frac{0.1723 \text{ APR}}{\text{year}}\right)\left(\frac{1 \text{ year}}{365 \text{ days}}\right) = 0.000472$ per day

$$\frac{P_n}{P_0} = \underbrace{(1+j\tau)^n}_{\text{per quarter}} = \underbrace{(1+j'\tau')^{n'}}_{\text{per year}}$$

$$j' = \frac{1}{\tau'}\left[(1+j\tau)^{n/n'} - 1\right] = \frac{1}{\text{year}}\left[\left(1 + \frac{0.000472}{\text{day}} \cdot \text{day}\right)^{365/1} - 1\right] = \frac{0.188}{\text{year}}$$

The effective annual interest rate is 18.8%.

8.1.3 Uniform Series Formula (Equal Payment Annuity Formula)

The uniform series formula expresses the growth of a quantity due to a fractional increase plus a regular annuity or payment. This may be used to determine the amount of money that should be collected per time interval in order to recoup capital costs.

The symbols used are the same as for the compound interest formula with the addition of the fixed payment, S, per time interval.

P: quantity which increases by a fractional rate at fixed time intervals
S: payment or annuity per time interval τ; S will be added after each τ
τ: time interval; hr, day, month, year, quarter, ...
j: growth rate, fractional increase in value per time interval [%/τ]
n: number of time intervals

The uniform series formula can be derived in a similar manner as the compound interest formula; by examining the increase in P after each time interval. At the end of the first time interval, an initial value of zero will increase by S. At the end of the second time interval, the value, P_1, will increased by a fractional amount plus another S.[2]

n	
0	$P_0 = 0$
1	$P_1 = S$ note that the first payment occurred after the first period
2	$P_2 = P_1 + (j\tau)P_1 + S = P_1(1 + j\tau) + S = S(1 + j\tau) + S$
3	$P_3 = P_2 + (j\tau)P_2 + S = P_2(1 + j\tau) + S = S(1 + j\tau)^2 + S(1 + j\tau) + S$
$\vdots$	
n	$P_n = P_{n-1} + (j\tau)P_{n-1} + S = S\left\{(1 + j\tau)^{n-1} + (1 + j\tau)^{n-2} + \cdots + (1 + j\tau) + 1\right\}$

To simplify this series, multiply P_n by $(1 + j\tau)$ and then subtract P_n:

$$\begin{array}{rcl} P_n(1 + j\tau) & = & S\left\{(1 + j\tau)^n + (1 + j\tau)^{n-1} + (1 + j\tau)^{n-2} + \cdots + (1 + j\tau) \qquad \right\} \\ -\ P_n & = & S\left\{ \qquad - (1 + j\tau)^{n-1} - (1 + j\tau)^{n-2} - \cdots - (1 + j\tau) - 1\right\} \\ \hline P_n(j\tau) & = & S\left\{(1 + j\tau)^n - 1\right\} \end{array}$$

$$\textbf{Uniform Series Formula: } P_n = S\left\{\frac{(1 + j\tau)^n - 1}{j\tau}\right\}$$

[2]The symbol A is often used to represent the equal payment annuity instead of S.

8.1.3.1 Starting Point for First Payment

The derivation of the Uniform Series Formula was based on the first payment beginning at the end of the first time interval. For payments that start at the beginning of the first time interval, there is an extra compounding period,

$$P_n = S\left\{\frac{(1+j\tau)^{n+1}-1}{j\tau}\right\}$$

8.1.3.2 Gains & Losses

Gains (compounding interest) and losses (inflation) can both be accounted for when determining the required uniform series payments. If d is the fractional decrease per time interval, then:

gain: $(j\tau_1)\,P_n = S\left\{(1+j\tau_1)^n - 1\right\}$

loss: $(d\tau_2)\,P_m = S\left\{(1+d\tau_2)^m - 1\right\}$

Losses can be considered as a negative gain. If the compounding periods are the same, then $n = m$ and $\tau_1 = \tau_2 = \tau$ and the gains and losses may be added together.

$$(j\tau)\,P_n - (d\tau)\,P_n = S\left\{(1+j\tau)^n - (1+d\tau)^n\right\}$$

$$\text{Gains \& Losses: } P_n = S\left\{\frac{(1+j\tau)^n-(1+d\tau)^n}{(j-d)\,\tau}\right\}$$

8.1.4 Examples using Future Value

8.1.4.1 Example 4-3. Effective Heating Costs – Alternative Investment

A solar-powered home heating system can be built for \$8000 and will supply all of the heating requirements for 20 years. Assume that the salvage value of the solar heating system just compensates for the maintenance and operational costs over the 20 year period. If the interest on money is 8%, compounded annually, what is the effective cost of heating the house? Another way to ask this question is "How much would you have to save per year to equal the future value of \$8,000 invested for 20 years at 8%?"[3]

The discount rate is based on the lost future value of the \$8000 at 8% compounded annually. Therefore, the total capital cost is based on the build cost plus the lost future value of an alternative investment.

$$P_{20} = P_0\left(1 + j\tau\right)^n = \$8000\left[1 + \left(\frac{0.08}{\mathrm{yr}}\right)(\mathrm{yr})\right]^{20} = \$37,287.66$$

The capital cost is considered to be \$37,287.66 and not just the immediate cost of \$8,000.

The current annual heating costs which just offsets the capital cost are found using the uniform series formula where S is the yearly heating cost saved with the new solar heating system.

$$P_{20} = S\left\{\frac{\left(1 + j\tau\right)^n - 1}{j\tau}\right\} = S\left\{\frac{\left(1 + \left(\frac{0.08}{\mathrm{yr}}\right)(\mathrm{yr})\right)^{20} - 1}{\left(\frac{0.08}{\mathrm{yr}}\right)(\mathrm{yr})}\right\} = S\left\{\frac{1.08^{20} - 1}{0.08}\right\}$$

$$S = \frac{P_{20}}{45.76} = \$814.82 \text{ per year}$$

The annual savings in heating costs required to offset the capital cost, including lost potential earnings, is \$815 per year. This analysis does not account for inflation or any escalation in fuel prices.

[3] This is an example of a Levelized Cost Of Energy (LCOE) analysis.

8.1.4.2 Example 4-4. Effective Heating Costs – Minimum Payments

A proposed solar-heating system for a home costs \$6,000 and has a rated operational life of 20 years. Purchase and installation of the system is to be financed by a 60-month loan with an annual percentage rate of 7.2%. The salvage value of the solar-heating system will essentially be zero. Determine the maximum effective yearly heating costs for the system to pay for itself if the annual savings could be invested at 6% interest compounded annually.[4]

In order to determine the required annual savings in heating costs, the capital cost must be determined. The initial loan is \$6000 with a 7.2% APR for 60 months. The monthly interest rate on the loan is:

$$j = \left(\frac{0.072}{\text{year}}\right)\left(\frac{1\,\text{year}}{12\,\text{months}}\right) = \frac{0.006}{\text{month}}$$

Therefore, the capital cost associated with the borrowed money is:

$$P_{60} = \$6000\left[1 + \frac{0.006}{\text{month}}\cdot\text{month}\right]^{60} = \$8,590.73$$

This value, however, is not the complete capital cost because the funds could have been invested at 6% per annum. Thus, the total capital cost includes the lost future value of the initial loan plus interest.

$$P_{20} = P_0\,(1 + j\tau)^n = \$8,590.73\left[1 + \frac{0.06}{\text{year}}\cdot\text{year}\right]^{20} = \$27,551.63$$

Note the change in time interval to reflect the 20 year lifetime and 20 years of lost future revenue. The total capital cost of the solar-heating system is estimated to be \$27,551.63.

The annual savings in heating costs required to break even with the total capital cost is:

$$P_{20} = \$27,551.63 = S\left\{\frac{(1 + j\tau)^n - 1}{j\tau}\right\} = S\left\{\frac{(1.06)^{20} - 1}{0.06}\right\}$$

The annual heating costs must be $S \geq \$749$ per year.

If the opportunity cost of the capital is not considered, then the annual heating costs would have to be less than

$$P_{20} = \$8,590.72 = S\left\{\frac{(1.06)^{20} - 1}{0.06}\right\}$$

Now the annual heat cost must be $S \geq \$233.56$, which is likely too low to justify purchase of the solar-heating system. Both scenarios assume 0% inflation and no escalation of fuel costs.

[4] This is an example of a Levelized Cost Of Energy (LCOE) analysis.

8.1.4.3 Example 4-5. Effective Heating Costs – With & Without Inflation

What should be the annual savings in heating costs in order to "break even" after twenty years on an $8000 solar-powered residential heating system? Consider two cases, one without inflation and one with inflation.[5]

Case 1 (future value without inflation)

- long-term investment at 8% per year, compounded annually
- no operational or maintenance costs,
- no inflation
- no fuel cost escalation
- no salvage value or tax incentives
- savings reinvested at 6% APR, compounded monthly

First, the capital cost, which is based on the as-built cost plus lost future value on the money, must be determined.

$$P_{20} = \$8000\,(1+0.08)^{20} = \$37{,}287.66$$

The savings in heating costs required to "break even" after 20 years is based on investing the savings at 6% APR, compounded monthly.

monthly basis:

$$\left.\begin{aligned} P_n &= S\left\{\frac{(1+j\tau)^n-1}{j\tau}\right\} \\ \tau &\equiv \text{month} \\ n &= 12\times 20 = 240 \\ j &= \left(\frac{0.06}{\text{year}}\right)\left(\frac{1\text{ yr}}{12\text{ mo}}\right) = 0.005/\tau \end{aligned}\right\} \quad \begin{aligned} P_{240} &= \$37{,}287.66 = S\left\{\frac{(1+0.005)^{240}-1}{0.005}\right\} \\ S &= \$80.70 \text{ per month} \end{aligned}$$

yearly basis:

$$\left.\begin{aligned} P_{n'} &= S\left\{\frac{(1+j'\tau')^{n'}-1}{j'\tau'}\right\} \\ \tau' &\equiv \text{year} \\ n' &= 20 \\ j' &= \frac{1}{\tau'}\left[(1+j'\tau')^{n/n'}-1\right] \\ &= \frac{1}{\text{yr}}\left[\left(1+\frac{0.005}{\text{mo}}\cdot\text{mo}\right)^{240/20}-1\right] \\ &= 0.0617 \text{ per year} \end{aligned}\right\} \quad \begin{aligned} P_{20} &= \$37{,}287.66 = S\left\{\frac{(1+0.0617)^{20}-1}{0.0617}\right\} \\ S &= \$995.27 \text{ per year} \end{aligned}$$

[5] This is an example of a Levelized Cost Of Energy (LCOE) analysis.

The annual savings in heating costs should be at least \$995. This does not account for inflation. Notice that the required monthly savings multiplied by 12 does not equal the required annual savings.

Case 2 (future value with inflation)

- long-term investment at 8% per year, compounded annually
- no operational or maintenance costs
- inflation rate of 4% per year
- no salvage value
- no fuel cost escalation
- no tax incentives
- savings reinvested at 6% APR, compounded monthly

From Case 1, the capital costs are estimated at \$37,287.66 and the effective annual interest rate available for reinvestment of savings is 6.17%, compounded annually. The savings in heating costs required to "break even" after 20 years is:

$$\left.\begin{aligned} &P_n = S\left\{\frac{(1+j\tau)^n-(1+d\tau)^n}{(j-d)\tau}\right\}\\ &\tau \equiv \text{year}\\ &n = 20\\ &j = 0.0617/\tau\\ &d = 0.04/\tau \end{aligned}\right\} \begin{aligned} &P_{20} = \$37{,}287.66 = S\left\{\frac{1.0617^{20}-1.04^{20}}{0.0617-0.04}\right\}\\ &S_1 = \$722.15 \text{ (savings in first year)} \end{aligned}$$

The calculated required annual savings of \$722.15 is only valid for the first year. After that inflation will increase the cost of home heating by 4% per year. The required savings in heating costs for future years is:

$$S_n = S_0(1+d\tau)^n$$

but what is S_0? We can rewrite this in terms of S_1 by recognizing that $S_0 = S_1(1+d\tau)^{-1}$. Subsequently,

$$S_n = S_1(1+d\tau)^{n-1}$$

The required savings in heating costs for future years are:

$$\begin{aligned} 10^{\text{th}} \text{ year:} \quad & S_{10} = S_1(1.04)^9 = \$1{,}027.84\\ 20^{\text{th}} \text{ year:} \quad & S_{20} = S_1(1.04)^{19} = \$1{,}521.46 \end{aligned}$$

8.2 Discounting Factors

Many energy economic analysis books or articles will begin with six basic discounting factors that can be tabulated for various discount rates. Discounting factors are almost always tabulated on an annual basis. In the previous articles, we have used two basic growth rate expressions instead of annualized discount factors:

$$\text{compounding: } P_n = P_0(1 + j\tau)^n \tag{8.1}$$

and

$$\text{uniform series: } P_n = S\left\{\frac{(1 + j\tau)^n - 1}{j\tau}\right\} \tag{8.2}$$

The six basic discounting factors are simply algebraic manipulations of these two equations. While there is no common language or nomenclature, a commonly used nomenclature is F for future value (P_n), P for present value (P_0), and A for uniform series payment (S); the annual growth rate is i% over m years.

Annual Discounting Factors		*Solution*
Future Value FV) Future Worth Factor (FW) Future Sum (FS) Single Compound Amount (SCA)	given P, find F (F/P, i%, m)	equation (8.1), solve for P_n
Present Value (FV) Present Worth Factor (FW) Single-Payment Present Worth Single Present Worth (SCW)	given F, find P (P/F, i%, m)	equation (8.1), solve for P_0
Equal-Payment Series Compound Uniform Compound Amount (UCA) Uniform-Series Compound Factor	given A, find F (F/A, i%, m)	equation (8.2), solve for P_n
Sinking-Fund Payment Uniform Sinking Fund (USF) Sinking Fund Factor	given F, find A (A/F, i%, m)	equation (8.2), solve for S
Capital Recovery Equal-Payment Capital Recovery Uniform Capital Recovery (UCR)	given A, find P (P/A, i%, m)	substitute eq. (8.1) into eq. (8.2), solve for P_0
Uniform-Series Present Worth Uniform Present Worth (UPW) Series Present Worth	given P, find A (A/P, i%, m)	substitute eq. (8.1) into eq. (8.2), solve for S

8.3 Discounting Tool - Present Value

The present value methods are used to bring all future costs, which may occur in different years, back to today's value of money. In this way, the cost effectiveness of different energy technologies can be compared on an equal basis.

8.3.1 Compounding – Present Value

Compounding in terms of present value is the inverse of future value.

$$P_0 = P_n \frac{1}{(1 + j\tau)^n} = P_n (1 + j\tau)^{-n}$$

In other words, use equation (8.1) and solve for P_0.

8.3.2 Uniform Series – Present Value

In terms of future value, a uniform series annuity was derived as [eq. (8.2)]:

$$P_n = S \left\{ \frac{(1 + j\tau)^n - 1}{j\tau} \right\}$$

The present value can be determined by substituting equation (8.1) into the uniform series formula:

$$P_0 = S \left\{ \frac{(1 + j\tau)^n - 1}{(j\tau)(1 + j\tau)^n} \right\}$$

Another way to look at this is that there are two equations (8.1 & 8.2) and two unknowns (P_0 and P_n).

8.3.3 Examples using Present Value

8.3.3.1 Example 4-6. Home Mortgage Payments & Present Value

A good example of the need to calculate present value is with a home mortgage. A lump sum is borrowed at a fixed annual interest rate and uniform series payments are made on the mortgage while interest is accruing. Consider a 30-year fixed-rate mortgage for \$250,000 at 6% per year.

The present value is $P_0 = \$250,000$. If no payments were made, the future value would be $P_{30} = P_0(1.06)^{30} = \$1,435,873$. With uniform annual payments of S, we could recalculate the future value. However, what we would like to determine is the value of S which pays off the mortgage while interest is accruing at 6% per year. The problem is that we have one equation and two unknowns P_{30} and S:

$$P_{30} = S\left\{\frac{(1+j\tau)^n - 1}{j\tau}\right\}$$

The future value, P_{30}, is related to the present value, P_0: $P_{30} = P_0\left(1+j\tau\right)^{30}$. Substituting this into the uniform series formula,

$$P_0 = P_{30}(1+j\tau)^{-30} = S\left\{\frac{(1+j\tau)^{30}-1}{(j\tau)(1+j\tau)^{30}}\right\} = S\left\{\frac{1.06^{30}-1}{0.06\cdot 1.06^{30}}\right\} = S\left(13.76\right)$$

Thus, the annual uniform series payment required to pay off in 30 years a present value of \$250,000 which grows at 6% per year is \$18,162.

The monthly payment required would have to be determined using the monthly growth rate, where $j'n'\tau' = jn\tau$.

$$j' = j\left(\frac{n\tau}{n'\tau'}\right) = 0.06/\text{yr}\left(\frac{1\ \text{yr}}{12\ \text{mo}}\right) = 0.005/\text{mo}$$

The uniform series in terms of present value on a monthly basis is:

$$P_0 = P_{360}(1+j\tau)^{-360} = S\left\{\frac{1.005^{360}-1}{0.005\cdot 1.005^{360}}\right\} = S\left(166.79\right)$$

The monthly uniform series payment required to pay off in 30 years a present value of \$250,000 which grows at 6% per year is approximately \$1,500.

8.3.3.2 Example 4-7. Least Current Cash Option

You are in charge of purchasing several new fleet vehicles. You are offered two payment options. Option A requires a $30,000 payment at the end of the year for four years. Option B requires a $39,000 payment at the end of the year for the next three years. Which is the least costly option if the long-term interest is 8%, compounded annually. In other words, what is the minimum amount of cash that should be set aside now to make the annual payments?[6]

The two options require uniform payments, but over different periods of time. In order to compare the two options, calculate how much money today (present value) would be required to make the payments if the lump sum was invested at 8%, compounded annually.

Option A:

The cash outlay in present value is $4 \times \$30,000 = \$120,000$. This calculation, however, does not account for the future value of this money, which is:

$$P_4 = S\left\{\frac{(1+j\tau)^4 - 1}{j\tau}\right\} = \$30,000\left\{\frac{1.08^4 - 1}{0.08}\right\} = \$135,183$$

The present value of the future value ($135,183) is:

$$P_0 = P_4(1+j\tau)^{-4} = \$135,183(1.08)^{-4} = \$99,364$$

If the $99,364 were invested now at 8%/year, then each year a $30,000 payment could be made with a balance of $0 at the end of year 4.

Option B:

The cash outlay in present value is $3 \times \$39,000 = \$117,000$, which appears to be the less expensive option. However, when considering the future value and converting that into the present:

$$P_3 = \$39,000\left\{\frac{(1.08)^3 - 1}{0.08}\right\} = \$126,610$$

The present value of P_3 is:

$$P_0 = P_3(1+j\tau)^3 = \$126,610(1.08)^{-3} = \$100,507$$

Option B (3 years) is less based on total actual dollar payments. However, when accounting for the future value of money, less cash would have to be set aside in order to make the payments with Option A. Thus, Option A (4 years) is more cost effective.

[6] This is an example of a Net Present Value (NPV) analysis.

8.3.3.3 Example 4-8. Alternative Energy Payback - Present Value

A small municipality is considering installing a 1 MW_e wind turbine that will cost \$6.5 million to install, and then generate a net annuity of \$400,000 per year for twenty-five years, with an estimated salvage value of \$1 million.[7] The inflation rate is estimated to be 5% per year.
(a) Use a simple payback method to assess the economic viability.
(b) Calculate the present value to assess the economic viability.

Part (a) Using a simple payback method, the net value of the project is the income plus salvage value less the capital cost.

$$\begin{array}{l} \$400,000/\text{year} \times 25 \text{ years (annual income)} \\ +\$1,000,000 \text{ (salvage)} \\ -\$6,500,000 \text{ (capital)} \\ \hline \$4,500,000 \text{ (net value of project)} \end{array}$$

The income from annuities and salvage exceed the initial cost by 69%. The presumes, however, that the value of money is the same twenty-five years from now.

Part (b) If the value of money changes in time, then the income gathered over time needs to be discounted. In this case, the easiest method would be to convert the income into present value since the project cost is already at present value.

Future Value of Income (subscript i)

$$P_{i_{25}} = S_i \left\{ \frac{(1+j\tau)^n - 1}{j\tau} \right\} = S_i \left\{ \frac{1.05^{25} - 1}{0.05} \right\}$$

Present Value of Income

$$P_{i_0} = P_{i_{25}} (1+j\tau)^{-n} = P_{i_{25}} (1.05)^{-25} = \$5.64 \text{ million}$$

Inflation has substantially reduced the present value of income.

Present Value of Salvage (subscript s)

$$P_{s_0} = P_{s_{25}} (1+j\tau)^{-n} = \$295,303$$

\$1 million 25 years from now is only worth \$295,000 today.

Present Value of Capital (subscript c)

$$P_{c_0} = \$6.5 \text{ million}$$

Present Value of Project

$$P_0 = P_{i_0} + P_{s_0} - P_{c_0} = -\$565,000$$

Based on the present value analysis, the project is not economically viable at 5% inflation. Economic viability will require a lower inflation rate, higher income, or tax incentives or subsidies.

[7] example adapted from Vanek and Albright [4, example 3-3]

8.4 Discounting Tool - Levelized Value

Levelized value is a technique used to convert a series of non-uniform payments into a uniform series payment per time per energy unit. In this way, the cost of a project relative to the energy produced can be examined through an equal payment or dividend per some time period; usually a year. This method is useful when comparing two different energy technologies; especially when comparing a fossil fuel technology with a renewable energy technology.

fossil fuel	**renewable energy**
relatively low capital	high capital cost
significant recurring costs in fuel; may escalate at different rate than inflation	low recurring costs

8.4.1 Levelization of Values

The levelization process is straightforward. First convert each value (future, present, series) into a present value and sum to find the total present value. Then convert the total equivalent present value into an equivalent uniform series of values over the anticipated life of the project; usually this is on an annual basis, but the time interval can be anything you choose. Finally, divide the equivalent cost for that time interval by the energy produced/consumed during that time interval. These steps will use only the two basic equations derived, compounding and uniform series, but will require some manipulation of the equations.

$$\textbf{Compounding:}\ P_n = P_0\,(1 + j_\tau)^n$$

$$\textbf{Uniform Series:}\ P_n = S\left\{\frac{(1 + j_\tau)^n - 1}{j_\tau}\right\}$$

8.4.2 Examples using Levelized Values

8.4.2.1 Example 4-9. Levelizing a Non-Uniform Series

A series of payments will be made annually for ten years. The initial payment is \$20,000 and each year the payment increases by \$5,000. The interest rate is 10%, compounded annually. Determine the equivalent uniform series value from the non-uniform series value.[8]

First, convert each payment into a present value based on a 10% escalation.

$$P_n = P_0\left(1 + j\tau\right)^n \longrightarrow P_0 = P_n\left(1 + j\tau\right)^{-n}$$

$$\left.\begin{array}{r} j = 0.10 \\ \tau \equiv \text{year} \\ n = 10 \end{array}\right\} \begin{array}{l} P_{0_1} = \$20\cdot 10^3(1.1)^{-1} = 18.2\cdot 10^3 \\ P_{0_2} = \$25\cdot 10^3(1.1)^{-2} = 20.7\cdot 10^3 \\ P_{0_3} = \$30\cdot 10^3(1.1)^{-3} = 22.5\cdot 10^3 \\ \vdots \\ P_{0_{10}} = \$65\cdot 10^3(1.1)^{-10} = 25.1\cdot 10^3 \\ \hline \Sigma P_0 = P_{0_T} = \quad \cdots \quad = \$237\cdot 10^3 \end{array}$$

Next, convert this total present value into a uniform series of annual payments.

$$\left.\begin{array}{c} P_{n_T} = S\left\{\dfrac{(1+j\tau)^n - 1}{j\tau}\right\} \\ \text{and} \\ P_{n_T} = P_{0_T}(1+j\tau)^n \end{array}\right\} \Rightarrow S = P_{0_T}\left\{\dfrac{(j\tau)(1+j\tau)^n}{(1+j\tau)^n - 1}\right\}$$

$$S = \$237\cdot 10^3\left\{\frac{(0.1)(1.1)^{10}}{(1.1)^{10} - 1}\right\} = \$38.6\cdot 10^3 \text{ per year}$$

Thus, the equivalent uniform series value is \$38,600 per year.

[8] example from Black & Veatch [5, p12]

8.4.2.2 Example 4-10. Levelizing Cost of Electricity

An electric power plant that produces 2 billion kWh_e per year has a capital cost of \$500 million and anticipated lifetime of 20 years. The salvage value is estimated to cover the cost of dismantling the plant. The capital cost of the plant is repaid at 7% interest, compounded annually. The total annual operational cost of the plant is \$25 million, and the annual return to investors is estimated at 10% of the operating cost plus the capital repayment cost. Determine the levelized cost of electricity for this plant, in $\$/kWh_e$.[9]

The levelization will be done on an annual time increment. First, annualize the capital cost by taking the initial loan plus the interest and converting it into a uniform series of annual costs. Then add the other annualized cost; operational costs and annual dividends to investors. The levelized cost is the total annualized (uniform series) cost divided by the annual energy output.

Total Capital Cost (subscript c)

$$P_{c_{20}} = P_{c_0}\left(1 + j_T\right)^n = P_{c_0}\left(1.07\right)^{20} = \$1,935 \cdot 10^6$$

Uniform Series Equivalent of Total Capital Cost

$$P_{c_{20}} = S_c \left\{ \frac{\left(1 + j_T\right)^n - 1}{j_T} \right\} = S_c\,(40.996)$$

$$S_c = \$47.2 \cdot 10^6$$

Annual Operating Cost (subsript o)

$$S_o = \$25 \cdot 10^6$$

Annual Return to Investors (subsript r)

$$S_r = 0.10\,(S_c + S_o) = \$7.2 \cdot 10^6$$

Total Annualized Cost

$$S = (\$47.2 + \$25 + \$7.2) \cdot 10^6 = \$79.4 \cdot 10^6 \text{ per year}$$

$$\text{Levelized Cost} = \frac{\$79.4 \cdot 10^6/\text{yr}}{2000 \cdot 10^6\,\text{kWh}_e/\text{yr}} = \$0.0397/\text{kWh}_e$$

In 2007, the average electrical energy price for all customers (residential, commercial, industrial and transportation) was $\$0.089/kWh_e$.[10] This price includes taxes and transmission costs. Based on the levelized cost of $\$0.0397/kWh_e$, this plant appears to be competitive in many U.S. markets.

[9] example adapted from Vanek and Albright [4, example 3-5]

[10] based on data from Energy Information Agency [6, p261]

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